



MATH110: Probability

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Learning Objectives

- Identify the sample space of a random experiment
- Compute classical (theoretical) probability
- Express probabilities as fractions and decimals

Simplify each expression completely. Show all steps and circle your final answer.

Binomial Probability

1. In 15 independent trials each with success probability 0.2, find $P(X = 1)$.

$$n = 15, \quad k = 1, \quad p = 0.2, \quad q = 0.8$$

Answer: _____

2. In 10 independent trials each with success probability 0.5, find $P(X = 1)$.

$$n = 10, \quad k = 1, \quad p = 0.5, \quad q = 0.5$$

Answer: _____

3. In 9 independent trials each with success probability 0.5, find $P(X = 1)$.

$$n = 9, \quad k = 1, \quad p = 0.5, \quad q = 0.5$$

Answer: _____

4. In 5 independent trials each with success probability 0.2, find $P(X = 3)$.

$$n = 5, \quad k = 3, \quad p = 0.2, \quad q = 0.8$$

Answer: _____

5. In 14 independent trials each with success probability 0.2, find $P(X = 2)$.

$$n = 14, \quad k = 2, \quad p = 0.2, \quad q = 0.8$$

Answer: _____

6. In 6 independent trials each with success probability 0.5, find $P(X = 1)$.

$$n = 6, \quad k = 1, \quad p = 0.5, \quad q = 0.5$$

Answer: _____

Classical (theoretical) probability

7. A bag contains 2 red, 6 blue, and 4 green marbles. One marble is drawn at random. Find $P(\text{red})$.

$$\text{red} = 2, \text{ blue} = 6, \text{ green} = 4$$

Answer: _____

8. A bag contains 7 red, 4 blue, and 6 green marbles. One marble is drawn at random. Find $P(\text{red})$.

$$\text{red} = 7, \text{ blue} = 4, \text{ green} = 6$$

Answer: _____

9. A bag contains 5 red, 9 blue, and 9 green marbles. One marble is drawn at random. Find $P(\text{red})$.

$$\text{red} = 5, \text{ blue} = 9, \text{ green} = 9$$

Answer: _____

10. A bag contains 5 red, 5 blue, and 5 green marbles. One marble is drawn at random. Find $P(\text{red})$.

$$\text{red} = 5, \text{ blue} = 5, \text{ green} = 5$$

Answer: _____

11. A bag contains 9 red, 9 blue, and 6 green marbles. One marble is drawn at random. Find $P(\text{red})$.

$$\text{red} = 9, \text{ blue} = 9, \text{ green} = 6$$

Answer: _____

12. A bag contains 9 red, 3 blue, and 5 green marbles. One marble is drawn at random. Find $P(\text{red})$.

$$\text{red} = 9, \text{ blue} = 3, \text{ green} = 5$$

Answer: _____

Geometric Distribution

13. Each trial has probability 0.5 of success. Find the probability that the first success occurs on trial 4.

$$k = 4, \quad p = 0.5, \quad q = 0.5$$

Answer: _____

14. Each trial has probability 0.3 of success. Find the probability that the first success occurs on trial 1.

$$k = 1, \quad p = 0.3, \quad q = 0.7$$

Answer: _____

15. Each trial has probability 0.4 of success. Find the probability that the first success occurs on trial 2.

$$k = 2, \quad p = 0.4, \quad q = 0.6$$

Answer: _____

16. Each trial has probability 0.2 of success. Find the probability that the first success occurs on trial 2.

$$k = 2, \quad p = 0.2, \quad q = 0.8$$

Answer: _____

17. Each trial has probability 0.2 of success. Find the probability that the first success occurs on trial 4.

$$k = 4, \quad p = 0.2, \quad q = 0.8$$

Answer: _____

18. Each trial has probability 0.2 of success. Find the probability that the first success occurs on trial 6.

$$k = 6, \quad p = 0.2, \quad q = 0.8$$

Answer: _____

Expected Value of a Discrete Random Variable

19. A random variable X has the following probability distribution. Values: [2, 4, 6, 8], Probabilities: [0.3, 0.3, 0.2, 0.2]. Find E(X).

