MATH110: Normal Distribution

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Name: ________________________________ Date: ________________ Score: / 30

Learning Objectives

- Compute and interpret z-scores
- Apply the empirical rule (68-95-99.7)
- Relate raw scores to a standard normal distribution

Simplify each expression completely. Show all steps and circle your final answer.

Empirical rule (68-95-99.7)

1. Test scores are normally distributed with $\mu = 90$ and $\sigma = 4$. By the empirical rule, what percent of scores lie within 1 standard deviation(s) of the mean — that is, in the interval [86, 94]?

$$\mu = 90, \quad \sigma = 4, \quad [86,\ 94]$$

Answer: ____

2. Test scores are normally distributed with $\mu = 54$ and $\sigma = 8$. By the empirical rule, what percent of scores lie within 1 standard deviation(s) of the mean — that is, in the interval [46, 62]?

$$\mu = 54, \quad \sigma = 8, \quad [46,\ 62]$$

Answer: ____

3. Test scores are normally distributed with $\mu = 81$ and $\sigma = 6$. By the empirical rule, what percent of scores lie within 2 standard deviation(s) of the mean — that is, in the interval [69, 93]?

$$\mu = 81, \quad \sigma = 6, \quad [69,\ 93]$$

Answer: ____

4. Test scores are normally distributed with $\mu = 58$ and $\sigma = 8$. By the empirical rule, what percent of scores lie within 3 standard deviation(s) of the mean — that is, in the interval [34, 82]?

$$\mu = 58, \quad \sigma = 8, \quad [34,\ 82]$$

Answer: ____

5. Test scores are normally distributed with $\mu = 87$ and $\sigma = 5$. By the empirical rule, what percent of scores lie within 1 standard deviation(s) of the mean — that is, in the interval [82, 92]?

$$\mu = 87, \quad \sigma = 5, \quad [82,\ 92]$$

Answer: ____

6. Test scores are normally distributed with μ = 86 and σ = 5. By the empirical rule, what percent of scores lie within 1 standard deviation(s) of the mean — that is, in the interval [81, 91]?

$$\mu = 86, \quad \sigma = 5, \quad [81,\ 91]$$

Answer: ____

7. Test scores are normally distributed with μ = 54 and σ = 6. By the empirical rule, what percent of scores lie within 2 standard deviation(s) of the mean — that is, in the interval [42, 66]?

$$\mu = 54, \quad \sigma = 6, \quad [42,\ 66]$$

Answer: ____

8. Test scores are normally distributed with μ = 57 and σ = 6. By the empirical rule, what percent of scores lie within 3 standard deviation(s) of the mean — that is, in the interval [39, 75]?

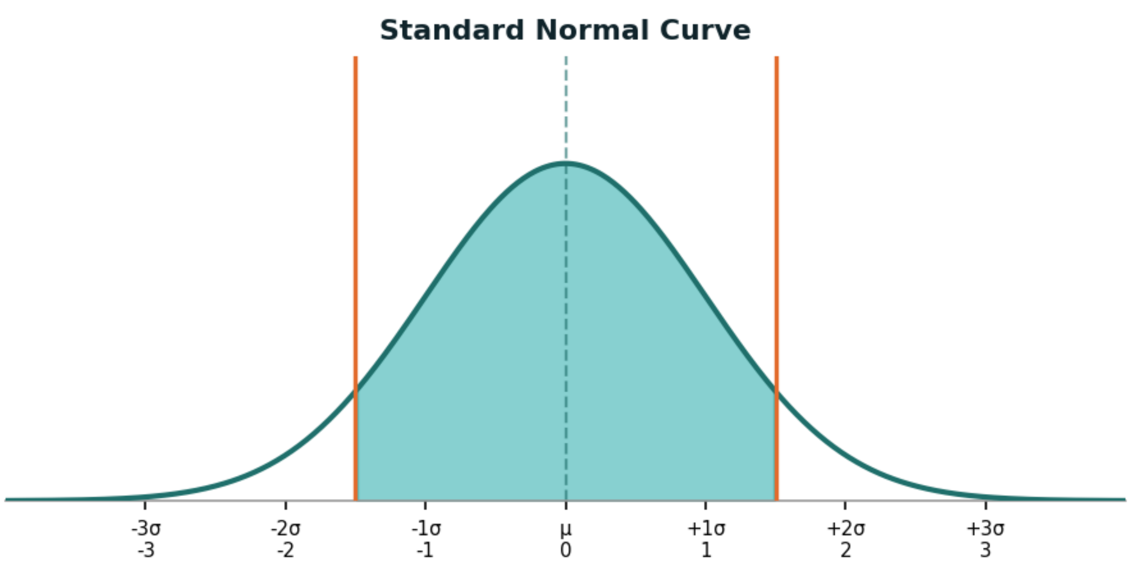
$$\mu = 57, \quad \sigma = 6, \quad [39,\ 75]$$

