MATH120: Applications of Derivatives

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Learning Objectives

- Find critical points by setting f'(x) = 0
- Classify intervals where f is increasing or decreasing
- Apply the First and Second Derivative Tests
- Solve optimization problems using calculus

Simplify each expression completely. Show all steps and circle your final answer.

Optimization

1. A company's profit function is P(x) = -4x^2 + 10x + 47, where x is units produced. Find the production level that maximizes profit.

$$-4x^2 + 10x + 47$$

Answer: _____

2. Revenue is R(x) = -3x^2 + 21x + 18. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-3x^2 + 21x + 18$$

Answer: _____

3. A ball is thrown upward. Its height is h(t) = -10t^2 + 53t + 8 feet. Find the time t at which it reaches maximum height, then find that maximum height.

$$-10x^2 + 53x + 8$$

Answer: _____

4. A company's profit function is P(x) = -4x^2 + 22x + 2, where x is units produced. Find the production level that maximizes profit.

$$-4x^2 + 22x + 2$$

Answer: _____

5. Revenue is R(x) = -2x^2 + 14x + 55. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-2x^2 + 14x + 55$$

Answer: _____

6. A ball is thrown upward. Its height is h(t) = -8t^2 + 40t + 2 feet. Find the time t at which it reaches maximum height, then find that maximum height.

$$-8x^2 + 40x + 2$$

Answer: _____

7. A company's profit function is P(x) = -1x^2 + 37x + 17, where x is units produced. Find the production level that maximizes profit.

$$-1x^2 + 37x + 17$$

Answer: _____

8. Revenue is R(x) = -3x^2 + 28x + 70. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-3x^2 + 28x + 70$$

Answer: _____

9. A ball is thrown upward. Its height is h(t) = -12t^2 + 23t + 8 feet. Find the time t at which it reaches maximum height, then find that maximum height.

$$-12x^2 + 23x + 8$$

Answer: _____

10. A company's profit function is P(x) = -3x^2 + 24x + 35, where x is units produced. Find the production level that maximizes profit.

$$-3x^2 + 24x + 35$$

Answer: _____

11. Revenue is R(x) = -3x^2 + 18x + 19. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-3x^2 + 18x + 19$$

Answer: _____

12. A ball is thrown upward. Its height is h(t) = -8t^2 + 20t + 7 feet. Find the time t at which it reaches maximum height, then find that maximum height.

$$-8x^2 + 20x + 7$$

Answer: _____

13. A company's profit function is P(x) = -3x^2 + 16x + 47, where x is units produced. Find the production level that maximizes profit.

$$-3x^2 + 16x + 47$$

Answer: _____

14. Revenue is R(x) = -3x^2 + 14x + 58. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-3x^2 + 14x + 58$$

Answer: _____

15. A ball is thrown upward. Its height is h(t) = -12t^2 + 38t + 9 feet. Find the time t at which it reaches maximum height, then find that maximum height.

$$-12x^2 + 38x + 9$$

Answer: _____

16. A company's profit function is P(x) = -3x^2 + 12x + 15, where x is units produced. Find the production level that maximizes profit.

$$-3x^2 + 12x + 15$$

Answer: _____

17. Revenue is R(x) = -2x^2 + 26x + 37. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-2x^2 + 26x + 37$$

Answer: _____

18. A ball is thrown upward. Its height is h(t) = -9t^2 + 27t + 10 feet. Find the time t at which it reaches maximum height, then find that maximum height.

$$-9x^2 + 27x + 10$$

Answer: _____

19. A company's profit function is P(x) = -4x^2 + 19x + 27, where x is units produced. Find the production level that maximizes profit.

$$-4x^2 + 19x + 27$$

Answer: _____

20. Revenue is R(x) = -3x^2 + 15x + 52. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-3x^2 + 15x + 52$$

Answer: _____

21. A ball is thrown upward. Its height is h(t) = -9t^2 + 52t + 1 feet. Find the time t at which it reaches maximum height, then find that maximum height.

$$-9x^2 + 52x + 1$$

Answer: _____

22. A company's profit function is P(x) = -4x^2 + 19x + 44, where x is units produced. Find the production level that maximizes profit.

