



# MATH110: Confidence Intervals

Introduction to Statistics · C-ID MATH110 · numberbender.com

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Score: / 30

## Learning Objectives

- Identify the critical value  $z^*$  for a confidence level
- Compute the standard error and margin of error
- Construct a confidence interval for a population mean

*Simplify each expression completely. Show all steps and circle your final answer.*

## Confidence interval for a mean ( $\sigma$ known)

1. A sample of  $n = 16$  has a sample mean of 131. The population standard deviation is  $\sigma = 10$ . Construct the 99% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 131, \quad \sigma = 10, \quad n = 16, \quad 99\% \text{ CI}$$

Answer: \_\_\_\_\_

2. A sample of  $n = 100$  has a sample mean of 84. The population standard deviation is  $\sigma = 20$ . Construct the 95% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 84, \quad \sigma = 20, \quad n = 100, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

3. A sample of  $n = 25$  has a sample mean of 120. The population standard deviation is  $\sigma = 15$ . Construct the 99% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 120, \quad \sigma = 15, \quad n = 25, \quad 99\% \text{ CI}$$

Answer: \_\_\_\_\_

4. A sample of  $n = 25$  has a sample mean of 81. The population standard deviation is  $\sigma = 12$ . Construct the 90% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 81, \quad \sigma = 12, \quad n = 25, \quad 90\% \text{ CI}$$

Answer: \_\_\_\_\_

5. A sample of  $n = 36$  has a sample mean of 67. The population standard deviation is  $\sigma = 20$ . Construct the 95% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 67, \quad \sigma = 20, \quad n = 36, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

6. A sample of  $n = 16$  has a sample mean of 55. The population standard deviation is  $\sigma = 15$ . Construct the 90% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 55, \quad \sigma = 15, \quad n = 16, \quad 90\% \text{ CI}$$

Answer: \_\_\_\_\_

---

7. A sample of  $n = 25$  has a sample mean of 89. The population standard deviation is  $\sigma = 15$ . Construct the 95% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 89, \quad \sigma = 15, \quad n = 25, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

---

8. A sample of  $n = 36$  has a sample mean of 106. The population standard deviation is  $\sigma = 20$ . Construct the 95% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 106, \quad \sigma = 20, \quad n = 36, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

---

9. A sample of  $n = 100$  has a sample mean of 59. The population standard deviation is  $\sigma = 15$ . Construct the 90% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 59, \quad \sigma = 15, \quad n = 100, \quad 90\% \text{ CI}$$

Answer: \_\_\_\_\_

---

10. A sample of  $n = 16$  has a sample mean of 137. The population standard deviation is  $\sigma = 10$ . Construct the 90% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 137, \quad \sigma = 10, \quad n = 16, \quad 90\% \text{ CI}$$

Answer: \_\_\_\_\_

---

### Confidence Interval for a Population Proportion

---

11. In a random sample of 246 people, 39 had a particular trait. Construct a 99% confidence interval for the true proportion.

$$x = 39, \quad n = 246, \quad 99\% \text{ CI}$$

Answer: \_\_\_\_\_

---

12. In a random sample of 304 people, 49 had a particular trait. Construct a 90% confidence interval for the true proportion.

$$x = 49, \quad n = 304, \quad 90\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**13.** In a random sample of 276 people, 47 had a particular trait. Construct a 95% confidence interval for the true proportion.

$$x = 47, \quad n = 276, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**14.** In a random sample of 127 people, 98 had a particular trait. Construct a 99% confidence interval for the true proportion.

$$x = 98, \quad n = 127, \quad 99\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**15.** In a random sample of 200 people, 53 had a particular trait. Construct a 90% confidence interval for the true proportion.

$$x = 53, \quad n = 200, \quad 90\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**16.** In a random sample of 200 people, 178 had a particular trait. Construct a 90% confidence interval for the true proportion.

