# MATH110: Hypothesis Testing

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Name: ________________________________ Date: ________________ Score: / 30

## Learning Objectives

- State null and alternative hypotheses
- Compute a one-sample z test statistic
- Make a decision at a given significance level

*Simplify each expression completely. Show all steps and circle your final answer.*

## Chi-Square Goodness of Fit

**1.** Observed frequencies: [18, 22, 20, 15, 25]. Expected frequencies: [20, 20, 20, 20, 20]. Compute the chi-square goodness-of-fit test statistic.

$$O = [18, 22, 20, 15, 25], \quad E = [20, 20, 20, 20, 20]$$

Answer: ____________________________________________

**2.** Observed frequencies: [30, 25, 20, 15, 10]. Expected frequencies: [20, 20, 20, 20, 20]. Compute the chi-square goodness-of-fit test statistic.

$$O = [30, 25, 20, 15, 10], \quad E = [20, 20, 20, 20, 20]$$

Answer: ____________________________________________

**3.** Observed frequencies: [12, 18, 24, 22, 24]. Expected frequencies: [20, 20, 20, 20, 20]. Compute the chi-square goodness-of-fit test statistic.

$$O = [12, 18, 24, 22, 24], \quad E = [20, 20, 20, 20, 20]$$

Answer: ____________________________________________

**4.** Observed frequencies: [18, 22, 20, 15, 25]. Expected frequencies: [20, 20, 20, 20, 20]. Compute the chi-square goodness-of-fit test statistic.

$$O = [18, 22, 20, 15, 25], \quad E = [20, 20, 20, 20, 20]$$

Answer: ____________________________________________

## Chi-Square Test for Independence

**5.** A 2x2 contingency table has counts: 12, 28 (row 1) and 19, 39 (row 2). Test for independence. Compute the chi-square statistic.

$$\text{Row1: } 12,\ 28 \quad \text{Row2: } 19,\ 39$$

Answer: ____________________________________________

**6.** A 2x2 contingency table has counts: 27, 13 (row 1) and 42, 40 (row 2). Test for independence. Compute the chi-square statistic.

$$\text{Row1: } 27,\ 13 \quad \text{Row2: } 42,\ 40$$

Answer: ____________________________________________

**7.** A 2x2 contingency table has counts: 41, 21 (row 1) and 45, 23 (row 2). Test for independence. Compute the chi-square statistic.

$$\text{Row1: } 41,\ 21 \quad \text{Row2: } 45,\ 23$$

Answer: ____________________________________________

**8.** A 2x2 contingency table has counts: 17, 28 (row 1) and 39, 38 (row 2). Test for independence. Compute the chi-square statistic.

$$\text{Row1: } 17,\ 28 \quad \text{Row2: } 39,\ 38$$

Answer: ____________________________________________

## Matched-Pairs t-Test

**9.** A matched-pairs study yields differences: [3, 5, 2, 4, 1, 3, 2, 4]. Test H0: mu_d = 0 at alpha = 0.05. Compute the t-statistic.

$$d = [3, 5, 2, 4, 1, 3, 2, 4], \quad \bar{d} = 3.0$$

Answer: ____________________________________________

**10.** A matched-pairs study yields differences: [5, 3, -1, 4, 2, 6, 1, 4]. Test H0: mu_d = 0 at alpha = 0.05. Compute the t-statistic.

$$d = [5, 3, -1, 4, 2, 6, 1, 4], \quad \bar{d} = 3.0$$

Answer: ____________________________________________

**11.** A matched-pairs study yields differences: [5, 3, -1, 4, 2, 6, 1, 4]. Test H0: mu_d = 0 at alpha = 0.05. Compute the t-statistic.

$$d = [5, 3, -1, 4, 2, 6, 1, 4], \quad \bar{d} = 3.0$$

Answer: ____________________________________________

**12.** A matched-pairs study yields differences: [-2, 1, -3, 0, 2, -1, 1, -2]. Test H0: mu_d = 0 at alpha = 0.05. Compute the t-statistic.

$$d = [-2, 1, -3, 0, 2, -1, 1, -2], \quad \bar{d} = -0.5$$

Answer: ____________________________________________

## Type I and Type II Errors

**13.** Identify the type of error: H0: A new drug has no effect. A researcher rejects H0 when in fact the drug truly has no effect.

