AP Calculus AB FRQ Practice · Unit 4

Particle Motion and a Growing Oil Spill · numberbender.com

Name: _____ Date: _____ Score: / 9

What this question practices

- Velocity and acceleration of a particle
- When a particle is at rest
- When a particle is speeding up
- Related rates

No calculator is allowed for this question. Show all your work, and justify your answers. Suggested time: 15 minutes.

A particle moves along the x-axis. Its velocity at time t seconds, for $0 \le t \le 5$, is given by

$$v(t) = t^2 - 6t + 8.$$

(a) Find all times t in the interval $0 < t < 5$ at which the particle is at rest.

(b) Find the acceleration of the particle at time $t = 3$. What does this value tell you about the velocity at $t = 3$?

(c) On what intervals of time is the speed of the particle increasing? Justify your answer.

(d) In a separate situation, the radius of a circular oil spill increases at a constant rate of 2 meters per minute. How fast is the area of the spill increasing at the instant the radius is 10 meters? Indicate units.

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This question is worth 9 points, scored the way AP Calculus readers score free-response questions. Compare your work with the model solution, then award yourself points using the scoring notes.

(a) Find all times t in the interval 0 < t < 5 at which the particle is at rest.

$$v(t) = (t-2)(t-4) = 0 \Rightarrow t = 2,\ t = 4$$

→ The particle is at rest when its velocity is 0.

Scoring. 1 point for t = 2 and t = 4.

(b) Find the acceleration of the particle at time t = 3. What does this value tell you about the velocity at t = 3?

$$a(t) = v'(t) = 2t - 6, \qquad a(3) = 0$$

→ At t = 3 the velocity is at its minimum value, v(3) = −1: the acceleration changes from negative to positive there.

Scoring. 2 points: 1 point for a(3) = 0, 1 point for interpreting it (the velocity has a minimum, or is momentarily not changing, at t = 3).

(c) On what intervals of time is the speed of the particle increasing? Justify your answer.

→ The speed increases when the velocity and the acceleration have the same sign.

→ On (2, 3), v < 0 and a < 0. On (4, 5), v > 0 and a > 0. On (0, 2) and (3, 4) the signs differ, so the particle is slowing down.

$$2 < t < 3 \ \text{ and } \ 4 < t < 5$$

Scoring. 3 points: 1 point for the sign analysis of v, 1 point for the sign analysis of a, 1 point for both intervals with the same-sign justification.

(d) In a separate situation, the radius of a circular oil spill increases at a constant rate of 2 meters per minute. How fast is the area of the spill increasing at the instant the radius is 10 meters? Indicate units.

→ Differentiate the area formula with respect to time, then substitute r = 10 and dr/dt = 2.

$$A = \pi r^2 \ \Rightarrow \ \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(10)(2) = 40\pi \text{ m}^2/\text{min} \approx 125.66 \text{ m}^2/\text{min}$$

Scoring. 3 points: 1 point for differentiating with respect to t, 1 point for the answer 40π, 1 point for units of square meters per minute.

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