$$x = 178, \quad n = 200, \quad 90\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**17.** In a random sample of 193 people, 156 had a particular trait. Construct a 99% confidence interval for the true proportion.

$$x = 156, \quad n = 193, \quad 99\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**18.** In a random sample of 163 people, 151 had a particular trait. Construct a 99% confidence interval for the true proportion.

$$x = 151, \quad n = 163, \quad 99\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**19.** In a random sample of 159 people, 49 had a particular trait. Construct a 95% confidence interval for the true proportion.

$$x = 49, \quad n = 159, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**20.** In a random sample of 250 people, 60 had a particular trait. Construct a 99% confidence interval for the true proportion.

$$x = 60, \quad n = 250, \quad 99\% \text{ CI}$$

Answer: \_\_\_\_\_

---

### Confidence Interval for a Mean (t-distribution)

---

**21.** A sample of  $n = 25$  gives  $\bar{x} = 86$  and  $s = 19$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 86, \quad s = 19, \quad n = 25, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**22.** A sample of  $n = 20$  gives  $\bar{x} = 82$  and  $s = 5$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 82, \quad s = 5, \quad n = 20, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**23.** A sample of  $n = 15$  gives  $\bar{x} = 91$  and  $s = 11$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 91, \quad s = 11, \quad n = 15, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**24.** A sample of  $n = 20$  gives  $\bar{x} = 99$  and  $s = 9$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 99, \quad s = 9, \quad n = 20, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**25.** A sample of  $n = 5$  gives  $\bar{x} = 114$  and  $s = 20$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 114, \quad s = 20, \quad n = 5, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**26.** A sample of  $n = 30$  gives  $\bar{x} = 74$  and  $s = 5$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 74, \quad s = 5, \quad n = 30, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**27.** A sample of  $n = 5$  gives  $\bar{x} = 119$  and  $s = 8$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 119, \quad s = 8, \quad n = 5, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**28.** A sample of  $n = 15$  gives  $\bar{x} = 86$  and  $s = 12$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 86, \quad s = 12, \quad n = 15, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**29.** A sample of  $n = 30$  gives  $\bar{x} = 107$  and  $s = 17$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 107, \quad s = 17, \quad n = 30, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

---

**30.** A sample of  $n = 25$  gives  $\bar{x} = 80$  and  $s = 19$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 80, \quad s = 19, \quad n = 25, \quad 95\% \text{ CI}$$

Answer: \_\_\_\_\_

---



# MATH110: Confidence Intervals

Introduction to Statistics · C-ID MATH110 · numberbender.com

ANSWER KEY & SOLUTIONS

*Topics: Confidence Interval for a Mean (t-distribution), Confidence Interval for a Population Proportion, Confidence interval for a mean ( $\sigma$  known). All answers verified by independent computation.*

## Solutions

---

## Confidence interval for a mean ( $\sigma$ known)

---

1. A sample of  $n = 16$  has a sample mean of 131. The population standard deviation is  $\sigma = 10$ . Construct the 99% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 131, \quad \sigma = 10, \quad n = 16, \quad 99\% \text{ CI}$$

- Critical value  $z^*$  for a 99% interval = 2.576.
- Standard error =  $\sigma / \sqrt{n} = 10/4 = 2.5$ .
- Margin of error  $E = z^* \times SE = 2.576 \times 2.5 = 6.44$ .
- CI = sample mean  $\pm E = 131 \pm 6.44 = (124.56, 137.44)$ .

**Answer:**  $E = z^* \frac{\sigma}{\sqrt{n}} = 2.576 \cdot \frac{10}{4} = 6.44 \Rightarrow (124.56, 137.44)$

---

2. A sample of  $n = 100$  has a sample mean of 84. The population standard deviation is  $\sigma = 20$ . Construct the 95% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 84, \quad \sigma = 20, \quad n = 100, \quad 95\% \text{ CI}$$

- Critical value  $z^*$  for a 95% interval = 1.96.
- Standard error =  $\sigma / \sqrt{n} = 20/10 = 2.0$ .
- Margin of error  $E = z^* \times SE = 1.96 \times 2.0 = 3.92$ .
- CI = sample mean  $\pm E = 84 \pm 3.92 = (80.08, 87.92)$ .

