



AP Calculus AB FRQ Practice · Unit 3

Implicit Differentiation and Inverses · numberbender.com

Name: _____

Date: _____

Score: / 9

What this question practices

- Implicit differentiation
- Tangent lines to a curve
- Horizontal tangents
- The derivative of an inverse function

No calculator is allowed for this question. Show all your work, and justify your answers. Suggested time: 15 minutes.

Consider the curve given by the equation

$$x^2 + xy + y^2 = 7.$$

(a) Show that the derivative is given by the expression below.

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

(b) Verify that the point (1, 2) is on the curve, and write an equation for the line tangent to the curve at (1, 2).

(c) Find the coordinates of every point on the curve at which the line tangent to the curve is horizontal.



(d) The function f is defined below, and g is the inverse function of f . Find $g'(4)$.

$$f(x) = x^3 + 2x + 1$$





This question is worth 9 points, scored the way AP Calculus readers score free-response questions. Compare your work with the model solution, then award yourself points using the scoring notes.

(a) Show that the derivative is given by the expression below.

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

→ Differentiate both sides with respect to x , using the product rule on xy and the chain rule on y^2 , then solve for dy/dx .

$$2x + \left(y + x \frac{dy}{dx}\right) + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x + 2y) = -2x - y \Rightarrow \frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

Scoring. 2 points: 1 point for correct implicit differentiation, 1 point for solving for dy/dx .

(b) Verify that the point $(1, 2)$ is on the curve, and write an equation for the line tangent to the curve at $(1, 2)$.

→ Substitute the point into the equation, then substitute it into dy/dx to get the slope.

$$1^2 + 1(2) + 2^2 = 7$$

$$\frac{dy}{dx} \text{ at } (1, 2): \frac{-2 - 2}{1 + 4} = -\frac{4}{5}$$

$$y - 2 = -\frac{4}{5}(x - 1)$$

Scoring. 2 points: 1 point for the slope $-4/5$, 1 point for the tangent line equation.

(c) Find the coordinates of every point on the curve at which the line tangent to the curve is horizontal.

→ A horizontal tangent needs the numerator of dy/dx to be 0 (with a nonzero denominator). Substitute $y = -2x$ into the curve's equation.

$$-2x - y = 0 \Rightarrow y = -2x$$

$$x^2 + x(-2x) + (-2x)^2 = 3x^2 = 7 \Rightarrow x = \pm \sqrt{\frac{7}{3}} = \pm \frac{\sqrt{21}}{3}$$

$$\left(\frac{\sqrt{21}}{3}, -\frac{2\sqrt{21}}{3}\right) \text{ and } \left(-\frac{\sqrt{21}}{3}, \frac{2\sqrt{21}}{3}\right)$$

→ At both points the denominator $x + 2y$ is $-\sqrt{21}$ or $\sqrt{21}$, not 0, so both tangents are horizontal.

Scoring. 3 points: 1 point for setting the numerator equal to 0, 1 point for solving with the curve's equation, 1 point for both points.



(d) The function f is defined below, and g is the inverse function of f . Find $g'(4)$.

$$f(x) = x^3 + 2x + 1$$

$$f(1) = 4 \Rightarrow g(4) = 1$$

$$g'(4) = \frac{1}{f'(g(4))} = \frac{1}{f'(1)} = \frac{1}{3(1)^2 + 2} = \frac{1}{5}$$

→ To use the inverse rule, first find the x -value where $f(x) = 4$.

Scoring. 2 points: 1 point for $g(4) = 1$, 1 point for $g'(4) = 1/5$.