Answer: ____

Area Under the Normal Curve

9. Find the proportion of values in a standard normal distribution between z = -1.5 and z = 1.5. Use the z-table.

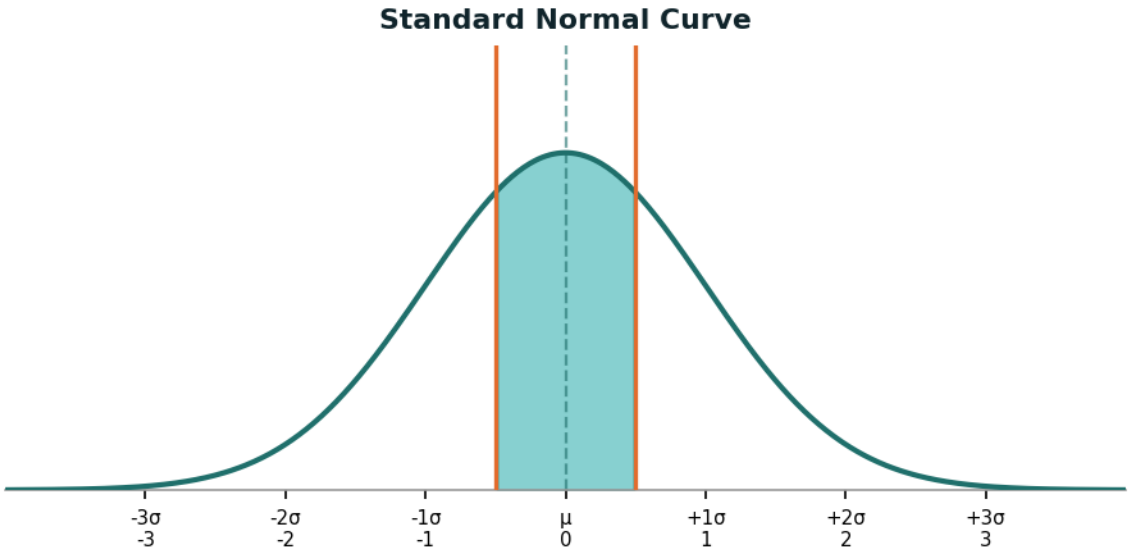
$$z_1 = -1.5, \quad z_2 = 1.5$$



Answer: ____

10. Find the proportion of values in a standard normal distribution between z = -0.5 and z = 0.5. Use the z-table.

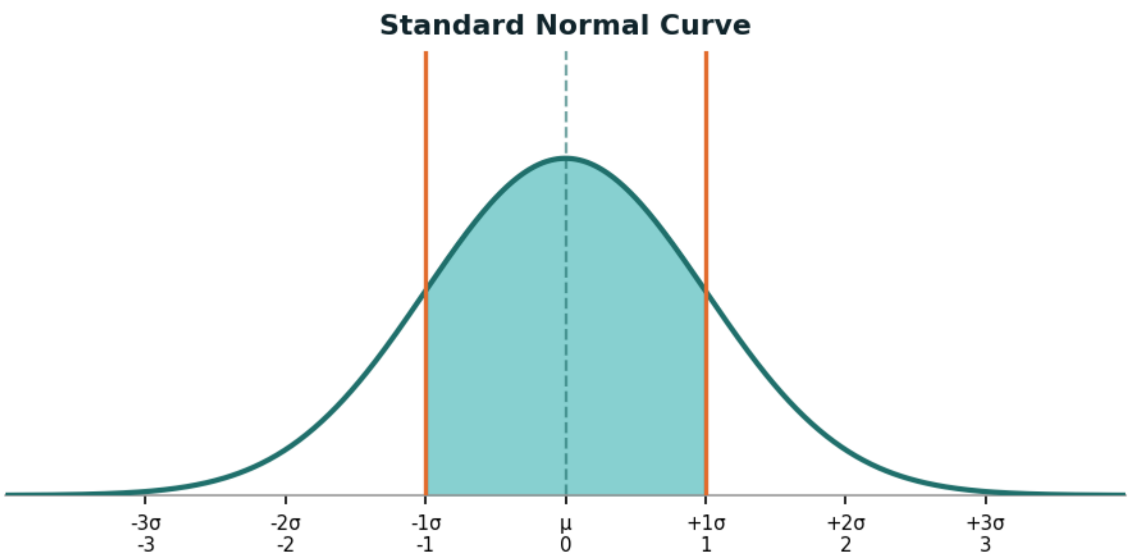
$$z_1 = -0.5, \quad z_2 = 0.5$$



Answer: ____

11. Find the proportion of values in a standard normal distribution between z = -1.0 and z = 1.0. Use the z-table.

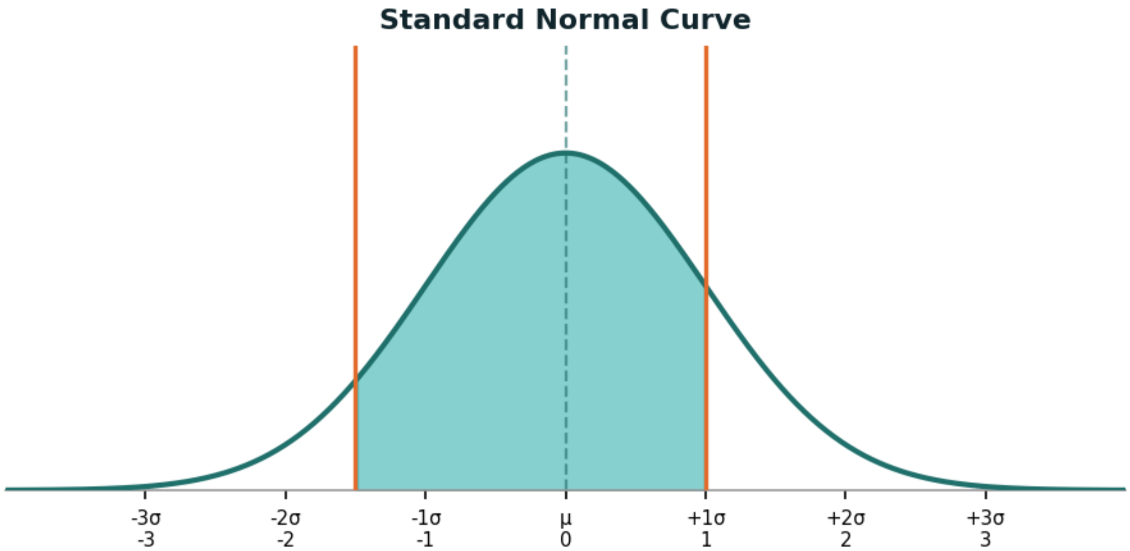
$$z_1 = -1.0, \quad z_2 = 1.0$$



Answer: ____

12. Find the proportion of values in a standard normal distribution between z = -1.5 and z = 1.0. Use the z-table.

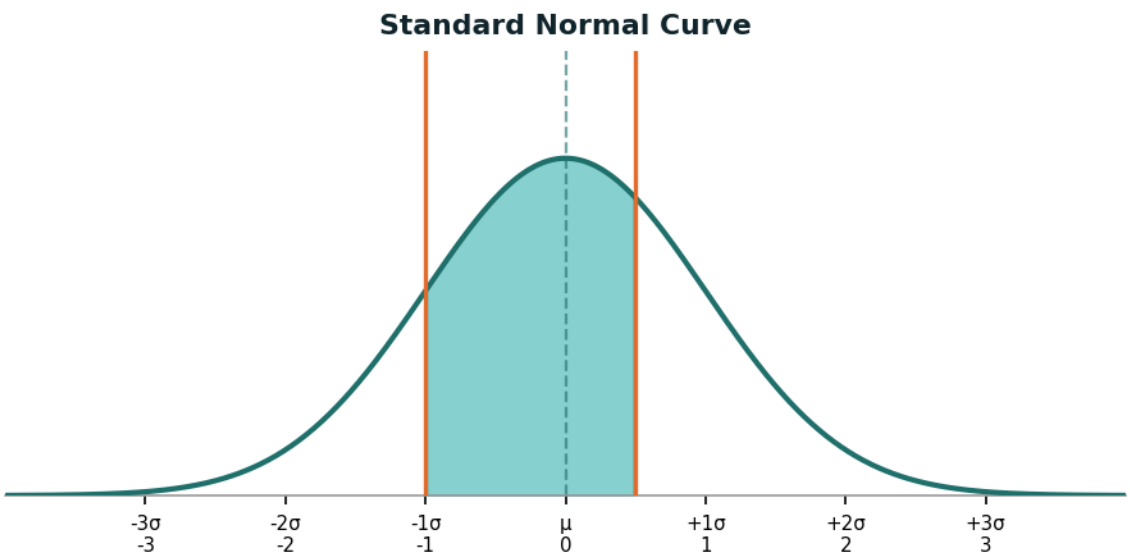
$$z_1 = -1.5, \quad z_2 = 1.0$$