**Answer:**  $E = z^* \frac{\sigma}{\sqrt{n}} = 1.96 \cdot \frac{20}{10} = 3.92 \Rightarrow (80.08, 87.92)$

---

3. A sample of  $n = 25$  has a sample mean of 120. The population standard deviation is  $\sigma = 15$ . Construct the 99% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 120, \quad \sigma = 15, \quad n = 25, \quad 99\% \text{ CI}$$

- Critical value  $z^*$  for a 99% interval = 2.576.
- Standard error =  $\sigma / \sqrt{n} = 15/5 = 3.0$ .
- Margin of error  $E = z^* \times SE = 2.576 \times 3.0 = 7.728$ .
- CI = sample mean  $\pm E = 120 \pm 7.728 = (112.272, 127.728)$ .

**Answer:**  $E = z^* \frac{\sigma}{\sqrt{n}} = 2.576 \cdot \frac{15}{5} = 7.728 \Rightarrow (112.272, 127.728)$

---

4. A sample of  $n = 25$  has a sample mean of 81. The population standard deviation is  $\sigma = 12$ . Construct the 90% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 81, \quad \sigma = 12, \quad n = 25, \quad 90\% \text{ CI}$$

- Critical value  $z^*$  for a 90% interval = 1.645.
- Standard error =  $\sigma / \sqrt{n} = 12/5 = 2.4$ .
- Margin of error  $E = z^* \times SE = 1.645 \times 2.4 = 3.948$ .
- CI = sample mean  $\pm E = 81 \pm 3.948 = (77.052, 84.948)$ .

**Answer:**  $E = z^* \frac{\sigma}{\sqrt{n}} = 1.645 \cdot \frac{12}{5} = 3.948 \Rightarrow (77.052, 84.948)$

---

5. A sample of  $n = 36$  has a sample mean of 67. The population standard deviation is  $\sigma = 20$ . Construct the 95% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 67, \quad \sigma = 20, \quad n = 36, \quad 95\% \text{ CI}$$

- Critical value  $z^*$  for a 95% interval = 1.96.
- Standard error =  $\sigma / \sqrt{n} = 20/6 = 3.3333$ .
- Margin of error  $E = z^* \times SE = 1.96 \times 3.3333 = 6.533$ .
- CI = sample mean  $\pm E = 67 \pm 6.533 = (60.467, 73.533)$ .

**Answer:**  $E = z^* \frac{\sigma}{\sqrt{n}} = 1.96 \cdot \frac{20}{6} = 6.533 \Rightarrow (60.467, 73.533)$

---

6. A sample of  $n = 16$  has a sample mean of 55. The population standard deviation is  $\sigma = 15$ . Construct the 90% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 55, \quad \sigma = 15, \quad n = 16, \quad 90\% \text{ CI}$$

- Critical value  $z^*$  for a 90% interval = 1.645.
- Standard error =  $\sigma / \sqrt{n} = 15/4 = 3.75$ .
- Margin of error  $E = z^* \times SE = 1.645 \times 3.75 = 6.169$ .
- CI = sample mean  $\pm E = 55 \pm 6.169 = (48.831, 61.169)$ .

**Answer:**  $E = z^* \frac{\sigma}{\sqrt{n}} = 1.645 \cdot \frac{15}{4} = 6.169 \Rightarrow (48.831, 61.169)$

---

7. A sample of  $n = 25$  has a sample mean of 89. The population standard deviation is  $\sigma = 15$ . Construct the 95% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 89, \quad \sigma = 15, \quad n = 25, \quad 95\% \text{ CI}$$

- Critical value  $z^*$  for a 95% interval = 1.96.
- Standard error =  $\sigma / \sqrt{n} = 15/5 = 3.0$ .
- Margin of error  $E = z^* \times SE = 1.96 \times 3.0 = 5.88$ .
- CI = sample mean  $\pm E = 89 \pm 5.88 = (83.12, 94.88)$ .