Answer: ________________________________________

**14.** Identify the type of error: H0: A machine is working correctly. An inspector fails to reject H0 even though the machine is actually broken.

Answer: ________________________________________

**15.** Identify the type of error: H0: A student is not cheating. A professor rejects H0 and accuses the student, but the student was actually innocent.

Answer: ________________________________________

**16.** Identify the type of error: H0: A new drug has no effect. A researcher rejects H0 when in fact the drug truly has no effect.

Answer: ________________________________________

**17.** Identify the type of error: H0: A machine is working correctly. An inspector fails to reject H0 even though the machine is actually broken.

Answer: ________________________________________

**18.** Identify the type of error: H0: A student is not cheating. A professor rejects H0 and accuses the student, but the student was actually innocent.

Answer: ________________________________________

**19.** Identify the type of error: H0: A new drug has no effect. A researcher rejects H0 when in fact the drug truly has no effect.

Answer: ________________________________________

**20.** Identify the type of error: H0: A machine is working correctly. An inspector fails to reject H0 even though the machine is actually broken.

Answer: ________________________________________

**21.** Identify the type of error: H0: A student is not cheating. A professor rejects H0 and accuses the student, but the student was actually innocent.

Answer: ________________________________________

## Two-Sample z-Test for Means

**22.** Group 1: x-bar1=95, sigma1=9, n1=100. Group 2: x-bar2=80, sigma2=14, n2=36. Test H0: mu1=mu2 at alpha=0.05.

$$\bar{x}_1 = 95, \bar{x}_2 = 80, \sigma_1 = 9, \sigma_2 = 14, n_1 = 100, n_2 = 36$$

Answer: ________________________________________

**23.** Group 1: x-bar1=62, sigma1=5, n1=100. Group 2: x-bar2=83, sigma2=12, n2=49. Test H0: mu1=mu2 at alpha=0.05.

$$\bar{x}_1 = 62, \bar{x}_2 = 83, \sigma_1 = 5, \sigma_2 = 12, n_1 = 100, n_2 = 49$$

Answer: ________________________________________

**24.** Group 1: x-bar1=64, sigma1=18, n1=64. Group 2: x-bar2=79, sigma2=12, n2=49. Test H0: mu1=mu2 at alpha=0.05.

$$\bar{x}_1 = 64, \bar{x}_2 = 79, \sigma_1 = 18, \sigma_2 = 12, n_1 = 64, n_2 = 49$$

Answer: ________________________________________

## One-sample z-test for a mean

**25.** Test H0: mu = 48 against H1: mu != 48. A sample of n = 16 gives a sample mean of 44 with sigma = 15. Compute the z test statistic and state the decision at alpha = 0.05.

$$H_0: \mu = 48, \quad \bar{x} = 44, \quad \sigma = 15, \quad n = 16$$

Answer: ________________________________________

**26.** Test H0: mu = 114 against H1: mu != 114. A sample of n = 25 gives a sample mean of 105 with sigma = 10. Compute the z test statistic and state the decision at alpha = 0.05.

$$H_0: \mu = 114, \quad \bar{x} = 105, \quad \sigma = 10, \quad n = 25$$

Answer: ________________________________________

**27.** Test H0: mu = 49 against H1: mu != 49. A sample of n = 100 gives a sample mean of 37 with sigma = 15. Compute the z test statistic and state the decision at alpha = 0.05.

$$H_0: \mu = 49, \quad \bar{x} = 37, \quad \sigma = 15, \quad n = 100$$

Answer: ________________________________________

## One-Sample z-Test for a Proportion

**28.** A researcher claims the true proportion is p0 = 0.5. A sample of n = 236 gave 157 successes. Test H0: p = 0.5 at alpha = 0.05.