$$(x_i, p_i) : (2, 0.3), (4, 0.3), (6, 0.2), (8, 0.2)$$

Answer: _____

20. A random variable X has the following probability distribution. Values: [0, 2, 4, 6], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find E(X).

$$(x_i, p_i) : (0, 0.25), (2, 0.25), (4, 0.25), (6, 0.25)$$

Answer: _____

21. A random variable X has the following probability distribution. Values: [0, 1, 2, 3], Probabilities: [0.2, 0.3, 0.3, 0.2]. Find E(X).

$$(x_i, p_i) : (0, 0.2), (1, 0.3), (2, 0.3), (3, 0.2)$$

Answer: _____

22. A random variable X has the following probability distribution. Values: [2, 4, 6, 8], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find E(X).

$$(x_i, p_i) : (2, 0.25), (4, 0.25), (6, 0.25), (8, 0.25)$$

Answer: _____

23. A random variable X has the following probability distribution. Values: [0, 2, 4, 6], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find E(X).

$$(x_i, p_i) : (0, 0.25), (2, 0.25), (4, 0.25), (6, 0.25)$$

Answer: _____

24. A random variable X has the following probability distribution. Values: [0, 1, 2, 3], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find E(X).

$$(x_i, p_i) : (0, 0.25), (1, 0.25), (2, 0.25), (3, 0.25)$$

Answer: _____

Conditional Probability from a Two-Way Table

25. A two-way table shows: Group A has 7 successes and 17 failures; Group B has 6 successes and 23 failures. Find P(success | Group A).

Row1: 7, 17 Row2: 6, 23

Two-Way Table

	Success	Failure	Total
Group A	7	17	24
Group B	6	23	29
Total	13	40	53

Answer: _____

26. A two-way table shows: Group A has 12 successes and 21 failures; Group B has 22 successes and 10 failures. Find $P(\text{success} \mid \text{Group A})$.

Row1: 12, 21 Row2: 22, 10

Two-Way Table

	Success	Failure	Total
Group A	12	21	33
Group B	22	10	32
Total	34	31	65

Answer: _____

27. A two-way table shows: Group A has 22 successes and 5 failures; Group B has 20 successes and 25 failures. Find $P(\text{success} \mid \text{Group A})$.

Row1: 22, 5 Row2: 20, 25

Two-Way Table

	Success	Failure	Total
Group A	22	5	27
Group B	20	25	45
Total	42	30	72

Answer: _____

28. A two-way table shows: Group A has 20 successes and 10 failures; Group B has 22 successes and 11 failures. Find $P(\text{success} \mid \text{Group A})$.

Row1: 20, 10 Row2: 22, 11

Two-Way Table

	Success	Failure	Total
Group A	20	10	30
Group B	22	11	33
Total	42	21	63

Answer: _____

29. A two-way table shows: Group A has 7 successes and 14 failures; Group B has 18 successes and 12 failures. Find $P(\text{success} \mid \text{Group A})$.

Row1: 7, 14 Row2: 18, 12

Two-Way Table

	Success	Failure	Total
Group A	7	14	21
Group B	18	12	30
Total	25	26	51

Answer: _____

30. A two-way table shows: Group A has 15 successes and 21 failures; Group B has 24 successes and 30 failures. Find $P(\text{success} \mid \text{Group A})$.

Row1: 15, 21 Row2: 24, 30

Two-Way Table

	Success	Failure	Total
Group A	15	21	36
Group B	24	30	54
Total	39	51	90

Answer: _____



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ANSWER KEY & SOLUTIONS

Topics: Expected Value of a Discrete Random Variable, Binomial Probability, Classical (theoretical) probability, Geometric Distribution, Conditional Probability from a Two-Way Table. All answers verified by independent computation.

Solutions

Binomial Probability

1. In 15 independent trials each with success probability 0.2, find $P(X = 1)$.

$$n = 15, \quad k = 1, \quad p = 0.2, \quad q = 0.8$$

→ Identify: $n=15, k=1, p=0.2, q=1-p=0.8$.