Answer: ____

13. Find the proportion of values in a standard normal distribution between z = -1.0 and z = 0.5. Use the z-table.

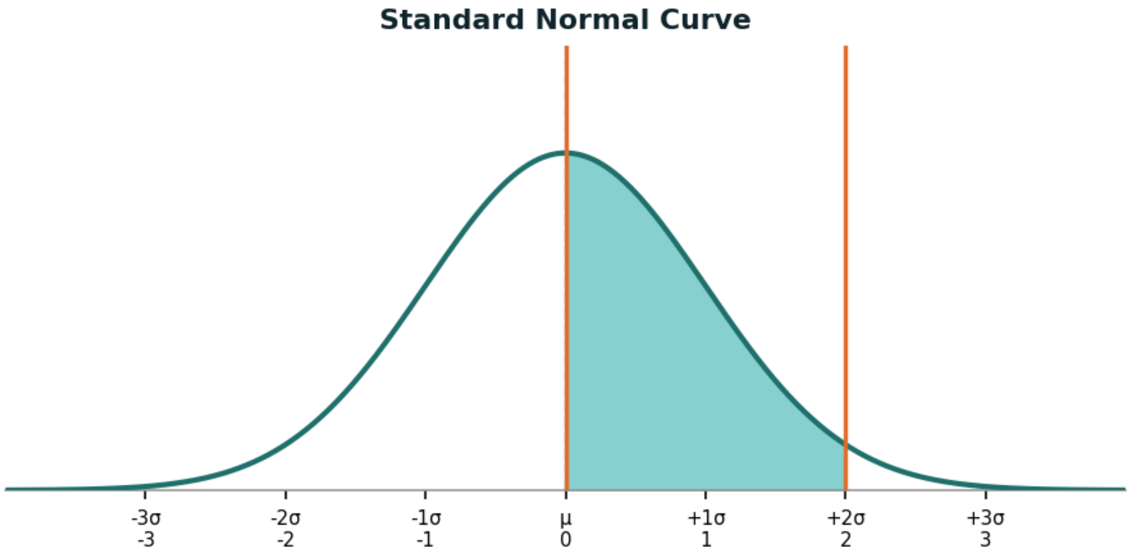
$$z_1 = -1.0, \quad z_2 = 0.5$$



Answer: ____

14. Find the proportion of values in a standard normal distribution between z = 0.0 and z = 2.0. Use the z-table.

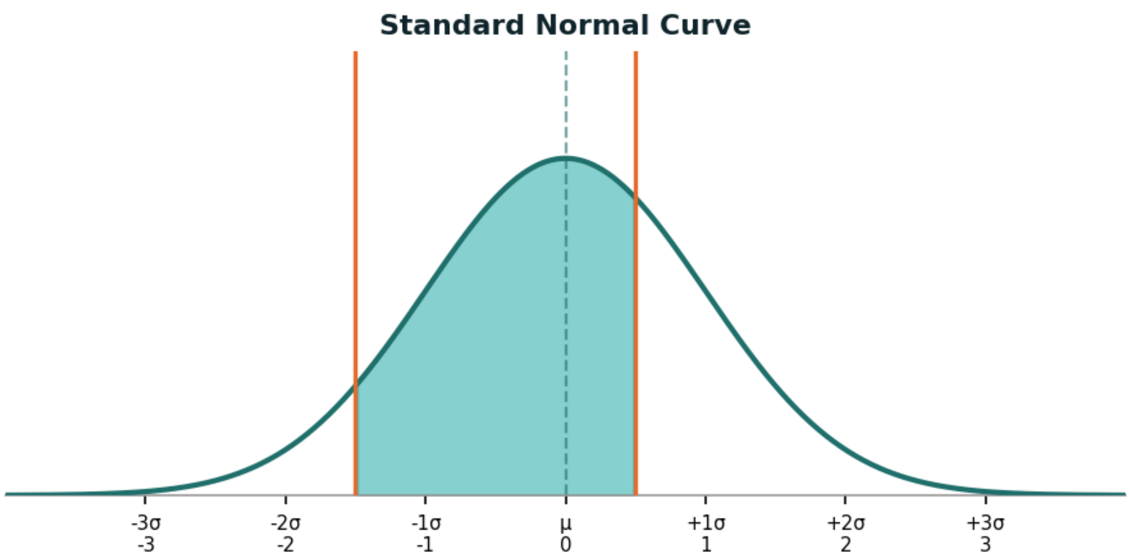
$$z_1 = 0.0, \quad z_2 = 2.0$$



Answer: ____

15. Find the proportion of values in a standard normal distribution between z = -1.5 and z = 0.5. Use the z-table.

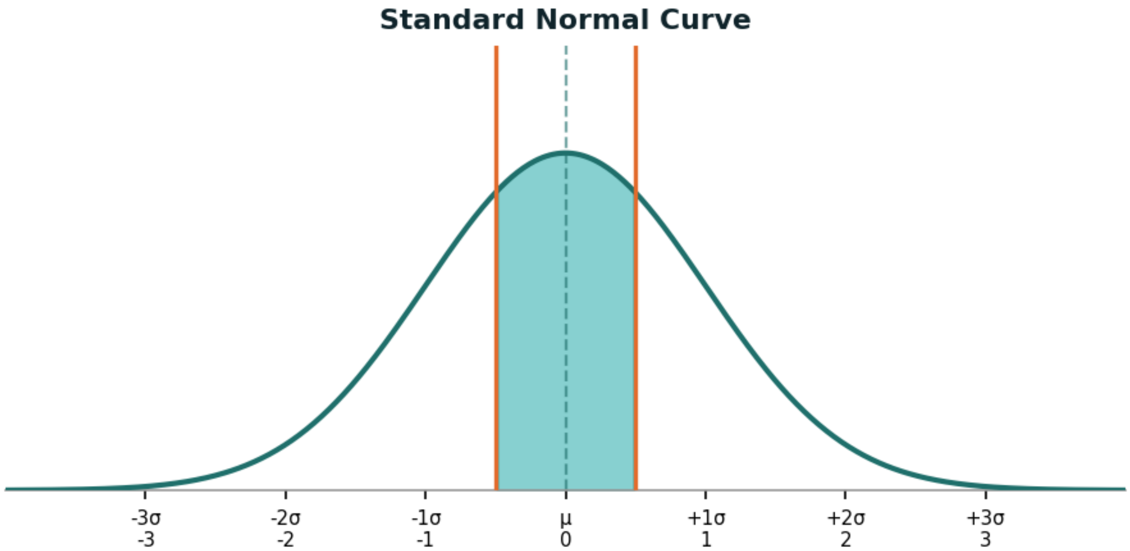
$$z_1 = -1.5, \quad z_2 = 0.5$$



Answer: ____

16. Find the proportion of values in a standard normal distribution between z = -0.5 and z = 0.5. Use the z-table.

$$z_1 = -0.5, \quad z_2 = 0.5$$



Answer: ____

Finding a Score from a Percentile (Inverse Normal)