$$H_0: p = 0.5, \quad \hat{p} = 0.6653, \quad n = 236$$

Answer: ________________________________________

**29.** A researcher claims the true proportion is p0 = 0.6. A sample of n = 143 gave 87 successes. Test H0: p = 0.6 at alpha = 0.05.

$$H_0: p = 0.6, \quad \hat{p} = 0.6084, \quad n = 143$$

Answer: ________________________________________

**30.** A researcher claims the true proportion is p0 = 0.7. A sample of n = 453 gave 149 successes. Test H0: p = 0.7 at alpha = 0.05.

$$H_0: p = 0.7, \quad \hat{p} = 0.3289, \quad n = 453$$

Answer: ________________________________________

# MATH110: Hypothesis Testing

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ANSWER KEY & SOLUTIONS

*Topics: Matched-Pairs t-Test, One-sample z-test for a mean, One-Sample z-Test for a Proportion, Two-Sample z-Test for Means, Chi-Square Test for Independence, Type I and Type II Errors, Chi-Square Goodness of Fit. All answers verified by independent computation.*

## Solutions

## Chi-Square Goodness of Fit

**1.** Observed frequencies: [18, 22, 20, 15, 25]. Expected frequencies: [20, 20, 20, 20, 20]. Compute the chi-square goodness-of-fit test statistic.

$$O = [18, 22, 20, 15, 25], \quad E = [20, 20, 20, 20, 20]$$

→ *Compute each (O-E)^2/E term: (18-20)^2/20=0.2; (22-20)^2/20=0.2; (20-20)^2/20=0.0; (15-20)^2/20=1.25; (25-20)^2/20=1.25.*
→ *Sum all terms: chi-square = 2.9.*
→ *Degrees of freedom: df = k - 1 = 4.*

**Answer:** $\chi^2 = 2.9, \quad df = 4$

**2.** Observed frequencies: [30, 25, 20, 15, 10]. Expected frequencies: [20, 20, 20, 20, 20]. Compute the chi-square goodness-of-fit test statistic.

$$O = [30, 25, 20, 15, 10], \quad E = [20, 20, 20, 20, 20]$$

→ *Compute each (O-E)^2/E term: (30-20)^2/20=5.0; (25-20)^2/20=1.25; (20-20)^2/20=0.0; (15-20)^2/20=1.25; (10-20)^2/20=5.0.*
→ *Sum all terms: chi-square = 12.5.*
→ *Degrees of freedom: df = k - 1 = 4.*

**Answer:** $\chi^2 = 12.5, \quad df = 4$

**3.** Observed frequencies: [12, 18, 24, 22, 24]. Expected frequencies: [20, 20, 20, 20, 20]. Compute the chi-square goodness-of-fit test statistic.

$$O = [12, 18, 24, 22, 24], \quad E = [20, 20, 20, 20, 20]$$

→ *Compute each (O-E)^2/E term: (12-20)^2/20=3.2; (18-20)^2/20=0.2; (24-20)^2/20=0.8; (22-20)^2/20=0.2; (24-20)^2/20=0.8.*
→ *Sum all terms: chi-square = 5.2.*
→ *Degrees of freedom: df = k - 1 = 4.*

**Answer:** $\chi^2 = 5.2, \quad df = 4$

**4.** Observed frequencies: [18, 22, 20, 15, 25]. Expected frequencies: [20, 20, 20, 20, 20]. Compute the chi-square goodness-of-fit test statistic.

$$O = [18, 22, 20, 15, 25], \quad E = [20, 20, 20, 20, 20]$$

→ *Compute each (O-E)^2/E term: (18-20)^2/20=0.2; (22-20)^2/20=0.2; (20-20)^2/20=0.0; (15-20)^2/20=1.25; (25-20)^2/20=1.25.*
→ *Sum all terms: chi-square = 2.9.*
→ *Degrees of freedom: df = k - 1 = 4.*