**Answer:**  $E = z^* \frac{\sigma}{\sqrt{n}} = 1.96 \cdot \frac{15}{5} = 5.88 \Rightarrow (83.12, 94.88)$

---

8. A sample of  $n = 36$  has a sample mean of 106. The population standard deviation is  $\sigma = 20$ . Construct the 95% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 106, \quad \sigma = 20, \quad n = 36, \quad 95\% \text{ CI}$$

- Critical value  $z^*$  for a 95% interval = 1.96.
- Standard error =  $\sigma / \sqrt{n} = 20/6 = 3.3333$ .
- Margin of error  $E = z^* \times SE = 1.96 \times 3.3333 = 6.533$ .
- CI = sample mean  $\pm E = 106 \pm 6.533 = (99.467, 112.533)$ .

**Answer:**  $E = z^* \frac{\sigma}{\sqrt{n}} = 1.96 \cdot \frac{20}{6} = 6.533 \Rightarrow (99.467, 112.533)$

---

9. A sample of  $n = 100$  has a sample mean of 59. The population standard deviation is  $\sigma = 15$ . Construct the 90% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 59, \quad \sigma = 15, \quad n = 100, \quad 90\% \text{ CI}$$

→ Critical value  $z^*$  for a 90% interval = 1.645.

→ Standard error =  $\sigma / \sqrt{n} = 15/10 = 1.5$ .

→ Margin of error  $E = z^* \times SE = 1.645 \times 1.5 = 2.468$ .

→ CI = sample mean  $\pm E = 59 \pm 2.468 = (56.532, 61.468)$ .

**Answer:**  $E = z^* \frac{\sigma}{\sqrt{n}} = 1.645 \cdot \frac{15}{10} = 2.468 \Rightarrow (56.532, 61.468)$

---

10. A sample of  $n = 16$  has a sample mean of 137. The population standard deviation is  $\sigma = 10$ . Construct the 90% confidence interval for the population mean  $\mu$ .

$$\bar{x} = 137, \quad \sigma = 10, \quad n = 16, \quad 90\% \text{ CI}$$

→ Critical value  $z^*$  for a 90% interval = 1.645.

→ Standard error =  $\sigma / \sqrt{n} = 10/4 = 2.5$ .

→ Margin of error  $E = z^* \times SE = 1.645 \times 2.5 = 4.112$ .

→ CI = sample mean  $\pm E = 137 \pm 4.112 = (132.887, 141.113)$ .

**Answer:**  $E = z^* \frac{\sigma}{\sqrt{n}} = 1.645 \cdot \frac{10}{4} = 4.112 \Rightarrow (132.887, 141.113)$

---

## Confidence Interval for a Population Proportion

---

11. In a random sample of 246 people, 39 had a particular trait. Construct a 99% confidence interval for the true proportion.

$$x = 39, \quad n = 246, \quad 99\% \text{ CI}$$

→ Sample proportion:  $p\text{-hat} = x/n = 39/246 = 0.1585$ .

→ Critical value:  $z^* = 2.576$  for 99% CI.

→ Standard error:  $SE = \sqrt{p\text{-hat}(1-p\text{-hat})/n} = 0.0233$ .

→ Margin of error:  $E = z^* * SE = 2.576 * 0.0233 = 0.06$ .

→ 99% CI:  $(0.1585 - 0.06, 0.1585 + 0.06) = (0.0985, 0.2185)$ .

**Answer:**  $\hat{p} = 0.1585, E = 0.06 \Rightarrow (0.0985, 0.2185)$

---

12. In a random sample of 304 people, 49 had a particular trait. Construct a 90% confidence interval for the true proportion.

$$x = 49, \quad n = 304, \quad 90\% \text{ CI}$$

→ Sample proportion:  $p\text{-hat} = x/n = 49/304 = 0.1612$ .