17. A distribution has mean mu = 76 and standard deviation sigma = 8. A score is at z = -1.5. Find the raw score X.

$$\mu = 76, \quad \sigma = 8, \quad z = -1.5$$

Answer: ____

18. A distribution has mean mu = 85 and standard deviation sigma = 15. A score is at z = -1.5. Find the raw score X.

$$\mu = 85, \quad \sigma = 15, \quad z = -1.5$$

Answer: ____

19. A distribution has mean mu = 67 and standard deviation sigma = 6. A score is at z = 1.5. Find the raw score X.

$$\mu = 67, \quad \sigma = 6, \quad z = 1.5$$

Answer: ____

20. A distribution has mean mu = 85 and standard deviation sigma = 5. A score is at z = 0.5. Find the raw score X.

$$\mu = 85, \quad \sigma = 5, \quad z = 0.5$$

Answer: ____

21. A distribution has mean mu = 69 and standard deviation sigma = 14. A score is at z = 1.0. Find the raw score X.

$$\mu = 69, \quad \sigma = 14, \quad z = 1.0$$

Answer: ____

22. A distribution has mean mu = 80 and standard deviation sigma = 8. A score is at z = 1.5. Find the raw score X.

$$\mu = 80, \quad \sigma = 8, \quad z = 1.5$$

Answer: ____

23. A distribution has mean mu = 97 and standard deviation sigma = 19. A score is at z = 1.5. Find the raw score X.

$$\mu = 97, \quad \sigma = 19, \quad z = 1.5$$

Answer: ____

Z-score from a normal distribution

24. A normal distribution has mean μ = 67 and standard deviation σ = 9. Find the z-score for x = 78 and interpret it.

$$x = 78, \quad \mu = 67, \quad \sigma = 9$$

Answer: ____

25. A normal distribution has mean μ = 54 and standard deviation σ = 8. Find the z-score for x = 60 and interpret it.

$$x = 60, \quad \mu = 54, \quad \sigma = 8$$

Answer: ____

26. A normal distribution has mean μ = 89 and standard deviation σ = 6. Find the z-score for x = 98 and interpret it.

$$x = 98, \quad \mu = 89, \quad \sigma = 6$$

Answer: ____

27. A normal distribution has mean μ = 52 and standard deviation σ = 8. Find the z-score for x = 69 and interpret it.

$$x = 69, \quad \mu = 52, \quad \sigma = 8$$

Answer: ____

28. A normal distribution has mean $\mu = 81$ and standard deviation $\sigma = 5$. Find the z-score for $x = 96$ and interpret it.

$$x = 96, \quad \mu = 81, \quad \sigma = 5$$

Answer: ____

29. A normal distribution has mean $\mu = 76$ and standard deviation $\sigma = 7$. Find the z-score for $x = 74$ and interpret it.

$$x = 74, \quad \mu = 76, \quad \sigma = 7$$

Answer: ____

30. A normal distribution has mean $\mu = 93$ and standard deviation $\sigma = 3$. Find the z-score for $x = 79$ and interpret it.

$$x = 79, \quad \mu = 93, \quad \sigma = 3$$

Answer: ____

MATH110: Normal Distribution

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Topics: Area Under the Normal Curve, Empirical rule (68-95-99.7), Finding a Score from a Percentile (Inverse Normal), Z-score from a normal distribution. All answers verified by independent computation.

Solutions

Empirical rule (68-95-99.7)

1. Test scores are normally distributed with μ = 90 and σ = 4. By the empirical rule, what percent of scores lie within 1 standard deviation(s) of the mean — that is, in the interval [86, 94]?

$$\mu = 90, \quad \sigma = 4, \quad [86,\ 94]$$

→ *Empirical rule: 68% within 1σ, 95% within 2σ, 99.7% within 3σ.*
→ *Within 1σ the interval is [86, 94] → 68%.*

Answer: $$86 = \mu - 1\sigma, \quad 94 = \mu + 1\sigma \ \Rightarrow\ 68\%$$

---

2. Test scores are normally distributed with μ = 54 and σ = 8. By the empirical rule, what percent of scores lie within 1 standard deviation(s) of the mean — that is, in the interval [46, 62]?

$$\mu = 54, \quad \sigma = 8, \quad [46,\ 62]$$

→ *Empirical rule: 68% within 1σ, 95% within 2σ, 99.7% within 3σ.*
→ *Within 1σ the interval is [46, 62] → 68%.*

Answer: $$46 = \mu - 1\sigma, \quad 62 = \mu + 1\sigma \ \Rightarrow\ 68\%$$

---

3. Test scores are normally distributed with μ = 81 and σ = 6. By the empirical rule, what percent of scores lie within 2 standard deviation(s) of the mean — that is, in the interval [69, 93]?

$$\mu = 81, \quad \sigma = 6, \quad [69,\ 93]$$

→ *Empirical rule: 68% within 1σ, 95% within 2σ, 99.7% within 3σ.*
→ *Within 2σ the interval is [69, 93] → 95%.*

Answer: $$69 = \mu - 2\sigma, \quad 93 = \mu + 2\sigma \ \Rightarrow\ 95\%$$

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4. Test scores are normally distributed with μ = 58 and σ = 8. By the empirical rule, what percent of scores lie within 3 standard deviation(s) of the mean — that is, in the interval [34, 82]?

$$\mu = 58, \quad \sigma = 8, \quad [34,\ 82]$$

→ *Empirical rule: 68% within 1σ, 95% within 2σ, 99.7% within 3σ.*
→ *Within 3σ the interval is [34, 82] → 99.7%.*

Answer: $$34 = \mu - 3\sigma, \quad 82 = \mu + 3\sigma \ \Rightarrow\ 99.7\%$$

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5. Test scores are normally distributed with μ = 87 and σ = 5. By the empirical rule, what percent of scores lie within 1 standard deviation(s) of the mean — that is, in the interval [82, 92]?

$$\mu = 87, \quad \sigma = 5, \quad [82,\ 92]$$

→ *Empirical rule: 68% within 1σ, 95% within 2σ, 99.7% within 3σ.*
→ *Within 1σ the interval is [82, 92] → 68%.*

Answer: $$82 = \mu - 1\sigma, \quad 92 = \mu + 1\sigma \ \Rightarrow\ 68\%$$

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6. Test scores are normally distributed with μ = 86 and σ = 5. By the empirical rule, what percent of scores lie within 1 standard deviation(s) of the mean — that is, in the interval [81, 91]?

$$\mu = 86, \quad \sigma = 5, \quad [81,\ 91]$$

→ *Empirical rule: 68% within 1σ, 95% within 2σ, 99.7% within 3σ.*
→ *Within 1σ the interval is [81, 91] → 68%.*

Answer: $81 = \mu - 1\sigma, \quad 91 = \mu + 1\sigma \Rightarrow 68\%$

7. Test scores are normally distributed with μ = 54 and σ = 6. By the empirical rule, what percent of scores lie within 2 standard deviation(s) of the mean — that is, in the interval [42, 66]?

$$\mu = 54, \quad \sigma = 6, \quad [42,\ 66]$$

→ *Empirical rule: 68% within 1σ, 95% within 2σ, 99.7% within 3σ.*
→ *Within 2σ the interval is [42, 66] → 95%.*