$$-4x^2 + 19x + 44$$

Answer: _____

23. Revenue is R(x) = -2x^2 + 28x + 79. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-2x^2 + 28x + 79$$

Answer: _____

Marginal analysis

24. Total cost is C(x) = 2x^3 + -1x^2 + 10x + 115. Find the marginal cost MC = C'(1), and determine whether cost is increasing or decreasing at that production level.

$$f(x) = 2x^3 + \ -1x^2 + 10x + 115,\ x = 1$$

Answer: _____

25. Total cost is C(x) = 1x^3 + -2x^2 + 9x + 78. Find the marginal cost MC = C'(2), and determine whether cost is increasing or decreasing at that production level.

$$f(x) = 1x^3 + \ -2x^2 + 9x + 78,\ x = 2$$

Answer: _____

26. Total cost is C(x) = 1x^3 + -1x^2 + 11x + 173. Find the marginal cost MC = C'(3), and determine whether cost is increasing or decreasing at that production level.

$$f(x) = 1x^3 + \ -1x^2 + 11x + 173,\ x = 3$$

Answer: _____

27. Total cost is C(x) = 1x^3 + -2x^2 + 12x + 54. Find the marginal cost MC = C'(1), and determine whether cost is increasing or decreasing at that production level.

$$f(x) = 1x^3 + -2x^2 + 12x + 54,\ x = 1$$

Answer: _____

28. Total cost is C(x) = 2x^3 + -3x^2 + 11x + 105. Find the marginal cost MC = C'(2), and determine whether cost is increasing or decreasing at that production level.

$$f(x) = 2x^3 + -3x^2 + 11x + 105,\ x = 2$$

Answer: _____

29. Total cost is C(x) = 2x^3 + -2x^2 + 7x + 181. Find the marginal cost MC = C'(1), and determine whether cost is increasing or decreasing at that production level.

$$f(x) = 2x^3 + -2x^2 + 7x + 181,\ x = 1$$

Answer: _____

30. Total cost is C(x) = 1x^3 + -4x^2 + 15x + 92. Find the marginal cost MC = C'(1), and determine whether cost is increasing or decreasing at that production level.

$$f(x) = 1x^3 + -4x^2 + 15x + 92,\ x = 1$$

Answer: _____

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ANSWER KEY & SOLUTIONS

Topics: Marginal analysis, Optimization. All answers verified by independent computation.

Solutions

Optimization

1. A company's profit function is P(x) = -4x^2 + 10x + 47, where x is units produced. Find the production level that maximizes profit.

$$-4x^2 + 10x + 47$$

→ *Take the derivative: P'(x) = -8x + 10.*
→ *Set P'(x) = 0: -8x + 10 = 0 → x = 5/4.*
→ *Since the leading coefficient is negative, x = 5/4 is a maximum. Max profit = 213/4.*

Answer: $x^* = -\frac{10}{-8} = \frac{5}{4},\ f(x^*) = \frac{213}{4}$

2. Revenue is R(x) = -3x^2 + 21x + 18. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-3x^2 + 21x + 18$$

→ *Differentiate: R'(x) = -6x + 21.*
→ *Set equal to zero and solve: x = 7/2.*
→ *Maximum revenue = R(7/2) = 219/4.*

Answer: $x^* = -\frac{21}{-6} = \frac{7}{2},\ f(x^*) = \frac{219}{4}$

3. A ball is thrown upward. Its height is h(t) = -10t^2 + 53t + 8 feet. Find the time t at which it reaches maximum height, then find that maximum height.

$$-10x^2 + 53x + 8$$

→ *h'(t) = -20t + 53. Set h'(t) = 0.*
→ *t = 53/20 seconds.*
→ *Maximum height = h(53/20) = 3129/40 feet.*

Answer: $x^* = -\frac{53}{-20} = \frac{53}{20},\ f(x^*) = \frac{3129}{40}$

4. A company's profit function is P(x) = -4x^2 + 22x + 2, where x is units produced. Find the production level that maximizes profit.

$$-4x^2 + 22x + 2$$

→ *Take the derivative: P'(x) = -8x + 22.*
→ *Set P'(x) = 0: -8x + 22 = 0 → x = 11/4.*
→ *Since the leading coefficient is negative, x = 11/4 is a maximum. Max profit = 129/4.*