→ Critical value:  $z^* = 1.645$  for 90% CI.

→ Standard error:  $SE = \sqrt{p\text{-hat}(1-p\text{-hat})/n} = 0.0211$ .

→ Margin of error:  $E = z^* * SE = 1.645 * 0.0211 = 0.0347$ .

→ 90% CI:  $(0.1612 - 0.0347, 0.1612 + 0.0347) = (0.1265, 0.1959)$ .

**Answer:**  $\hat{p} = 0.1612, E = 0.0347 \Rightarrow (0.1265, 0.1959)$

---

13. In a random sample of 276 people, 47 had a particular trait. Construct a 95% confidence interval for the true proportion.

$$x = 47, \quad n = 276, \quad 95\% \text{ CI}$$

→ Sample proportion:  $p\text{-hat} = x/n = 47/276 = 0.1703$ .

→ Critical value:  $z^* = 1.96$  for 95% CI.

→ Standard error:  $SE = \sqrt{p\text{-hat}(1-p\text{-hat})/n} = 0.0226$ .

→ Margin of error:  $E = z^* * SE = 1.96 * 0.0226 = 0.0443$ .

→ 95% CI:  $(0.1703 - 0.0443, 0.1703 + 0.0443) = (0.126, 0.2146)$ .

**Answer:**  $\hat{p} = 0.1703, E = 0.0443 \Rightarrow (0.126, 0.2146)$

---

14. In a random sample of 127 people, 98 had a particular trait. Construct a 99% confidence interval for the true proportion.

$$x = 98, \quad n = 127, \quad 99\% \text{ CI}$$

→ Sample proportion:  $p\text{-hat} = x/n = 98/127 = 0.7717$ .

→ Critical value:  $z^* = 2.576$  for 99% CI.

→ Standard error:  $SE = \sqrt{p\text{-hat}(1-p\text{-hat})/n} = 0.0372$ .

→ Margin of error:  $E = z^* * SE = 2.576 * 0.0372 = 0.0959$ .

→ 99% CI:  $(0.7717 - 0.0959, 0.7717 + 0.0959) = (0.6758, 0.8676)$ .

**Answer:**  $\hat{p} = 0.7717, E = 0.0959 \Rightarrow (0.6758, 0.8676)$

---

15. In a random sample of 200 people, 53 had a particular trait. Construct a 90% confidence interval for the true proportion.

$$x = 53, \quad n = 200, \quad 90\% \text{ CI}$$

- Sample proportion:  $p\text{-hat} = x/n = 53/200 = 0.265$ .
- Critical value:  $z^* = 1.645$  for 90% CI.
- Standard error:  $SE = \sqrt{p\text{-hat}(1-p\text{-hat})/n} = 0.0312$ .
- Margin of error:  $E = z^* * SE = 1.645 * 0.0312 = 0.0513$ .
- 90% CI:  $(0.265 - 0.0513, 0.265 + 0.0513) = (0.2137, 0.3163)$ .

**Answer:**  $\hat{p} = 0.265, E = 0.0513 \Rightarrow (0.2137, 0.3163)$

---

16. In a random sample of 200 people, 178 had a particular trait. Construct a 90% confidence interval for the true proportion.

$$x = 178, \quad n = 200, \quad 90\% \text{ CI}$$

- Sample proportion:  $p\text{-hat} = x/n = 178/200 = 0.89$ .
- Critical value:  $z^* = 1.645$  for 90% CI.
- Standard error:  $SE = \sqrt{p\text{-hat}(1-p\text{-hat})/n} = 0.0221$ .
- Margin of error:  $E = z^* * SE = 1.645 * 0.0221 = 0.0364$ .
- 90% CI:  $(0.89 - 0.0364, 0.89 + 0.0364) = (0.8536, 0.9264)$ .

