



AP Calculus AB FRQ Practice · Unit 6

Filling a Water Tank · numberbender.com

Name: _____

Date: _____

Score: / 9

What this question practices

- Trapezoidal sums from a table
- Interpreting a definite integral in context
- The Fundamental Theorem of Calculus with the chain rule
- Definite integrals by u-substitution

No calculator is allowed for this question. Show all your work, and justify your answers. Suggested time: 15 minutes.

Water flows into a tank at a rate of $R(t)$ gallons per hour, where R is differentiable and t is measured in hours. Selected values of $R(t)$ are shown in the table.

t (hours)	0	2	5	7	10
$R(t)$ (gallons per hour)	40	52	47	38	30

(a) Use a trapezoidal sum with the four subintervals indicated by the table to approximate the integral below. Explain the meaning of this integral in context, with units.

$$\int_0^{10} R(t) dt$$

(b) Must there be a time t , with $2 < t < 5$, at which $R'(t) = -5/3$? Justify your answer.

(c) Find the derivatives of the two functions below.

$$F(x) = \int_1^x \sqrt{t^3 + 1} dt \quad (\text{find } F'(2)), \quad G(x) = \int_0^{x^2} \cos t dt \quad (\text{find } G'(x))$$



(d) Evaluate the definite integral below. Show your work.

$$\int_0^2 x(x^2 + 1)^3 dx$$





This question is worth 9 points, scored the way AP Calculus readers score free-response questions. Compare your work with the model solution, then award yourself points using the scoring notes.

(a) Use a trapezoidal sum with the four subintervals indicated by the table to approximate the integral below. Explain the meaning of this integral in context, with units.

$$\int_0^{10} R(t) dt$$

→ The subintervals have different widths (2, 3, 2 and 3 hours), so each trapezoid uses its own width.

$$2 \cdot \frac{40 + 52}{2} + 3 \cdot \frac{52 + 47}{2} + 2 \cdot \frac{47 + 38}{2} + 3 \cdot \frac{38 + 30}{2} = 427.5$$

→ The integral is the total amount of water, in gallons, that flows into the tank from $t = 0$ to $t = 10$ hours: about 427.5 gallons.

Scoring. 3 points: 1 point for a trapezoidal sum setup, 1 point for 427.5, 1 point for the meaning with units.

(b) Must there be a time t , with $2 < t < 5$, at which $R'(t) = -5/3$? Justify your answer.

→ Yes. R is differentiable, so it is continuous on $[2, 5]$ and differentiable on $(2, 5)$.

$$\frac{R(5) - R(2)}{5 - 2} = \frac{47 - 52}{3} = -\frac{5}{3}$$

→ By the Mean Value Theorem, there is a time c in $(2, 5)$ with $R'(c)$ equal to the average rate of change, $-5/3$.

Scoring. 2 points: 1 point for the average rate of change, 1 point for invoking the Mean Value Theorem with its conditions.

(c) Find the derivatives of the two functions below.

$$F(x) = \int_1^x \sqrt{t^3 + 1} dt \quad (\text{find } F'(2)), \quad G(x) = \int_0^{x^2} \cos t dt \quad (\text{find } G'(x))$$

→ By the Fundamental Theorem of Calculus, the derivative of an integral with a variable upper limit is the integrand evaluated at that limit.

$$F'(x) = \sqrt{x^3 + 1} \Rightarrow F'(2) = \sqrt{9} = 3$$

$$G'(x) = \cos(x^2) \cdot 2x$$

→ For G , the upper limit is x^2 , so the chain rule multiplies by its derivative, $2x$.

Scoring. 2 points: 1 point for $F'(2) = 3$, 1 point for $G'(x) = 2x \cos(x^2)$.



(d) Evaluate the definite integral below. Show your work.

$$\int_0^2 x(x^2 + 1)^3 dx$$

$$u = x^2 + 1, \quad du = 2x dx: \quad \int_1^5 \frac{1}{2} u^3 du = \frac{5^4}{8} - \frac{1^4}{8} = \frac{625 - 1}{8} = 78$$

→ Substitute $u = x^2 + 1$ and change the limits: $x = 0$ gives $u = 1$, and $x = 2$ gives $u = 5$.

Scoring. 2 points: 1 point for the substitution with correct limits or antiderivative, 1 point for the answer 78.