Answer: $42 = \mu - 2\sigma, \quad 66 = \mu + 2\sigma \Rightarrow 95\%$

8. Test scores are normally distributed with μ = 57 and σ = 6. By the empirical rule, what percent of scores lie within 3 standard deviation(s) of the mean — that is, in the interval [39, 75]?

$$\mu = 57, \quad \sigma = 6, \quad [39,\ 75]$$

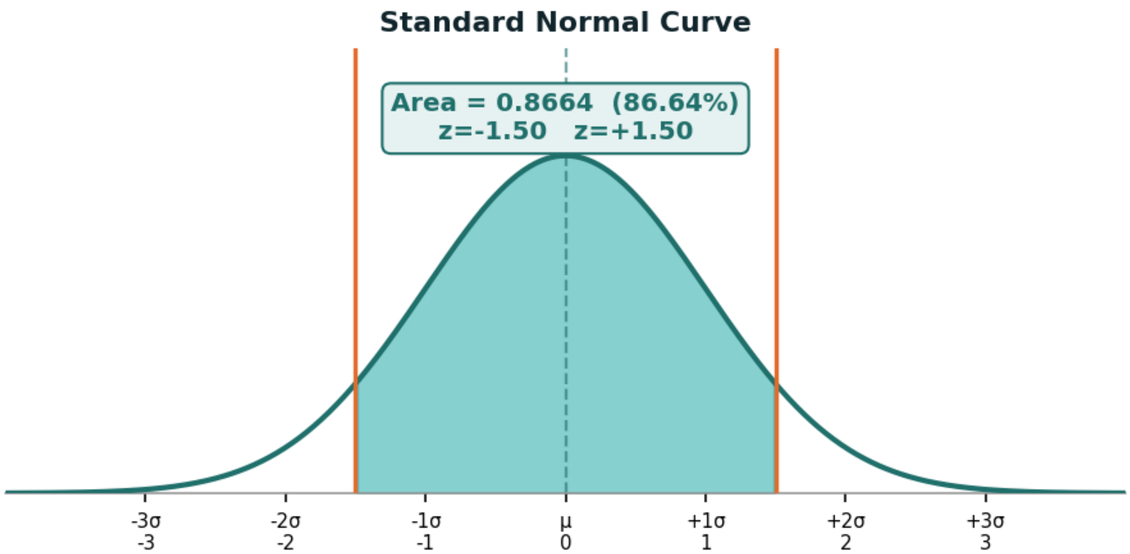
→ *Empirical rule: 68% within 1σ, 95% within 2σ, 99.7% within 3σ.*
→ *Within 3σ the interval is [39, 75] → 99.7%.*

Answer: $39 = \mu - 3\sigma, \quad 75 = \mu + 3\sigma \Rightarrow 99.7\%$

Area Under the Normal Curve

9. Find the proportion of values in a standard normal distribution between z = -1.5 and z = 1.5. Use the z-table.

$z_1 = -1.5, \quad z_2 = 1.5$

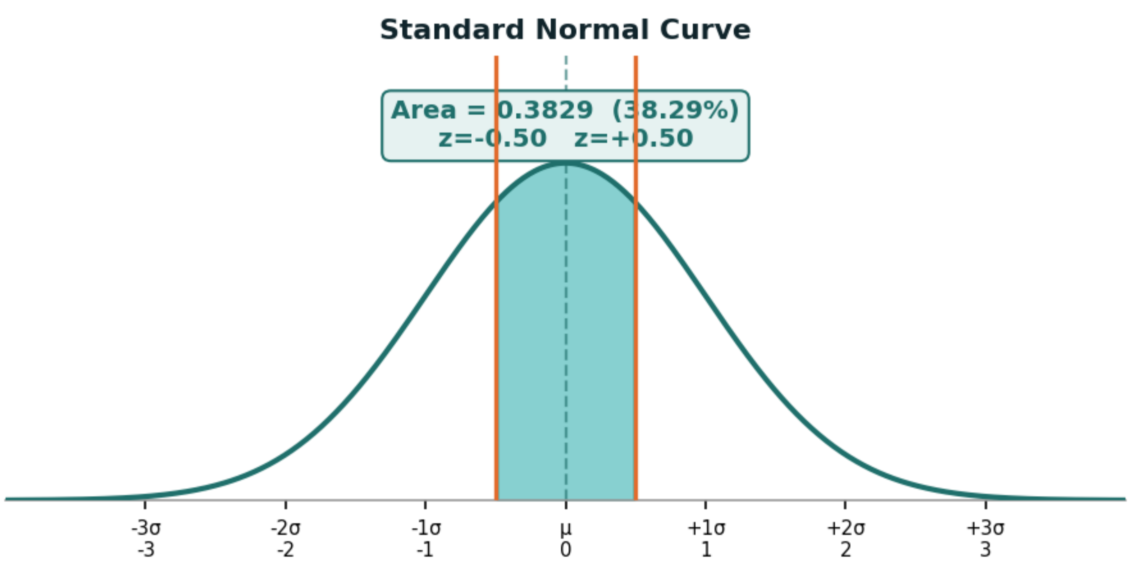


→ Look up Phi(-1.5) = 0.0668 (area to the left of z=-1.5).
→ Look up Phi(1.5) = 0.9332 (area to the left of z=1.5).
→ Area between z=-1.5 and z=1.5 = 0.9332 - 0.0668 = 0.8664.

Answer: $\Phi(1.5) - \Phi(-1.5) = 0.9332 - 0.0668 = 0.8664$

10. Find the proportion of values in a standard normal distribution between z = -0.5 and z = 0.5. Use the z-table.

$z_1 = -0.5, \quad z_2 = 0.5$

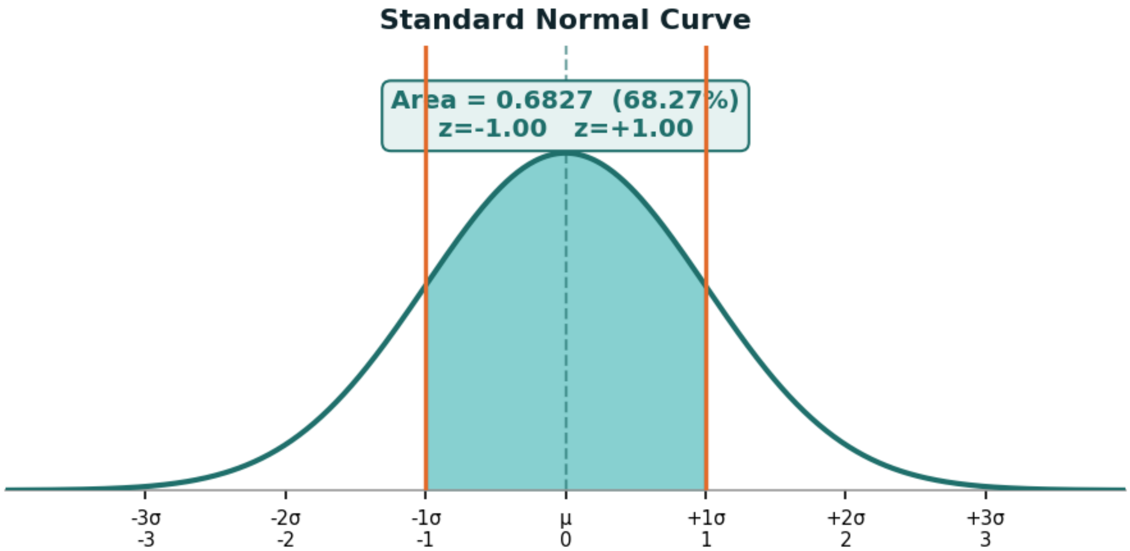


→ Look up Phi(-0.5) = 0.3085 (area to the left of z=-0.5).
→ Look up Phi(0.5) = 0.6915 (area to the left of z=0.5).
→ Area between z=-0.5 and z=0.5 = 0.6915 - 0.3085 = 0.3829.