**Answer:**  $\hat{p} = 0.89, E = 0.0364 \Rightarrow (0.8536, 0.9264)$

---

17. In a random sample of 193 people, 156 had a particular trait. Construct a 99% confidence interval for the true proportion.

$$x = 156, \quad n = 193, \quad 99\% \text{ CI}$$

- Sample proportion:  $p\text{-hat} = x/n = 156/193 = 0.8083$ .
- Critical value:  $z^* = 2.576$  for 99% CI.
- Standard error:  $SE = \sqrt{p\text{-hat}(1-p\text{-hat})/n} = 0.0283$ .
- Margin of error:  $E = z^* * SE = 2.576 * 0.0283 = 0.073$ .
- 99% CI:  $(0.8083 - 0.073, 0.8083 + 0.073) = (0.7353, 0.8813)$ .

**Answer:**  $\hat{p} = 0.8083, E = 0.073 \Rightarrow (0.7353, 0.8813)$

---

18. In a random sample of 163 people, 151 had a particular trait. Construct a 99% confidence interval for the true proportion.

$$x = 151, \quad n = 163, \quad 99\% \text{ CI}$$

- Sample proportion:  $p\text{-hat} = x/n = 151/163 = 0.9264$ .
- Critical value:  $z^* = 2.576$  for 99% CI.
- Standard error:  $SE = \sqrt{p\text{-hat}(1-p\text{-hat})/n} = 0.0205$ .
- Margin of error:  $E = z^* * SE = 2.576 * 0.0205 = 0.0527$ .
- 99% CI:  $(0.9264 - 0.0527, 0.9264 + 0.0527) = (0.8737, 0.9791)$ .

**Answer:**  $\hat{p} = 0.9264, E = 0.0527 \Rightarrow (0.8737, 0.9791)$

---

**19.** In a random sample of 159 people, 49 had a particular trait. Construct a 95% confidence interval for the true proportion.

$$x = 49, \quad n = 159, \quad 95\% \text{ CI}$$

→ Sample proportion:  $p\text{-hat} = x/n = 49/159 = 0.3082$ .

→ Critical value:  $z^* = 1.96$  for 95% CI.

→ Standard error:  $SE = \sqrt{p\text{-hat}(1-p\text{-hat})/n} = 0.0366$ .

→ Margin of error:  $E = z^* * SE = 1.96 * 0.0366 = 0.0718$ .

→ 95% CI:  $(0.3082 - 0.0718, 0.3082 + 0.0718) = (0.2364, 0.38)$ .

**Answer:**  $\hat{p} = 0.3082, E = 0.0718 \Rightarrow (0.2364, 0.38)$

---

**20.** In a random sample of 250 people, 60 had a particular trait. Construct a 99% confidence interval for the true proportion.

$$x = 60, \quad n = 250, \quad 99\% \text{ CI}$$

→ Sample proportion:  $p\text{-hat} = x/n = 60/250 = 0.24$ .

→ Critical value:  $z^* = 2.576$  for 99% CI.

→ Standard error:  $SE = \sqrt{p\text{-hat}(1-p\text{-hat})/n} = 0.027$ .

→ Margin of error:  $E = z^* * SE = 2.576 * 0.027 = 0.0696$ .

→ 99% CI:  $(0.24 - 0.0696, 0.24 + 0.0696) = (0.1704, 0.3096)$ .

**Answer:**  $\hat{p} = 0.24, E = 0.0696 \Rightarrow (0.1704, 0.3096)$

---

## Confidence Interval for a Mean (t-distribution)

---

**21.** A sample of  $n = 25$  gives  $\bar{x} = 86$  and  $s = 19$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 86, \quad s = 19, \quad n = 25, \quad 95\% \text{ CI}$$

→ Degrees of freedom:  $df = n - 1 = 25 - 1 = 24$ .

→ Critical value:  $t^* = 2.064$  (95% CI,  $df=24$ ).