**Answer:** $\chi^2 = 2.9, \quad df = 4$

## Chi-Square Test for Independence

**5.** A 2x2 contingency table has counts: 12, 28 (row 1) and 19, 39 (row 2). Test for independence. Compute the chi-square statistic.

$$\text{Row1: } 12,\ 28 \quad \text{Row2: } 19,\ 39$$

→ *Row totals: row1=40, row2=58. Column totals: col1=31, col2=67. Grand total: N=98.*
→ *Expected counts: E11=12.65, E12=27.35, E21=18.35, E22=39.65.*
→ *chi-square = (12-12.65)^2/12.65 + (28-27.35)^2/27.35 + (19-18.35)^2/18.35 + (39-39.65)^2/39.65 = 0.083.*

**Answer:** $\chi^2 = 0.083$

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**6.** A 2x2 contingency table has counts: 27, 13 (row 1) and 42, 40 (row 2). Test for independence. Compute the chi-square statistic.

$$\text{Row1: } 27,\ 13 \quad \text{Row2: } 42,\ 40$$

→ *Row totals: row1=40, row2=82. Column totals: col1=69, col2=53. Grand total: N=122.*
→ *Expected counts: E11=22.62, E12=17.38, E21=46.38, E22=35.62.*
→ *chi-square = (27-22.62)^2/22.62 + (13-17.38)^2/17.38 + (42-46.38)^2/46.38 + (40-35.62)^2/35.62 = 2.904.*

**Answer:** $\chi^2 = 2.904$

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**7.** A 2x2 contingency table has counts: 41, 21 (row 1) and 45, 23 (row 2). Test for independence. Compute the chi-square statistic.

$$\text{Row1: } 41,\ 21 \quad \text{Row2: } 45,\ 23$$

→ *Row totals: row1=62, row2=68. Column totals: col1=86, col2=44. Grand total: N=130.*
→ *Expected counts: E11=41.02, E12=20.98, E21=44.98, E22=23.02.*
→ *chi-square = (41-41.02)^2/41.02 + (21-20.98)^2/20.98 + (45-44.98)^2/44.98 + (23-23.02)^2/23.02 = 0.0.*

**Answer:** $\chi^2 = 0.0$

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**8.** A 2x2 contingency table has counts: 17, 28 (row 1) and 39, 38 (row 2). Test for independence. Compute the chi-square statistic.

$$\text{Row1: } 17,\ 28 \quad \text{Row2: } 39,\ 38$$

→ *Row totals: row1=45, row2=77. Column totals: col1=56, col2=66. Grand total: N=122.*
→ *Expected counts: E11=20.66, E12=24.34, E21=35.34, E22=41.66.*
→ *chi-square = (17-20.66)^2/20.66 + (28-24.34)^2/24.34 + (39-35.34)^2/35.34 + (38-41.66)^2/41.66 = 1.899.*

**Answer:** $\chi^2 = 1.899$

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## Matched-Pairs t-Test

**9.** A matched-pairs study yields differences: [3, 5, 2, 4, 1, 3, 2, 4]. Test H0: mu_d = 0 at alpha = 0.05. Compute the t-statistic.

$$d = [3, 5, 2, 4, 1, 3, 2, 4], \quad \bar{d} = 3.0$$

→ *Mean of differences: d-bar = 3.0.*
→ *Standard deviation of differences: sd = 1.3093.*
→ *Standard error: SE = sd / sqrt(n) = 1.3093 / sqrt(7+1) = 0.4629.*
→ *t = d-bar / SE = 3.0 / 0.4629 = 6.48.*
→ *Decision (df=7): |t|=6.48>2.093, reject H0 at alpha=0.05.*

**Answer:** $t = 6.48, \quad df = 7; \quad |t| = 6.48 > 2.093, reject H0 at alpha = 0.05$

**10.** A matched-pairs study yields differences: [5, 3, -1, 4, 2, 6, 1, 4]. Test H0: mu_d = 0 at alpha = 0.05. Compute the t-statistic.