Answer: $\Phi(0.5) - \Phi(-0.5) = 0.6915 - 0.3085 = 0.3829$

11. Find the proportion of values in a standard normal distribution between z = -1.0 and z = 1.0. Use the z-table.

$z_1 = -1.0, \quad z_2 = 1.0$

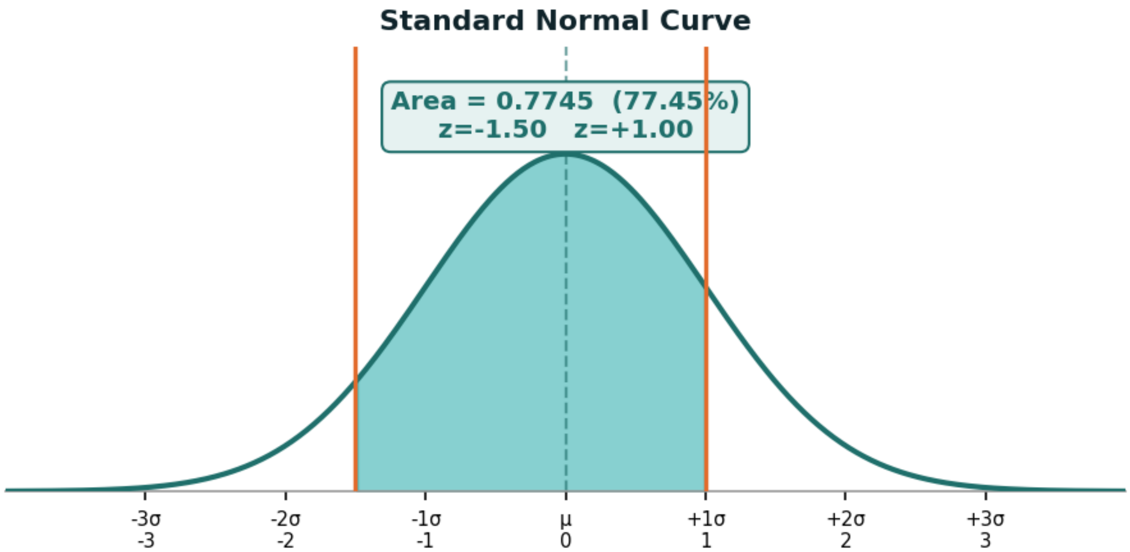


→ *Look up Phi(-1.0) = 0.1587 (area to the left of z=-1.0).*
→ *Look up Phi(1.0) = 0.8413 (area to the left of z=1.0).*
→ *Area between z=-1.0 and z=1.0 = 0.8413 - 0.1587 = 0.6827.*

Answer: $\Phi(1.0) - \Phi(-1.0) = 0.8413 - 0.1587 = 0.6827$

12. Find the proportion of values in a standard normal distribution between z = -1.5 and z = 1.0. Use the z-table.

$z_1 = -1.5, \quad z_2 = 1.0$

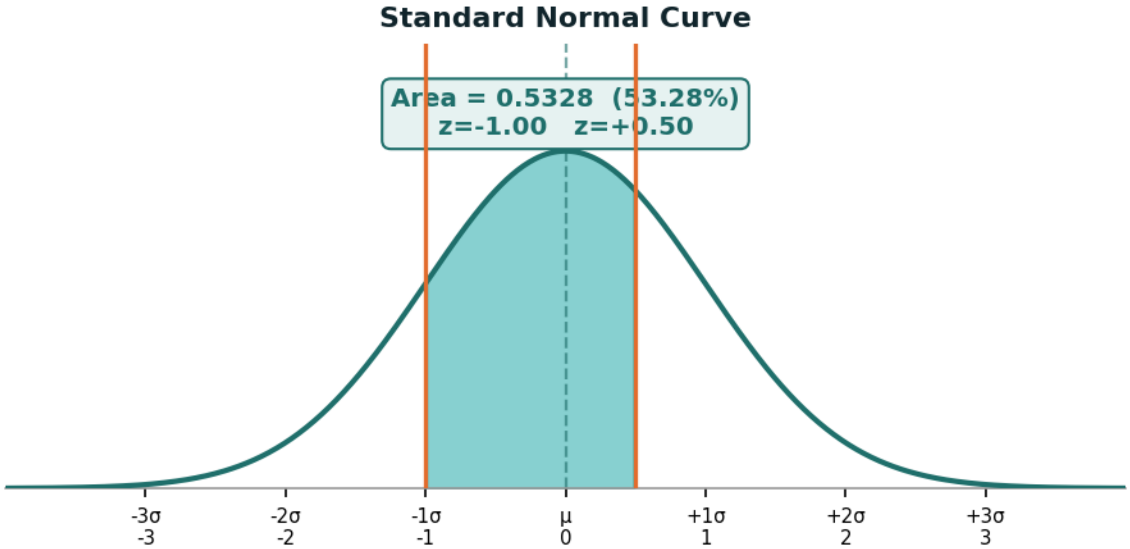


→ *Look up Phi(-1.5) = 0.0668 (area to the left of z=-1.5).*
→ *Look up Phi(1.0) = 0.8413 (area to the left of z=1.0).*
→ *Area between z=-1.5 and z=1.0 = 0.8413 - 0.0668 = 0.7745.*