→ Standard error:  $SE = s / \sqrt{n} = 19 / \sqrt{25} = 3.8$ .

→ Margin of error:  $E = t^* * SE = 2.064 * 3.8 = 7.843$ .

→ 95% CI:  $(86 - 7.843, 86 + 7.843) = (78.157, 93.843)$ .

**Answer:**  $t^* = 2.064, SE = 3.8, E = 7.843 \Rightarrow (78.157, 93.843)$

---

**22.** A sample of  $n = 20$  gives  $\bar{x} = 82$  and  $s = 5$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 82, \quad s = 5, \quad n = 20, \quad 95\% \text{ CI}$$

→ Degrees of freedom:  $df = n - 1 = 20 - 1 = 19$ .

→ Critical value:  $t^* = 2.093$  (95% CI,  $df=19$ ).

→ Standard error:  $SE = s / \sqrt{n} = 5 / \sqrt{20} = 1.118$ .

→ Margin of error:  $E = t^* * SE = 2.093 * 1.118 = 2.34$ .

→ 95% CI:  $(82 - 2.34, 82 + 2.34) = (79.66, 84.34)$ .

**Answer:**  $t^* = 2.093, SE = 1.118, E = 2.34 \Rightarrow (79.66, 84.34)$

---

**23.** A sample of  $n = 15$  gives  $\bar{x} = 91$  and  $s = 11$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 91, \quad s = 11, \quad n = 15, \quad 95\% \text{ CI}$$

→ Degrees of freedom:  $df = n - 1 = 15 - 1 = 14$ .

→ Critical value:  $t^* = 2.145$  (95% CI,  $df=14$ ).

→ Standard error:  $SE = s / \sqrt{n} = 11 / \sqrt{15} = 2.8402$ .

→ Margin of error:  $E = t^* * SE = 2.145 * 2.8402 = 6.092$ .

→ 95% CI:  $(91 - 6.092, 91 + 6.092) = (84.908, 97.092)$ .

**Answer:**  $t^* = 2.145, SE = 2.8402, E = 6.092 \Rightarrow (84.908, 97.092)$

---

**24.** A sample of  $n = 20$  gives  $\bar{x} = 99$  and  $s = 9$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 99, \quad s = 9, \quad n = 20, \quad 95\% \text{ CI}$$

→ Degrees of freedom:  $df = n - 1 = 20 - 1 = 19$ .

→ Critical value:  $t^* = 2.093$  (95% CI,  $df=19$ ).

→ Standard error:  $SE = s / \sqrt{n} = 9 / \sqrt{20} = 2.0125$ .

→ Margin of error:  $E = t^* * SE = 2.093 * 2.0125 = 4.212$ .

→ 95% CI:  $(99 - 4.212, 99 + 4.212) = (94.788, 103.212)$ .

**Answer:**  $t^* = 2.093, SE = 2.0125, E = 4.212 \Rightarrow (94.788, 103.212)$

---

25. A sample of  $n = 5$  gives  $\bar{x} = 114$  and  $s = 20$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 114, \quad s = 20, \quad n = 5, \quad 95\% \text{ CI}$$

- Degrees of freedom:  $df = n - 1 = 5 - 1 = 4$ .
- Critical value:  $t^* = 2.776$  (95% CI,  $df=4$ ).
- Standard error:  $SE = s / \sqrt{n} = 20 / \sqrt{5} = 8.9443$ .
- Margin of error:  $E = t^* * SE = 2.776 * 8.9443 = 24.829$ .
- 95% CI:  $(114 - 24.829, 114 + 24.829) = (89.171, 138.829)$ .