Answer: $x^* = -\frac{22}{-8} = \frac{11}{4},\ f(x^*) = \frac{129}{4}$

5. Revenue is R(x) = -2x^2 + 14x + 55. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-2x^2 + 14x + 55$$

→ *Differentiate: R'(x) = -4x + 14.*
→ *Set equal to zero and solve: x = 7/2.*
→ *Maximum revenue = R(7/2) = 159/2.*

Answer: $x^* = -\frac{14}{-4} = \frac{7}{2},\ f(x^*) = \frac{159}{2}$

6. A ball is thrown upward. Its height is h(t) = -8t^2 + 40t + 2 feet. Find the time t at which it reaches maximum height, then find that maximum height.

$$-8x^2 + 40x + 2$$

→ h'(t) = -16t + 40. Set h'(t) = 0.
→ t = 5/2 seconds.
→ Maximum height = h(5/2) = 52 feet.

Answer: $x^* = -\frac{40}{-16} = \frac{5}{2},\ f(x^*) = 52$

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7. A company's profit function is P(x) = -1x^2 + 37x + 17, where x is units produced. Find the production level that maximizes profit.

$$-1x^2 + 37x + 17$$

→ Take the derivative: P'(x) = -2x + 37.
→ Set P'(x) = 0: -2x + 37 = 0 → x = 37/2.
→ Since the leading coefficient is negative, x = 37/2 is a maximum. Max profit = 1437/4.

Answer: $x^* = -\frac{37}{-2} = \frac{37}{2},\ f(x^*) = \frac{1437}{4}$

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8. Revenue is R(x) = -3x^2 + 28x + 70. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-3x^2 + 28x + 70$$

→ Differentiate: R'(x) = -6x + 28.
→ Set equal to zero and solve: x = 14/3.
→ Maximum revenue = R(14/3) = 406/3.

Answer: $x^* = -\frac{28}{-6} = \frac{14}{3},\ f(x^*) = \frac{406}{3}$

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9. A ball is thrown upward. Its height is h(t) = -12t^2 + 23t + 8 feet. Find the time t at which it reaches maximum height, then find that maximum height.

$$-12x^2 + 23x + 8$$

→ h'(t) = -24t + 23. Set h'(t) = 0.
→ t = 23/24 seconds.
→ Maximum height = h(23/24) = 913/48 feet.

Answer: $x^* = -\frac{23}{-24} = \frac{23}{24},\ f(x^*) = \frac{913}{48}$

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10. A company's profit function is P(x) = -3x^2 + 24x + 35, where x is units produced. Find the production level that maximizes profit.

$$-3x^2 + 24x + 35$$

→ Take the derivative: P'(x) = -6x + 24.
→ Set P'(x) = 0: -6x + 24 = 0 → x = 4.
→ Since the leading coefficient is negative, x = 4 is a maximum. Max profit = 83.

Answer: $x^* = -\frac{24}{-6} = 4,\ f(x^*) = 83$

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11. Revenue is R(x) = -3x^2 + 18x + 19. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-3x^2 + 18x + 19$$

→ *Differentiate: R'(x) = -6x + 18.*
→ *Set equal to zero and solve: x = 3.*
→ *Maximum revenue = R(3) = 46.*

Answer: $x^* = -\frac{18}{-6} = 3,\ f(x^*) = 46$

12. A ball is thrown upward. Its height is h(t) = -8t^2 + 20t + 7 feet. Find the time t at which it reaches maximum height, then find that maximum height.

$$-8x^2 + 20x + 7$$

→ *h'(t) = -16t + 20. Set h'(t) = 0.*
→ *t = 5/4 seconds.*
→ *Maximum height = h(5/4) = 39/2 feet.*

Answer: $x^* = -\frac{20}{-16} = \frac{5}{4},\ f(x^*) = \frac{39}{2}$

13. A company's profit function is P(x) = -3x^2 + 16x + 47, where x is units produced. Find the production level that maximizes profit.

$$-3x^2 + 16x + 47$$

→ *Take the derivative: P'(x) = -6x + 16.*
→ *Set P'(x) = 0: -6x + 16 = 0 → x = 8/3.*
→ *Since the leading coefficient is negative, x = 8/3 is a maximum. Max profit = 205/3.*