Answer: $\Phi(1.0) - \Phi(-1.5) = 0.8413 - 0.0668 = 0.7745$

13. Find the proportion of values in a standard normal distribution between z = -1.0 and z = 0.5. Use the z-table.

$z_1 = -1.0, \quad z_2 = 0.5$

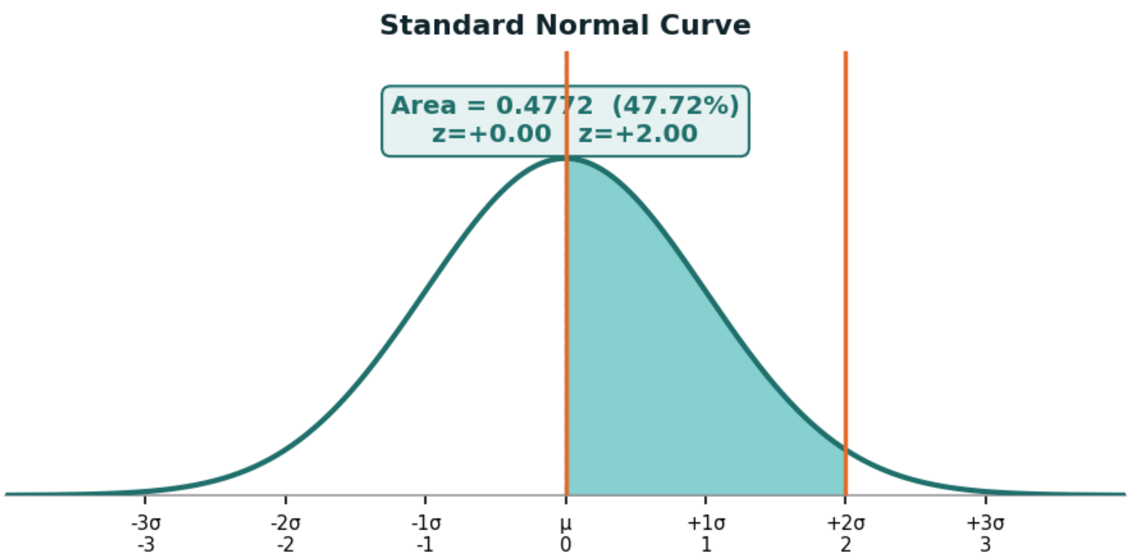


→ *Look up Phi(-1.0) = 0.1587 (area to the left of z=-1.0).*
→ *Look up Phi(0.5) = 0.6915 (area to the left of z=0.5).*
→ *Area between z=-1.0 and z=0.5 = 0.6915 - 0.1587 = 0.5328.*

Answer: $\Phi(0.5) - \Phi(-1.0) = 0.6915 - 0.1587 = 0.5328$

14. Find the proportion of values in a standard normal distribution between z = 0.0 and z = 2.0. Use the z-table.

$z_1 = 0.0, \quad z_2 = 2.0$

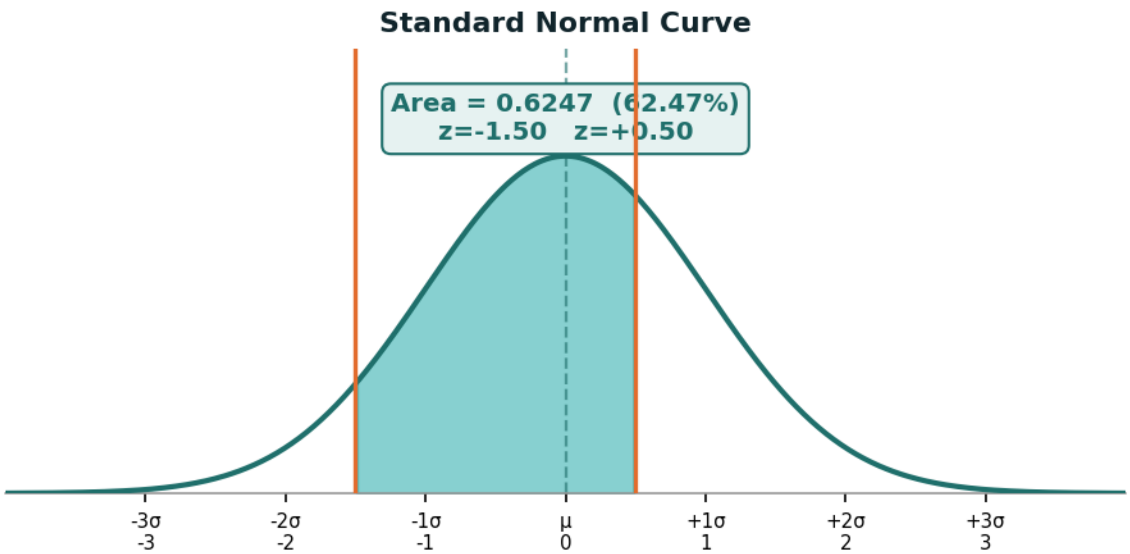


→ *Look up Phi(0.0) = 0.5 (area to the left of z=0.0).*
→ *Look up Phi(2.0) = 0.9772 (area to the left of z=2.0).*
→ *Area between z=0.0 and z=2.0 = 0.9772 - 0.5 = 0.4772.*

Answer: $\Phi(2.0) - \Phi(0.0) = 0.9772 - 0.5 = 0.4772$

15. Find the proportion of values in a standard normal distribution between z = -1.5 and z = 0.5. Use the z-table.

$z_1 = -1.5, \quad z_2 = 0.5$

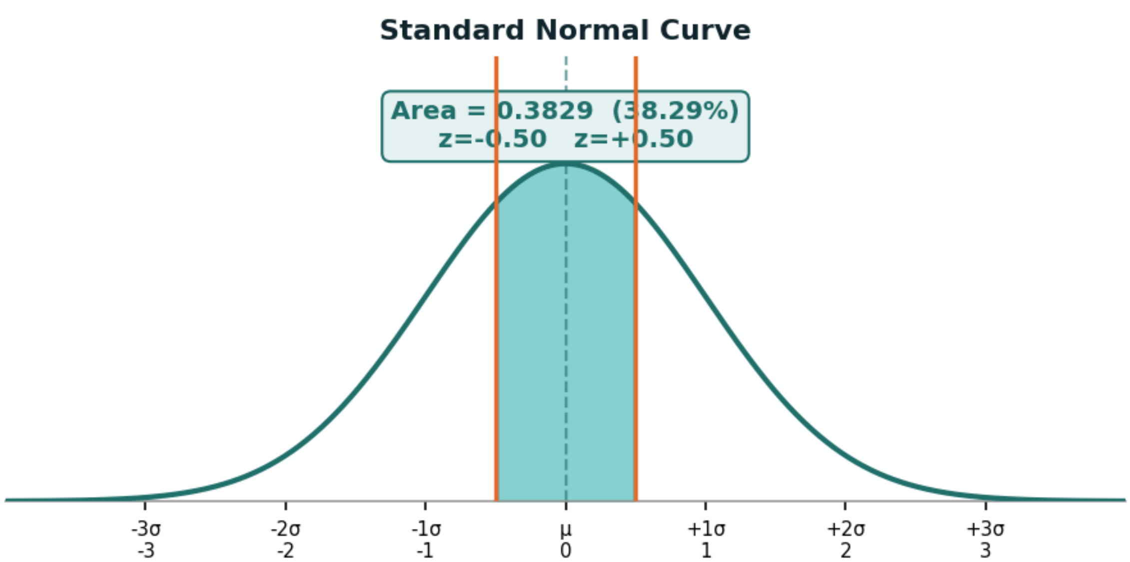


→ *Look up Phi(-1.5) = 0.0668 (area to the left of z=-1.5).*
→ *Look up Phi(0.5) = 0.6915 (area to the left of z=0.5).*
→ *Area between z=-1.5 and z=0.5 = 0.6915 - 0.0668 = 0.6247.*