**Answer:**  $t^* = 2.776, SE = 8.9443, E = 24.829 \Rightarrow (89.171, 138.829)$

---

26. A sample of  $n = 30$  gives  $\bar{x} = 74$  and  $s = 5$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 74, \quad s = 5, \quad n = 30, \quad 95\% \text{ CI}$$

- Degrees of freedom:  $df = n - 1 = 30 - 1 = 29$ .
- Critical value:  $t^* = 2.045$  (95% CI,  $df=29$ ).
- Standard error:  $SE = s / \sqrt{n} = 5 / \sqrt{30} = 0.9129$ .
- Margin of error:  $E = t^* * SE = 2.045 * 0.9129 = 1.867$ .
- 95% CI:  $(74 - 1.867, 74 + 1.867) = (72.133, 75.867)$ .

**Answer:**  $t^* = 2.045, SE = 0.9129, E = 1.867 \Rightarrow (72.133, 75.867)$

---

27. A sample of  $n = 5$  gives  $\bar{x} = 119$  and  $s = 8$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 119, \quad s = 8, \quad n = 5, \quad 95\% \text{ CI}$$

- Degrees of freedom:  $df = n - 1 = 5 - 1 = 4$ .
- Critical value:  $t^* = 2.776$  (95% CI,  $df=4$ ).
- Standard error:  $SE = s / \sqrt{n} = 8 / \sqrt{5} = 3.5777$ .
- Margin of error:  $E = t^* * SE = 2.776 * 3.5777 = 9.932$ .
- 95% CI:  $(119 - 9.932, 119 + 9.932) = (109.068, 128.932)$ .

**Answer:**  $t^* = 2.776, SE = 3.5777, E = 9.932 \Rightarrow (109.068, 128.932)$

---

28. A sample of  $n = 15$  gives  $\bar{x} = 86$  and  $s = 12$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 86, \quad s = 12, \quad n = 15, \quad 95\% \text{ CI}$$

- Degrees of freedom:  $df = n - 1 = 15 - 1 = 14$ .
- Critical value:  $t^* = 2.145$  (95% CI,  $df=14$ ).
- Standard error:  $SE = s / \sqrt{n} = 12 / \sqrt{15} = 3.0984$ .
- Margin of error:  $E = t^* * SE = 2.145 * 3.0984 = 6.646$ .
- 95% CI:  $(86 - 6.646, 86 + 6.646) = (79.354, 92.646)$ .

**Answer:**  $t^* = 2.145, SE = 3.0984, E = 6.646 \Rightarrow (79.354, 92.646)$

---

**29.** A sample of  $n = 30$  gives  $\bar{x} = 107$  and  $s = 17$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 107, \quad s = 17, \quad n = 30, \quad 95\% \text{ CI}$$

→ Degrees of freedom:  $df = n - 1 = 30 - 1 = 29$ .

→ Critical value:  $t^* = 2.045$  (95% CI,  $df=29$ ).

→ Standard error:  $SE = s / \sqrt{n} = 17 / \sqrt{30} = 3.1038$ .

→ Margin of error:  $E = t^* * SE = 2.045 * 3.1038 = 6.347$ .

→ 95% CI:  $(107 - 6.347, 107 + 6.347) = (100.653, 113.347)$ .

**Answer:**  $t^* = 2.045, SE = 3.1038, E = 6.347 \Rightarrow (100.653, 113.347)$

---

**30.** A sample of  $n = 25$  gives  $\bar{x} = 80$  and  $s = 19$ . Build a 95% confidence interval for the population mean using the t-distribution.

$$\bar{x} = 80, \quad s = 19, \quad n = 25, \quad 95\% \text{ CI}$$

→ Degrees of freedom:  $df = n - 1 = 25 - 1 = 24$ .

→ Critical value:  $t^* = 2.064$  (95% CI,  $df=24$ ).

→ Standard error:  $SE = s / \sqrt{n} = 19 / \sqrt{25} = 3.8$ .

→ Margin of error:  $E = t^* * SE = 2.064 * 3.8 = 7.843$ .

→ 95% CI:  $(80 - 7.843, 80 + 7.843) = (72.157, 87.843)$ .

**Answer:**  $t^* = 2.064, SE = 3.8, E = 7.843 \Rightarrow (72.157, 87.843)$

---