$$d = [5, 3, -1, 4, 2, 6, 1, 4], \quad \bar{d} = 3.0$$

→ *Mean of differences: d-bar = 3.0.*
→ *Standard deviation of differences: sd = 2.2678.*
→ *Standard error: SE = sd / sqrt(n) = 2.2678 / sqrt(7+1) = 0.8018.*
→ *t = d-bar / SE = 3.0 / 0.8018 = 3.74.*
→ *Decision (df=7): |t|=3.74>2.093, reject H0 at alpha=0.05.*

**Answer:** $t = 3.74, \quad df = 7; \quad |t| = 3.74 > 2.093, reject H0 at alpha = 0.05$

**11.** A matched-pairs study yields differences: [5, 3, -1, 4, 2, 6, 1, 4]. Test H0: mu_d = 0 at alpha = 0.05. Compute the t-statistic.

$$d = [5, 3, -1, 4, 2, 6, 1, 4], \quad \bar{d} = 3.0$$

→ *Mean of differences: d-bar = 3.0.*
→ *Standard deviation of differences: sd = 2.2678.*
→ *Standard error: SE = sd / sqrt(n) = 2.2678 / sqrt(7+1) = 0.8018.*
→ *t = d-bar / SE = 3.0 / 0.8018 = 3.74.*
→ *Decision (df=7): |t|=3.74>2.093, reject H0 at alpha=0.05.*

**Answer:** $t = 3.74, \quad df = 7; \quad |t| = 3.74 > 2.093, reject H0 at alpha = 0.05$

**12.** A matched-pairs study yields differences: [-2, 1, -3, 0, 2, -1, 1, -2]. Test H0: mu_d = 0 at alpha = 0.05. Compute the t-statistic.

$$d = [-2, 1, -3, 0, 2, -1, 1, -2], \quad \bar{d} = -0.5$$

→ *Mean of differences: d-bar = -0.5.*
→ *Standard deviation of differences: sd = 1.7728.*
→ *Standard error: SE = sd / sqrt(n) = 1.7728 / sqrt(7+1) = 0.6268.*
→ *t = d-bar / SE = -0.5 / 0.6268 = -0.8.*
→ *Decision (df=7): |t|=0.8<=2.093, fail to reject H0 at alpha=0.05.*

**Answer:** $t = -0.8, \quad df = 7; \quad |t| = 0.8 < = 2.093, fail to reject H0 at alpha = 0.05$

## Type I and Type II Errors

**13.** Identify the type of error: H0: A new drug has no effect. A researcher rejects H0 when in fact the drug truly has no effect.

→ *H0 is true but was rejected — this is a Type I error (false positive).*
→ *Answer: Type I Error.*

**Answer:** $Type I Error$

**14.** Identify the type of error: H0: A machine is working correctly. An inspector fails to reject H0 even though the machine is actually broken.

→ *H0 is false but was not rejected — this is a Type II error (false negative).*
→ *Answer: Type II Error.*

**Answer:** $Type II Error$

**15.** Identify the type of error: H0: A student is not cheating. A professor rejects H0 and accuses the student, but the student was actually innocent.

→ *H0 is true (student is innocent) but was rejected — this is a Type I error (false positive).*
→ *Answer: Type I Error.*

**Answer:** $Type I Error$

**16.** Identify the type of error: H0: A new drug has no effect. A researcher rejects H0 when in fact the drug truly has no effect.

→ *H0 is true but was rejected — this is a Type I error (false positive).*
→ *Answer: Type I Error.*

**Answer:** $Type I Error$

**17.** Identify the type of error: H0: A machine is working correctly. An inspector fails to reject H0 even though the machine is actually broken.

→ *H0 is false but was not rejected — this is a Type II error (false negative).*
→ *Answer: Type II Error.*

**Answer:** $Type II Error$

**18.** Identify the type of error: H0: A student is not cheating. A professor rejects H0 and accuses the student, but the student was actually innocent.