Answer: $\Phi(0.5) - \Phi(-1.5) = 0.6915 - 0.0668 = 0.6247$

16. Find the proportion of values in a standard normal distribution between z = -0.5 and z = 0.5. Use the z-table.

$z_1 = -0.5, \quad z_2 = 0.5$



→ *Look up Phi(-0.5) = 0.3085 (area to the left of z=-0.5).*
→ *Look up Phi(0.5) = 0.6915 (area to the left of z=0.5).*
→ *Area between z=-0.5 and z=0.5 = 0.6915 - 0.3085 = 0.3829.*

Answer: $\Phi(0.5) - \Phi(-0.5) = 0.6915 - 0.3085 = 0.3829$

Finding a Score from a Percentile (Inverse Normal)

17. A distribution has mean mu = 76 and standard deviation sigma = 8. A score is at z = -1.5. Find the raw score X.

$$\mu = 76, \quad \sigma = 8, \quad z = -1.5$$

*→ Use the formula: X = mu + z * sigma.*
→ X = 76 + (-1.5)(8).
→ X = 64.0.

Answer: $X = 76 + (-1.5)(8) = 64.0$

18. A distribution has mean mu = 85 and standard deviation sigma = 15. A score is at z = -1.5. Find the raw score X.

$$\mu = 85, \quad \sigma = 15, \quad z = -1.5$$

*→ Use the formula: X = mu + z * sigma.*
→ X = 85 + (-1.5)(15).
→ X = 62.5.

Answer: $X = 85 + (-1.5)(15) = 62.5$

19. A distribution has mean mu = 67 and standard deviation sigma = 6. A score is at z = 1.5. Find the raw score X.

$$\mu = 67, \quad \sigma = 6, \quad z = 1.5$$

*→ Use the formula: X = mu + z * sigma.*
→ X = 67 + (1.5)(6).
→ X = 76.0.

Answer: $X = 67 + (1.5)(6) = 76.0$

20. A distribution has mean mu = 85 and standard deviation sigma = 5. A score is at z = 0.5. Find the raw score X.

$$\mu = 85, \quad \sigma = 5, \quad z = 0.5$$

*→ Use the formula: X = mu + z * sigma.*
→ X = 85 + (0.5)(5).
→ X = 87.5.

Answer: $X = 85 + (0.5)(5) = 87.5$

21. A distribution has mean mu = 69 and standard deviation sigma = 14. A score is at z = 1.0. Find the raw score X.

$$\mu = 69, \quad \sigma = 14, \quad z = 1.0$$

*→ Use the formula: X = mu + z * sigma.*
→ X = 69 + (1.0)(14).
→ X = 83.0.

Answer: $X = 69 + (1.0)(14) = 83.0$

22. A distribution has mean mu = 80 and standard deviation sigma = 8. A score is at z = 1.5. Find the raw score X.

$$\mu = 80, \quad \sigma = 8, \quad z = 1.5$$

→ *Use the formula: X = mu + z * sigma.*
→ *X = 80 + (1.5)(8).*
→ *X = 92.0.*

Answer: $X = 80 + (1.5)(8) = 92.0$

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23. A distribution has mean mu = 97 and standard deviation sigma = 19. A score is at z = 1.5. Find the raw score X.

$$\mu = 97, \quad \sigma = 19, \quad z = 1.5$$

→ *Use the formula: X = mu + z * sigma.*
→ *X = 97 + (1.5)(19).*
→ *X = 125.5.*

Answer: $X = 97 + (1.5)(19) = 125.5$

Z-score from a normal distribution

24. A normal distribution has mean $\mu = 67$ and standard deviation $\sigma = 9$. Find the z-score for x = 78 and interpret it.

$$x = 78, \quad \mu = 67, \quad \sigma = 9$$

→ z = (x – μ) / σ.
→ z = (78 – 67) / 9 = 11 / 9 = 1.22.
→ z = 1.22: score is 1.22 standard deviation(s) above the mean.

Answer: $$z = \frac{78 - 67}{9} = 1.22$$

25. A normal distribution has mean $\mu = 54$ and standard deviation $\sigma = 8$. Find the z-score for x = 60 and interpret it.

$$x = 60, \quad \mu = 54, \quad \sigma = 8$$

→ z = (x – μ) / σ.
→ z = (60 – 54) / 8 = 6 / 8 = 0.75.
→ z = 0.75: score is 0.75 standard deviation(s) above the mean.

Answer: $$z = \frac{60 - 54}{8} = 0.75$$

26. A normal distribution has mean $\mu = 89$ and standard deviation $\sigma = 6$. Find the z-score for x = 98 and interpret it.

$$x = 98, \quad \mu = 89, \quad \sigma = 6$$

→ z = (x – μ) / σ.
→ z = (98 – 89) / 6 = 9 / 6 = 1.5.
→ z = 1.5: score is 1.5 standard deviation(s) above the mean.

Answer: $$z = \frac{98 - 89}{6} = 1.5$$

27. A normal distribution has mean $\mu = 52$ and standard deviation $\sigma = 8$. Find the z-score for x = 69 and interpret it.

$$x = 69, \quad \mu = 52, \quad \sigma = 8$$

→ z = (x – μ) / σ.
→ z = (69 – 52) / 8 = 17 / 8 = 2.12.
→ z = 2.12: score is 2.12 standard deviation(s) above the mean.

Answer: $$z = \frac{69 - 52}{8} = 2.12$$

28. A normal distribution has mean $\mu = 81$ and standard deviation $\sigma = 5$. Find the z-score for x = 96 and interpret it.

$$x = 96, \quad \mu = 81, \quad \sigma = 5$$

→ z = (x – μ) / σ.
→ z = (96 – 81) / 5 = 15 / 5 = 3.0.
→ z = 3.0: score is 3.0 standard deviation(s) above the mean.

Answer: $$z = \frac{96 - 81}{5} = 3.0$$

29. A normal distribution has mean $\mu = 76$ and standard deviation $\sigma = 7$. Find the z-score for $x = 74$ and interpret it.

$$x = 74, \quad \mu = 76, \quad \sigma = 7$$

→ z = (x – μ) / σ.
→ z = (74 – 76) / 7 = -2 / 7 = -0.29.
→ z = -0.29: score is 0.29 standard deviation(s) below the mean.

Answer: $$z = \frac{74 - 76}{7} = -0.29$$

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30. A normal distribution has mean $\mu = 93$ and standard deviation $\sigma = 3$. Find the z-score for $x = 79$ and interpret it.

$$x = 79, \quad \mu = 93, \quad \sigma = 3$$

→ z = (x – μ) / σ.
→ z = (79 – 93) / 3 = -14 / 3 = -4.67.
→ z = -4.67: score is 4.67 standard deviation(s) below the mean.

Answer: $$z = \frac{79 - 93}{3} = -4.67$$