→ Compute $C(n,k) = C(15,1) = 15$.

→ Compute $p^k = 0.2^1$.

→ Compute $q^{(n-k)} = 0.8^{(15-1)}$.

→ $P(X=1) = 15 * 0.2^1 * 0.8^{(15-1)} = 0.1319$.

Answer: $P(X = 1) = 0.1319$

2. In 10 independent trials each with success probability 0.5, find $P(X = 1)$.

$$n = 10, \quad k = 1, \quad p = 0.5, \quad q = 0.5$$

→ Identify: $n=10, k=1, p=0.5, q=1-p=0.5$.

→ Compute $C(n,k) = C(10,1) = 10$.

→ Compute $p^k = 0.5^1$.

→ Compute $q^{(n-k)} = 0.5^{(10-1)}$.

→ $P(X=1) = 10 * 0.5^1 * 0.5^{(10-1)} = 0.0098$.

Answer: $P(X = 1) = 0.0098$

3. In 9 independent trials each with success probability 0.5, find $P(X = 1)$.

$$n = 9, \quad k = 1, \quad p = 0.5, \quad q = 0.5$$

→ Identify: $n=9, k=1, p=0.5, q=1-p=0.5$.

→ Compute $C(n,k) = C(9,1) = 9$.

→ Compute $p^k = 0.5^1$.

→ Compute $q^{(n-k)} = 0.5^{(9-1)}$.

→ $P(X=1) = 9 * 0.5^1 * 0.5^{(9-1)} = 0.0176$.

Answer: $P(X = 1) = 0.0176$

4. In 5 independent trials each with success probability 0.2, find $P(X = 3)$.

$$n = 5, \quad k = 3, \quad p = 0.2, \quad q = 0.8$$

→ Identify: $n=5, k=3, p=0.2, q=1-p=0.8$.

→ Compute $C(n,k) = C(5,3) = 10$.

→ Compute $p^k = 0.2^3$.

→ Compute $q^{(n-k)} = 0.8^{(5-3)}$.

→ $P(X=3) = 10 * 0.2^3 * 0.8^{(5-3)} = 0.0512$.

Answer: $P(X = 3) = 0.0512$

5. In 14 independent trials each with success probability 0.2, find $P(X = 2)$.

$$n = 14, \quad k = 2, \quad p = 0.2, \quad q = 0.8$$

→ Identify: $n=14, k=2, p=0.2, q=1-p=0.8$.

→ Compute $C(n,k) = C(14,2) = 91$.

→ Compute $p^k = 0.2^2$.

→ Compute $q^{(n-k)} = 0.8^{(14-2)}$.

→ $P(X=2) = 91 * 0.2^2 * 0.8^{(14-2)} = 0.2501$.

Answer: $P(X = 2) = 0.2501$

6. In 6 independent trials each with success probability 0.5, find $P(X = 1)$.

$$n = 6, \quad k = 1, \quad p = 0.5, \quad q = 0.5$$

→ Identify: $n=6, k=1, p=0.5, q=1-p=0.5$.

→ Compute $C(n,k) = C(6,1) = 6$.

→ Compute $p^k = 0.5^1$.

→ Compute $q^{(n-k)} = 0.5^{(6-1)}$.

→ $P(X=1) = 6 * 0.5^1 * 0.5^{(6-1)} = 0.0938$.

Answer: $P(X = 1) = 0.0938$

Classical (theoretical) probability

7. A bag contains 2 red, 6 blue, and 4 green marbles. One marble is drawn at random. Find $P(\text{red})$.

$$\text{red} = 2, \text{ blue} = 6, \text{ green} = 4$$

→ Count the total outcomes: $2 + 6 + 4 = 12$.

→ $P(\text{red}) = \text{favorable} / \text{total} = 2/12 = 1/6$.

→ As a decimal: $P(\text{red}) = 0.1667$ (approx).