→ *H0 is true (student is innocent) but was rejected — this is a Type I error (false positive).*
→ *Answer: Type I Error.*

**Answer:** $Type I Error$

**19.** Identify the type of error: H0: A new drug has no effect. A researcher rejects H0 when in fact the drug truly has no effect.

→ *H0 is true but was rejected — this is a Type I error (false positive).*
→ *Answer: Type I Error.*

**Answer:** $Type I Error$

**20.** Identify the type of error: H0: A machine is working correctly. An inspector fails to reject H0 even though the machine is actually broken.

→ *H0 is false but was not rejected — this is a Type II error (false negative).*
→ *Answer: Type II Error.*

**Answer:** $Type II Error$

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**21.** Identify the type of error: H0: A student is not cheating. A professor rejects H0 and accuses the student, but the student was actually innocent.

→ *H0 is true (student is innocent) but was rejected — this is a Type I error (false positive).*
→ *Answer: Type I Error.*

**Answer:** $Type I Error$

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## Two-Sample z-Test for Means

**22.** Group 1: x-bar1=95, sigma1=9, n1=100. Group 2: x-bar2=80, sigma2=14, n2=36. Test H0: mu1=mu2 at alpha=0.05.

$$\bar{x}_1 = 95, \bar{x}_2 = 80, \sigma_1 = 9, \sigma_2 = 14, n_1 = 100, n_2 = 36$$

→ *SE = sqrt(sigma1^2/n1 + sigma2^2/n2) = sqrt(9^2/100 + 14^2/36) = 2.5009.*
→ *z = (x-bar1 - x-bar2) / SE = (95 - 80) / 2.5009 = 6.0.*
→ *Decision: |z|=6.0>1.96, reject H0 at alpha=0.05.*

**Answer:** $z = 6.0; |z| = 6.0 > 1.96, rejectH0atalpha = 0.05$

**23.** Group 1: x-bar1=62, sigma1=5, n1=100. Group 2: x-bar2=83, sigma2=12, n2=49. Test H0: mu1=mu2 at alpha=0.05.

$$\bar{x}_1 = 62, \bar{x}_2 = 83, \sigma_1 = 5, \sigma_2 = 12, n_1 = 100, n_2 = 49$$

→ *SE = sqrt(sigma1^2/n1 + sigma2^2/n2) = sqrt(5^2/100 + 12^2/49) = 1.7857.*
→ *z = (x-bar1 - x-bar2) / SE = (62 - 83) / 1.7857 = -11.76.*
→ *Decision: |z|=11.76>1.96, reject H0 at alpha=0.05.*

**Answer:** $z = -11.76; |z| = 11.76 > 1.96, rejectH0atalpha = 0.05$

**24.** Group 1: x-bar1=64, sigma1=18, n1=64. Group 2: x-bar2=79, sigma2=12, n2=49. Test H0: mu1=mu2 at alpha=0.05.

$$\bar{x}_1 = 64, \bar{x}_2 = 79, \sigma_1 = 18, \sigma_2 = 12, n_1 = 64, n_2 = 49$$

→ *SE = sqrt(sigma1^2/n1 + sigma2^2/n2) = sqrt(18^2/64 + 12^2/49) = 2.8287.*
→ *z = (x-bar1 - x-bar2) / SE = (64 - 79) / 2.8287 = -5.3.*
→ *Decision: |z|=5.3>1.96, reject H0 at alpha=0.05.*

**Answer:** $z = -5.3; |z| = 5.3 > 1.96, rejectH0atalpha = 0.05$

## One-sample z-test for a mean

**25.** Test H0: mu = 48 against H1: mu != 48. A sample of n = 16 gives a sample mean of 44 with sigma = 15. Compute the z test statistic and state the decision at alpha = 0.05.

$$H_0 : \mu = 48, \quad \bar{x} = 44, \quad \sigma = 15, \quad n = 16$$

→ *Standard error = sigma / sqrt(n) = 15/4 = 3.75.*
→ *z = (sample mean - mu0) / SE = (44 - 48)/3.75 = -1.07.*
→ *Compare to +/-1.96: |z| = 1.07 <= 1.96, so fail to reject H0 at alpha = 0.05.*

