



AP Calculus AB FRQ Practice · Unit 5

Analyzing a Function From Its Derivative · numberbender.com

Name: _____

Date: _____

Score: / 9

What this question practices

- Relative extrema with the First Derivative Test
- Intervals of increase and decrease
- Concavity and points of inflection
- Absolute extrema on a closed interval

No calculator is allowed for this question. Show all your work, and justify your answers. Suggested time: 15 minutes.

A function f is differentiable on the closed interval $[0, 6]$, and its derivative is given by

$$f'(x) = (x - 1)^2(x - 4).$$

(a) Find the x -coordinate of each relative extremum of f on the open interval $0 < x < 6$, and classify each as a relative maximum or minimum. Justify your answer.

(b) On what intervals is f decreasing? Justify your answer.

(c) Find the intervals on which the graph of f is concave up, and find the x -coordinates of all points of inflection.

(d) Given that $f(0) = 5$, find the absolute minimum value of f on $[0, 6]$. Justify your answer.





This question is worth 9 points, scored the way AP Calculus readers score free-response questions. Compare your work with the model solution, then award yourself points using the scoring notes.

(a) Find the x-coordinate of each relative extremum of f on the open interval $0 < x < 6$, and classify each as a relative maximum or minimum. Justify your answer.

$$f'(x) = 0 \Rightarrow x = 1, x = 4$$

relative minimum at $x = 4$

→ At $x = 4$, f' changes from negative to positive, so f has a relative minimum there.

→ At $x = 1$, f' is negative on both sides (the factor $(x - 1)^2$ does not change sign), so there is no extremum at $x = 1$.

Scoring. 2 points: 1 point for the relative minimum at $x = 4$ with justification, 1 point for explaining why $x = 1$ is not an extremum.

(b) On what intervals is f decreasing? Justify your answer.

$$f'(x) \leq 0 \text{ on } [0, 4] \Rightarrow f \text{ is decreasing on } [0, 4]$$

→ For $0 \leq x < 4$, $x - 4$ is negative and $(x - 1)^2 \geq 0$, so $f'(x) \leq 0$, with $f'(x) = 0$ only at the single point $x = 1$.

Scoring. 2 points: 1 point for the interval $(0, 4)$ or $[0, 4]$, 1 point for the sign of f' as justification.

(c) Find the intervals on which the graph of f is concave up, and find the x-coordinates of all points of inflection.

$$f''(x) = 2(x - 1)(x - 4) + (x - 1)^2 = 3(x - 1)(x - 3)$$

$$f'' > 0 \text{ on } (0, 1) \text{ and } (3, 6); \quad \text{inflection points at } x = 1, x = 3$$

→ The graph is concave up where $f'' > 0$, and has a point of inflection wherever f'' changes sign.

Scoring. 3 points: 1 point for f'' , 1 point for the intervals of concave up, 1 point for both points of inflection.

(d) Given that $f(0) = 5$, find the absolute minimum value of f on $[0, 6]$. Justify your answer.

→ The absolute minimum occurs at a critical point or an endpoint. Compare $f(0) = 5$, $f(4)$ and $f(6)$.

$$f(4) = f(0) + \int_0^4 (x - 1)^2(x - 4) dx = 5 + (-8) = -3$$

$$f(6) = f(0) + \int_0^6 (x - 1)^2(x - 4) dx = 5 + 30 = 35$$

→ The absolute minimum value of f on $[0, 6]$ is -3 , at $x = 4$.

Scoring. 2 points: 1 point for using the Fundamental Theorem to find $f(4)$, 1 point for the answer -3 with a candidates test or the sign of f' as justification.