Answer: $P(\text{red}) = \frac{2}{12} = 1/6 \approx 0.1667$

8. A bag contains 7 red, 4 blue, and 6 green marbles. One marble is drawn at random. Find $P(\text{red})$.

$$\text{red} = 7, \text{ blue} = 4, \text{ green} = 6$$

→ Count the total outcomes: $7 + 4 + 6 = 17$.

→ $P(\text{red}) = \text{favorable} / \text{total} = 7/17 = 7/17$.

→ As a decimal: $P(\text{red}) = 0.4118$ (approx).

Answer: $P(\text{red}) = \frac{7}{17} = 7/17 \approx 0.4118$

9. A bag contains 5 red, 9 blue, and 9 green marbles. One marble is drawn at random. Find $P(\text{red})$.

$$\text{red} = 5, \text{ blue} = 9, \text{ green} = 9$$

→ Count the total outcomes: $5 + 9 + 9 = 23$.

→ $P(\text{red}) = \text{favorable} / \text{total} = 5/23 = 5/23$.

→ As a decimal: $P(\text{red}) = 0.2174$ (approx).

Answer: $P(\text{red}) = \frac{5}{23} = 5/23 \approx 0.2174$

10. A bag contains 5 red, 5 blue, and 5 green marbles. One marble is drawn at random. Find $P(\text{red})$.

$$\text{red} = 5, \text{ blue} = 5, \text{ green} = 5$$

→ Count the total outcomes: $5 + 5 + 5 = 15$.

→ $P(\text{red}) = \text{favorable} / \text{total} = 5/15 = 1/3$.

→ As a decimal: $P(\text{red}) = 0.3333$ (approx).

Answer: $P(\text{red}) = \frac{5}{15} = 1/3 \approx 0.3333$

11. A bag contains 9 red, 9 blue, and 6 green marbles. One marble is drawn at random. Find $P(\text{red})$.

$$\text{red} = 9, \text{ blue} = 9, \text{ green} = 6$$

→ Count the total outcomes: $9 + 9 + 6 = 24$.

→ $P(\text{red}) = \text{favorable} / \text{total} = 9/24 = 3/8$.

→ As a decimal: $P(\text{red}) = 0.375$ (approx).

Answer: $P(\text{red}) = \frac{9}{24} = 3/8 \approx 0.375$

12. A bag contains 9 red, 3 blue, and 5 green marbles. One marble is drawn at random. Find $P(\text{red})$.

red = 9, blue = 3, green = 5

→ Count the total outcomes: $9 + 3 + 5 = 17$.

→ $P(\text{red}) = \text{favorable} / \text{total} = 9/17 = 9/17$.

→ As a decimal: $P(\text{red}) = 0.5294$ (approx).

Answer: $P(\text{red}) = \frac{9}{17} = 9/17 \approx 0.5294$

Geometric Distribution

13. Each trial has probability 0.5 of success. Find the probability that the first success occurs on trial 4.

$$k = 4, \quad p = 0.5, \quad q = 0.5$$

→ Identify: $p=0.5$, $q=1-p=0.5$, $k=4$.

$$\rightarrow P(X=4) = q^{k-1} * p = 0.5^{(4-1)} * 0.5.$$

$$\rightarrow P(X=4) = 0.0625.$$

Answer: $P(X = 4) = 0.0625$

14. Each trial has probability 0.3 of success. Find the probability that the first success occurs on trial 1.

$$k = 1, \quad p = 0.3, \quad q = 0.7$$

→ Identify: $p=0.3$, $q=1-p=0.7$, $k=1$.

$$\rightarrow P(X=1) = q^{k-1} * p = 0.7^{(1-1)} * 0.3.$$

$$\rightarrow P(X=1) = 0.3.$$

Answer: $P(X = 1) = 0.3$

15. Each trial has probability 0.4 of success. Find the probability that the first success occurs on trial 2.

$$k = 2, \quad p = 0.4, \quad q = 0.6$$

→ Identify: $p=0.4$, $q=1-p=0.6$, $k=2$.