Answer: $x^* = -\frac{16}{-6} = \frac{8}{3},\ f(x^*) = \frac{205}{3}$

14. Revenue is R(x) = -3x^2 + 14x + 58. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-3x^2 + 14x + 58$$

→ *Differentiate: R'(x) = -6x + 14.*
→ *Set equal to zero and solve: x = 7/3.*
→ *Maximum revenue = R(7/3) = 223/3.*

Answer: $x^* = -\frac{14}{-6} = \frac{7}{3},\ f(x^*) = \frac{223}{3}$

15. A ball is thrown upward. Its height is h(t) = -12t^2 + 38t + 9 feet. Find the time t at which it reaches maximum height, then find that maximum height.

$$-12x^2 + 38x + 9$$

→ *h'(t) = -24t + 38. Set h'(t) = 0.*
→ *t = 19/12 seconds.*
→ *Maximum height = h(19/12) = 469/12 feet.*

Answer: $x^* = -\frac{38}{-24} = \frac{19}{12},\ f(x^*) = \frac{469}{12}$

16. A company's profit function is P(x) = -3x^2 + 12x + 15, where x is units produced. Find the production level that maximizes profit.

$$-3x^2 + 12x + 15$$

→ *Take the derivative: P'(x) = -6x + 12.*
→ *Set P'(x) = 0: -6x + 12 = 0 → x = 2.*
→ *Since the leading coefficient is negative, x = 2 is a maximum. Max profit = 27.*

Answer: $x^* = -\frac{12}{-6} = 2,\ f(x^*) = 27$

17. Revenue is R(x) = -2x^2 + 26x + 37. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-2x^2 + 26x + 37$$

→ *Differentiate: R'(x) = -4x + 26.*
→ *Set equal to zero and solve: x = 13/2.*
→ *Maximum revenue = R(13/2) = 243/2.*

Answer: $x^* = -\frac{26}{-4} = \frac{13}{2},\ f(x^*) = \frac{243}{2}$

18. A ball is thrown upward. Its height is h(t) = -9t^2 + 27t + 10 feet. Find the time t at which it reaches maximum height, then find that maximum height.

$$-9x^2 + 27x + 10$$

→ *h'(t) = -18t + 27. Set h'(t) = 0.*
→ *t = 3/2 seconds.*
→ *Maximum height = h(3/2) = 121/4 feet.*

Answer: $x^* = -\frac{27}{-18} = \frac{3}{2},\ f(x^*) = \frac{121}{4}$

19. A company's profit function is P(x) = -4x^2 + 19x + 27, where x is units produced. Find the production level that maximizes profit.

$$-4x^2 + 19x + 27$$

→ *Take the derivative: P'(x) = -8x + 19.*
→ *Set P'(x) = 0: -8x + 19 = 0 → x = 19/8.*
→ *Since the leading coefficient is negative, x = 19/8 is a maximum. Max profit = 793/16.*

Answer: $x^* = -\frac{19}{-8} = \frac{19}{8},\ f(x^*) = \frac{793}{16}$

20. Revenue is R(x) = -3x^2 + 15x + 52. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-3x^2 + 15x + 52$$

→ *Differentiate: R'(x) = -6x + 15.*
→ *Set equal to zero and solve: x = 5/2.*
→ *Maximum revenue = R(5/2) = 283/4.*

Answer: $x^* = -\frac{15}{-6} = \frac{5}{2},\ f(x^*) = \frac{283}{4}$

21. A ball is thrown upward. Its height is h(t) = -9t^2 + 52t + 1 feet. Find the time t at which it reaches maximum height, then find that maximum height.

$$-9x^2 + 52x + 1$$

→ *h'(t) = -18t + 52. Set h'(t) = 0.*
→ *t = 26/9 seconds.*
→ *Maximum height = h(26/9) = 685/9 feet.*

Answer: $x^* = -\frac{52}{-18} = \frac{26}{9},\ f(x^*) = \frac{685}{9}$

22. A company's profit function is P(x) = -4x^2 + 19x + 44, where x is units produced. Find the production level that maximizes profit.

$$-4x^2 + 19x + 44$$

→ *Take the derivative: P'(x) = -8x + 19.*
→ *Set P'(x) = 0: -8x + 19 = 0 → x = 19/8.*
→ *Since the leading coefficient is negative, x = 19/8 is a maximum. Max profit = 1065/16.*

