AP Calculus AB FRQ Practice · Unit 1

A Piecewise Function and Continuity · numberbender.com

Name: _____ Date: _____ Score: / 9

What this question practices

- One-sided limits by algebra
- Choosing constants to make a limit exist
- The definition of continuity
- Limits at infinity and horizontal asymptotes

No calculator is allowed for this question. Show all your work, and justify your answers. Suggested time: 15 minutes.

The function f is defined by

$$f(x) = \frac{x^2 - 9}{x - 3} \text{ for } x < 3, \qquad f(3) = k, \qquad f(x) = ax + 1 \text{ for } x > 3$$

where a and k are constants.

(a) Find the left-hand limit below. Show the work that leads to your answer.

$$\lim_{x \to 3^-} f(x)$$

(b) Find the value of a for which the limit of f(x) as x approaches 3 exists. Justify your answer.

(c) Using the value of a from part (b), find the value of k that makes f continuous at x = 3. Use the definition of continuity to justify your answer.

AP® is a trademark registered by the College Board, which is not affiliated with, and does not endorse, this product.

Scan to watch

(d) Another function g is given below. Find the limit of g(x) as x approaches infinity, and state the equation of the horizontal asymptote of the graph of g.

$$g(x) = \frac{3x^2 - 5x}{6x^2 + 1}$$

AP® is a trademark registered by the College Board, which is not affiliated with, and does not endorse, this product.

This question is worth 9 points, scored the way AP Calculus readers score free-response questions. Compare your work with the model solution, then award yourself points using the scoring notes.

(a) Find the left-hand limit below. Show the work that leads to your answer.

$$\lim_{x \to 3^-} f(x)$$

$$\lim_{x \to 3^-} \frac{(x-3)(x+3)}{x-3} = \lim_{x \to 3^-} (x+3) = 6$$

→ Factor the numerator and cancel x − 3, which is allowed because x ≠ 3 inside the limit.

Scoring. 2 points: 1 point for factoring and cancelling (or an equivalent method), 1 point for the answer 6.

(b) Find the value of a for which the limit of f(x) as x approaches 3 exists. Justify your answer.

$$\lim_{x \to 3^+} (ax+1) = 3a+1$$

$$3a+1 = 6 \Rightarrow a = \frac{5}{3}$$

→ The two-sided limit exists only if the left-hand and right-hand limits are equal.

Scoring. 2 points: 1 point for setting the right-hand limit equal to the left-hand limit, 1 point for a = 5/3.

(c) Using the value of a from part (b), find the value of k that makes f continuous at x = 3. Use the definition of continuity to justify your answer.

→ f is continuous at x = 3 if f(3) is defined, the limit as x approaches 3 exists, and the two are equal.

$$k = f(3) = \lim_{x \to 3} f(x) = 6$$

→ With a = 5/3 the limit exists and equals 6, so f is continuous at 3 exactly when k = 6.

Scoring. 3 points: 1 point for the answer k = 6, 2 points for a justification that addresses all three conditions of continuity.

(d) Another function g is given below. Find the limit of g(x) as x approaches infinity, and state the equation of the horizontal asymptote of the graph of g.

$$g(x) = \frac{3x^2 - 5x}{6x^2 + 1}$$

$$\lim_{x \to \infty} \frac{3x^2 - 5x}{6x^2 + 1} = \lim_{x \to \infty} \frac{3 - \frac{5}{x}}{6 + \frac{1}{x^2}} = \frac{3}{6} = \frac{1}{2}$$

$$y = \frac{1}{2}$$

→ Divide the numerator and the denominator by x^2, the highest power in the denominator.

Scoring. 2 points: 1 point for the limit 1/2 with supporting work, 1 point for the asymptote y = 1/2.

AP® is a trademark registered by the College Board, which is not affiliated with, and does not endorse, this product.
Scan to watch