$$\rightarrow P(X=2) = q^{k-1} * p = 0.6^{(2-1)} * 0.4.$$

$$\rightarrow P(X=2) = 0.24.$$

Answer: $P(X = 2) = 0.24$

16. Each trial has probability 0.2 of success. Find the probability that the first success occurs on trial 2.

$$k = 2, \quad p = 0.2, \quad q = 0.8$$

→ Identify: $p=0.2$, $q=1-p=0.8$, $k=2$.

$$\rightarrow P(X=2) = q^{k-1} * p = 0.8^{(2-1)} * 0.2.$$

$$\rightarrow P(X=2) = 0.16.$$

Answer: $P(X = 2) = 0.16$

17. Each trial has probability 0.2 of success. Find the probability that the first success occurs on trial 4.

$$k = 4, \quad p = 0.2, \quad q = 0.8$$

→ Identify: $p=0.2$, $q=1-p=0.8$, $k=4$.

$$\rightarrow P(X=4) = q^{k-1} * p = 0.8^{(4-1)} * 0.2.$$

$$\rightarrow P(X=4) = 0.1024.$$

Answer: $P(X = 4) = 0.1024$

18. Each trial has probability 0.2 of success. Find the probability that the first success occurs on trial 6.

$$k = 6, \quad p = 0.2, \quad q = 0.8$$

→ Identify: $p=0.2$, $q=1-p=0.8$, $k=6$.

→ $P(X=6) = q^{(k-1)} * p = 0.8^{(6-1)} * 0.2$.

→ $P(X=6) = 0.0655$.

Answer: $P(X = 6) = 0.0655$

Expected Value of a Discrete Random Variable

19. A random variable X has the following probability distribution. Values: [2, 4, 6, 8], Probabilities: [0.3, 0.3, 0.2, 0.2]. Find $E(X)$.

$$(x_i, p_i) : (2, 0.3), (4, 0.3), (6, 0.2), (8, 0.2)$$

→ Multiply each value by its probability: $2(0.3) + 4(0.3) + 6(0.2) + 8(0.2)$.

→ Sum the products: $E(X) = 4.6$.

Answer: $E(X) = 4.6$

20. A random variable X has the following probability distribution. Values: [0, 2, 4, 6], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find $E(X)$.

$$(x_i, p_i) : (0, 0.25), (2, 0.25), (4, 0.25), (6, 0.25)$$

→ Multiply each value by its probability: $0(0.25) + 2(0.25) + 4(0.25) + 6(0.25)$.

→ Sum the products: $E(X) = 3.0$.

Answer: $E(X) = 3.0$

21. A random variable X has the following probability distribution. Values: [0, 1, 2, 3], Probabilities: [0.2, 0.3, 0.3, 0.2]. Find $E(X)$.

$$(x_i, p_i) : (0, 0.2), (1, 0.3), (2, 0.3), (3, 0.2)$$

→ Multiply each value by its probability: $0(0.2) + 1(0.3) + 2(0.3) + 3(0.2)$.

→ Sum the products: $E(X) = 1.5$.

Answer: $E(X) = 1.5$

22. A random variable X has the following probability distribution. Values: [2, 4, 6, 8], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find $E(X)$.

$$(x_i, p_i) : (2, 0.25), (4, 0.25), (6, 0.25), (8, 0.25)$$

→ Multiply each value by its probability: $2(0.25) + 4(0.25) + 6(0.25) + 8(0.25)$.

→ Sum the products: $E(X) = 5.0$.

Answer: $E(X) = 5.0$

23. A random variable X has the following probability distribution. Values: [0, 2, 4, 6], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find $E(X)$.

$$(x_i, p_i) : (0, 0.25), (2, 0.25), (4, 0.25), (6, 0.25)$$

→ Multiply each value by its probability: $0(0.25) + 2(0.25) + 4(0.25) + 6(0.25)$.

→ Sum the products: $E(X) = 3.0$.

Answer: $E(X) = 3.0$

24. A random variable X has the following probability distribution. Values: [0, 1, 2, 3], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find $E(X)$.