**Answer:** $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{44 - 48}{15/4} = -1.07$$

**26.** Test H0: mu = 114 against H1: mu != 114. A sample of n = 25 gives a sample mean of 105 with sigma = 10. Compute the z test statistic and state the decision at alpha = 0.05.

$$H_0 : \mu = 114, \quad \bar{x} = 105, \quad \sigma = 10, \quad n = 25$$

→ *Standard error = sigma / sqrt(n) = 10/5 = 2.0.*
→ *z = (sample mean - mu0) / SE = (105 - 114)/2.0 = -4.5.*
→ *Compare to +/-1.96: |z| = 4.5 > 1.96, so reject H0 at alpha = 0.05.*

**Answer:** $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{105 - 114}{10/5} = -4.5$$

**27.** Test H0: mu = 49 against H1: mu != 49. A sample of n = 100 gives a sample mean of 37 with sigma = 15. Compute the z test statistic and state the decision at alpha = 0.05.

$$H_0 : \mu = 49, \quad \bar{x} = 37, \quad \sigma = 15, \quad n = 100$$

→ *Standard error = sigma / sqrt(n) = 15/10 = 1.5.*
→ *z = (sample mean - mu0) / SE = (37 - 49)/1.5 = -8.0.*
→ *Compare to +/-1.96: |z| = 8.0 > 1.96, so reject H0 at alpha = 0.05.*

**Answer:** $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{37 - 49}{15/10} = -8.0$$

## One-Sample z-Test for a Proportion

**28.** A researcher claims the true proportion is p0 = 0.5. A sample of n = 236 gave 157 successes. Test H0: p = 0.5 at alpha = 0.05.

$$H_0: p = 0.5, \quad \hat{p} = 0.6653, \quad n = 236$$

→ *Sample proportion: p-hat = 157/236 = 0.6653.*
→ *Standard error under H0: SE = sqrt(0.5*(1-0.5)/236) = 0.0325.*
→ *z = (p-hat - p0) / SE = (0.6653 - 0.5) / 0.0325 = 5.08.*
→ *Decision: |z|=5.08>1.96, reject H0 at alpha=0.05.*

**Answer:** $z = 5.08; |z| = 5.08 > 1.96, rejectH0atalpha = 0.05$

**29.** A researcher claims the true proportion is p0 = 0.6. A sample of n = 143 gave 87 successes. Test H0: p = 0.6 at alpha = 0.05.

$$H_0: p = 0.6, \quad \hat{p} = 0.6084, \quad n = 143$$

→ *Sample proportion: p-hat = 87/143 = 0.6084.*
→ *Standard error under H0: SE = sqrt(0.6*(1-0.6)/143) = 0.041.*
→ *z = (p-hat - p0) / SE = (0.6084 - 0.6) / 0.041 = 0.21.*
→ *Decision: |z|=0.21<=1.96, fail to reject H0 at alpha=0.05.*

**Answer:** $z = 0.21; |z| = 0.21 < = 1.96, failtorejectH0atalpha = 0.05$

**30.** A researcher claims the true proportion is p0 = 0.7. A sample of n = 453 gave 149 successes. Test H0: p = 0.7 at alpha = 0.05.

$$H_0: p = 0.7, \quad \hat{p} = 0.3289, \quad n = 453$$

→ *Sample proportion: p-hat = 149/453 = 0.3289.*
→ *Standard error under H0: SE = sqrt(0.7*(1-0.7)/453) = 0.0215.*
→ *z = (p-hat - p0) / SE = (0.3289 - 0.7) / 0.0215 = -17.24.*
→ *Decision: |z|=17.24>1.96, reject H0 at alpha=0.05.*

**Answer:** $z = -17.24; |z| = 17.24 > 1.96, rejectH0atalpha = 0.05$