



AP Calculus AB FRQ Practice · Unit 8

Area, Volume and Average Value · numberbender.com

Name: _____

Date: _____

Score: / 9

What this question practices

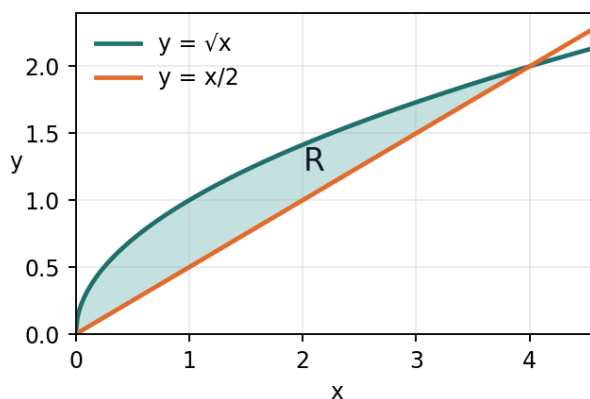
- Area between two curves
- Volume of revolution by washers
- Volume with known cross sections
- Average value of a function

No calculator is allowed for this question. Show all your work, and justify your answers. Suggested time: 15 minutes.

Let R be the region in the first quadrant bounded by the graphs of

$$y = \sqrt{x} \quad \text{and} \quad y = \frac{x}{2},$$

as shown in the figure. The graphs intersect at $x = 0$ and $x = 4$.



(a) Find the area of R .

(b) Find the volume of the solid generated when R is revolved about the x -axis.



(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

(d) Find the average value of the function below on the interval $[0, 4]$.

$$f(x) = \sqrt{x}$$





This question is worth 9 points, scored the way AP Calculus readers score free-response questions. Compare your work with the model solution, then award yourself points using the scoring notes.

(a) Find the area of R.

$$\int_0^4 \left(\sqrt{x} - \frac{x}{2} \right) dx = \frac{2}{3}(4)^{3/2} - \frac{4^2}{4} = \frac{16}{3} - 4 = \frac{4}{3}$$

→ On $[0, 4]$ the graph of $\sqrt{x}$ is above the line $y = x/2$.

Scoring. 2 points: 1 point for the integral, 1 point for the answer $4/3$.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

$$\pi \int_0^4 \left[(\sqrt{x})^2 - \left(\frac{x}{2} \right)^2 \right] dx = \pi \int_0^4 \left(x - \frac{x^2}{4} \right) dx = \pi \left(8 - \frac{16}{3} \right) = \frac{8\pi}{3}$$

→ Use washers: the outer radius is $\sqrt{x}$ and the inner radius is $x/2$.

Scoring. 3 points: 1 point for the limits and constant π , 1 point for the integrand, 1 point for the answer $8\pi/3$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

$$\int_0^4 \left(\sqrt{x} - \frac{x}{2} \right)^2 dx = \int_0^4 \left(x - x^{3/2} + \frac{x^2}{4} \right) dx = 8 - \frac{64}{5} + \frac{16}{3} = \frac{8}{15}$$

→ Each square has side length $\sqrt{x} - x/2$, so its area is the square of that length.

Scoring. 2 points: 1 point for the integral of the square of the side length, 1 point for the answer $8/15$.

(d) Find the average value of the function below on the interval $[0, 4]$.

$$f(x) = \sqrt{x}$$

$$\frac{1}{4-0} \int_0^4 \sqrt{x} dx = \frac{1}{4} \cdot \frac{16}{3} = \frac{4}{3}$$

→ Divide the definite integral by the length of the interval.

Scoring. 2 points: 1 point for the average value formula, 1 point for the answer $4/3$.