Answer: $x^* = -\frac{19}{-8} = \frac{19}{8},\ f(x^*) = \frac{1065}{16}$

23. Revenue is R(x) = -2x^2 + 28x + 79. Find the number of units x that maximizes revenue, and compute that maximum revenue.

$$-2x^2 + 28x + 79$$

→ *Differentiate: R'(x) = -4x + 28.*
→ *Set equal to zero and solve: x = 7.*
→ *Maximum revenue = R(7) = 177.*

Answer: $x^* = -\frac{28}{-4} = 7,\ f(x^*) = 177$

Marginal analysis

24. Total cost is C(x) = 2x^3 + -1x^2 + 10x + 115. Find the marginal cost MC = C'(1), and determine whether cost is increasing or decreasing at that production level.

$$f(x) = 2x^3 + -1x^2 + 10x + 115,\ x = 1$$

→ *C'(x) = 6x^2 + -2x + 10.*
→ *MC = C'(1) = 14.*
→ *Since MC > 0, cost is increasing at x = 1.*

Answer: $f'(x) = 6x^2 + -2x + 10, \quad f'(1) = 14$

25. Total cost is C(x) = 1x^3 + -2x^2 + 9x + 78. Find the marginal cost MC = C'(2), and determine whether cost is increasing or decreasing at that production level.

$$f(x) = 1x^3 + -2x^2 + 9x + 78,\ x = 2$$

→ *C'(x) = 3x^2 + -4x + 9.*
→ *MC = C'(2) = 13.*
→ *Since MC > 0, cost is increasing at x = 2.*

Answer: $f'(x) = 3x^2 + -4x + 9, \quad f'(2) = 13$

26. Total cost is C(x) = 1x^3 + -1x^2 + 11x + 173. Find the marginal cost MC = C'(3), and determine whether cost is increasing or decreasing at that production level.

$$f(x) = 1x^3 + -1x^2 + 11x + 173,\ x = 3$$

→ *C'(x) = 3x^2 + -2x + 11.*
→ *MC = C'(3) = 32.*
→ *Since MC > 0, cost is increasing at x = 3.*

Answer: $f'(x) = 3x^2 + -2x + 11, \quad f'(3) = 32$

27. Total cost is C(x) = 1x^3 + -2x^2 + 12x + 54. Find the marginal cost MC = C'(1), and determine whether cost is increasing or decreasing at that production level.

$$f(x) = 1x^3 + -2x^2 + 12x + 54,\ x = 1$$

→ *C'(x) = 3x^2 + -4x + 12.*
→ *MC = C'(1) = 11.*
→ *Since MC > 0, cost is increasing at x = 1.*

Answer: $f'(x) = 3x^2 + -4x + 12, \quad f'(1) = 11$

28. Total cost is C(x) = 2x^3 + -3x^2 + 11x + 105. Find the marginal cost MC = C'(2), and determine whether cost is increasing or decreasing at that production level.

$$f(x) = 2x^3 + -3x^2 + 11x + 105,\ x = 2$$

→ *C'(x) = 6x^2 + -6x + 11.*
→ *MC = C'(2) = 23.*
→ *Since MC > 0, cost is increasing at x = 2.*

Answer: $f'(x) = 6x^2 + -6x + 11, \quad f'(2) = 23$

29. Total cost is C(x) = 2x^3 + -2x^2 + 7x + 181. Find the marginal cost MC = C'(1), and determine whether cost is increasing or decreasing at that production level.

$$f(x) = 2x^3 + -2x^2 + 7x + 181,\ x = 1$$

→ *C'(x) = 6x^2 + -4x + 7.*
→ *MC = C'(1) = 9.*
→ *Since MC > 0, cost is increasing at x = 1.*

Answer: $f'(x) = 6x^2 + -4x + 7, \quad f'(1) = 9$

30. Total cost is C(x) = 1x^3 + -4x^2 + 15x + 92. Find the marginal cost MC = C'(1), and determine whether cost is increasing or decreasing at that production level.

$$f(x) = 1x^3 + -4x^2 + 15x + 92,\ x = 1$$

→ *C'(x) = 3x^2 + -8x + 15.*
→ *MC = C'(1) = 10.*
→ *Since MC > 0, cost is increasing at x = 1.*

Answer: $f'(x) = 3x^2 + -8x + 15, \quad f'(1) = 10$