$(x_i, p_i) : (0, 0.25), (1, 0.25), (2, 0.25), (3, 0.25)$

→ Multiply each value by its probability: $0(0.25) + 1(0.25) + 2(0.25) + 3(0.25)$.

→ Sum the products: $E(X) = 1.5$.

Answer: $E(X) = 1.5$

Conditional Probability from a Two-Way Table

25. A two-way table shows: Group A has 7 successes and 17 failures; Group B has 6 successes and 23 failures. Find $P(\text{success} \mid \text{Group A})$.

Row1: 7, 17 Row2: 6, 23

Two-Way Table

	Success	Failure	Total
Group A	7	17	24
Group B	6	23	29
Total	13	40	53

→ Column total for Group A: $7 + 6 = 13$.

→ Successes in Group A: 7.

→ $P(\text{success} \mid \text{Group A}) = 7 / 13 = 0.5385$.

Answer: $P(\text{success} \mid \text{Group A}) = \frac{7}{13} = 0.5385$

26. A two-way table shows: Group A has 12 successes and 21 failures; Group B has 22 successes and 10 failures. Find $P(\text{success} | \text{Group A})$.

Row1: 12, 21 Row2: 22, 10

Two-Way Table

	Success	Failure	Total
Group A	12	21	33
Group B	22	10	32
Total	34	31	65

→ Column total for Group A: $12 + 22 = 34$.

→ Successes in Group A: 12.

→ $P(\text{success} | \text{Group A}) = 12 / 34 = 0.3529$.

Answer: $P(\text{success} | \text{Group A}) = \frac{12}{34} = 0.3529$

27. A two-way table shows: Group A has 22 successes and 5 failures; Group B has 20 successes and 25 failures. Find $P(\text{success} | \text{Group A})$.

Row1: 22, 5 Row2: 20, 25

Two-Way Table

	Success	Failure	Total
Group A	22	5	27
Group B	20	25	45
Total	42	30	72

→ Column total for Group A: $22 + 20 = 42$.

→ Successes in Group A: 22.

→ $P(\text{success} | \text{Group A}) = 22 / 42 = 0.5238$.

Answer: $P(\text{success} | \text{Group A}) = \frac{22}{42} = 0.5238$

28. A two-way table shows: Group A has 20 successes and 10 failures; Group B has 22 successes and 11 failures. Find $P(\text{success} \mid \text{Group A})$.

Row1: 20, 10 Row2: 22, 11

Two-Way Table

	Success	Failure	Total
Group A	20	10	30
Group B	22	11	33
Total	42	21	63

→ Column total for Group A: $20 + 22 = 42$.

→ Successes in Group A: 20.

→ $P(\text{success} \mid \text{Group A}) = 20 / 42 = 0.4762$.

Answer: $P(\text{success} \mid \text{Group A}) = \frac{20}{42} = 0.4762$

29. A two-way table shows: Group A has 7 successes and 14 failures; Group B has 18 successes and 12 failures. Find $P(\text{success} \mid \text{Group A})$.

Row1: 7, 14 Row2: 18, 12

Two-Way Table

	Success	Failure	Total
Group A	7	14	21
Group B	18	12	30
Total	25	26	51

→ Column total for Group A: $7 + 18 = 25$.

→ Successes in Group A: 7.

→ $P(\text{success} \mid \text{Group A}) = 7 / 25 = 0.28$.

Answer: $P(\text{success} \mid \text{Group A}) = \frac{7}{25} = 0.28$

30. A two-way table shows: Group A has 15 successes and 21 failures; Group B has 24 successes and 30 failures. Find $P(\text{success} | \text{Group A})$.

Row1: 15, 21 Row2: 24, 30

Two-Way Table

	Success	Failure	Total
Group A	15	21	36
Group B	24	30	54
Total	39	51	90

→ Column total for Group A: $15 + 24 = 39$.

→ Successes in Group A: 15.

→ $P(\text{success} | \text{Group A}) = 15 / 39 = 0.3846$.

Answer: $P(\text{success} | \text{Group A}) = \frac{15}{39} = 0.3846$
