



Name: \_\_\_\_\_

Date: \_\_\_\_\_

Score: / 30

## Learning Objectives

- Find the area between two curves using integration
- Compute the average value of a function over an interval
- Set up and evaluate volumes of revolution (disk and washer methods)

*Simplify each expression completely. Show all steps and circle your final answer.*

## Area under a curve

1. Find the area of the region bounded by  $f(x) = x^2$ ,  $x = 1$ ,  $x = 2$ , and the x-axis.

$$\int_1^2 x^2 dx$$

Answer: \_\_\_\_\_

2. Find the area of the region bounded by  $f(x) = x^3$ ,  $x = 1$ ,  $x = 3$ , and the x-axis.

$$\int_1^3 x^3 dx$$

Answer: \_\_\_\_\_

3. Find the area of the region bounded by  $f(x) = x^2$ ,  $x = 1$ ,  $x = 4$ , and the x-axis.

$$\int_1^4 x^2 dx$$

Answer: \_\_\_\_\_

4. Find the area of the region bounded by  $f(x) = x^2$ ,  $x = 0$ ,  $x = 2$ , and the x-axis.

$$\int_0^2 x^2 dx$$

Answer: \_\_\_\_\_

5. Find the area of the region bounded by  $f(x) = x^3$ ,  $x = 1$ ,  $x = 3$ , and the x-axis.

$$\int_1^3 x^3 dx$$

Answer: \_\_\_\_\_

6. Find the area of the region bounded by  $f(x) = x^3$ ,  $x = 0$ ,  $x = 2$ , and the  $x$ -axis.

$$\int_0^2 x^3 dx$$

Answer: \_\_\_\_\_

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7. Find the area of the region bounded by  $f(x) = x^3$ ,  $x = 0$ ,  $x = 3$ , and the  $x$ -axis.

$$\int_0^3 x^3 dx$$

Answer: \_\_\_\_\_

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8. Find the area of the region bounded by  $f(x) = x^3$ ,  $x = 1$ ,  $x = 3$ , and the  $x$ -axis.

$$\int_1^3 x^3 dx$$

Answer: \_\_\_\_\_

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9. Find the area of the region bounded by  $f(x) = x^3$ ,  $x = 1$ ,  $x = 2$ , and the  $x$ -axis.

$$\int_1^2 x^3 dx$$

Answer: \_\_\_\_\_

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10. Find the area of the region bounded by  $f(x) = x^2$ ,  $x = 0$ ,  $x = 2$ , and the  $x$ -axis.

$$\int_0^2 x^2 dx$$

Answer: \_\_\_\_\_

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### Total accumulated change

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11. The rate of sales is  $r(t) = 2t^2$  units/day. Find the total number of units sold from day 0 to day 6.

$$\int_0^6 x^2 dx$$

Answer: \_\_\_\_\_

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12. A factory's output rate is  $O(t) = x^1$  (hundreds of units/hour). Find the total output over hours 0 to 6.

$$\int_0^6 x^1 dx$$

Answer: \_\_\_\_\_

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13. The rate of sales is  $r(t) = 2t^2$  units/day. Find the total number of units sold from day 0 to day 4.

$$\int_0^4 x^2 dx$$

Answer: \_\_\_\_\_

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14. A factory's output rate is  $O(t) = x^1$  (hundreds of units/hour). Find the total output over hours 1 to 6.

$$\int_1^6 x^1 dx$$

Answer: \_\_\_\_\_

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15. The rate of sales is  $r(t) = 2t^2$  units/day. Find the total number of units sold from day 1 to day 3.

$$\int_1^3 x^2 dx$$

Answer: \_\_\_\_\_

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16. A factory's output rate is  $O(t) = x^2$  (hundreds of units/hour). Find the total output over hours 1 to 4.

$$\int_1^4 x^2 dx$$

Answer: \_\_\_\_\_

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17. The rate of sales is  $r(t) = 4t^1$  units/day. Find the total number of units sold from day 1 to day 5.

$$\int_1^5 x^1 dx$$

Answer: \_\_\_\_\_

---

18. A factory's output rate is  $O(t) = x^2$  (hundreds of units/hour). Find the total output over hours 2 to 6.

$$\int_2^6 x^2 dx$$

Answer: \_\_\_\_\_

---

19. The rate of sales is  $r(t) = 2t^1$  units/day. Find the total number of units sold from day 0 to day 5.

$$\int_0^5 x^1 dx$$

Answer: \_\_\_\_\_

---

20. A factory's output rate is  $O(t) = x^2$  (hundreds of units/hour). Find the total output over hours 2 to 5.

$$\int_2^5 x^2 dx$$

Answer: \_\_\_\_\_

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21. The rate of sales is  $r(t) = 6t^1$  units/day. Find the total number of units sold from day 0 to day 4.

$$\int_0^4 x^1 dx$$

Answer: \_\_\_\_\_

---

22. A factory's output rate is  $O(t) = x^1$  (hundreds of units/hour). Find the total output over hours 2 to 6.

$$\int_2^6 x^1 dx$$

Answer: \_\_\_\_\_

---

23. The rate of sales is  $r(t) = 5t^1$  units/day. Find the total number of units sold from day 0 to day 5.

$$\int_0^5 x^1 dx$$

Answer: \_\_\_\_\_

---

24. A factory's output rate is  $O(t) = x^1$  (hundreds of units/hour). Find the total output over hours 0 to 6.

$$\int_0^6 x^1 dx$$

Answer: \_\_\_\_\_

---

25. The rate of sales is  $r(t) = 5t^1$  units/day. Find the total number of units sold from day 1 to day 3.

$$\int_1^3 x^1 dx$$

Answer: \_\_\_\_\_

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26. A factory's output rate is  $O(t) = x^2$  (hundreds of units/hour). Find the total output over hours 1 to 4.

$$\int_1^4 x^2 dx$$

Answer: \_\_\_\_\_

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27. The rate of sales is  $r(t) = 2t^1$  units/day. Find the total number of units sold from day 1 to day 6.

$$\int_1^6 x^1 dx$$

Answer: \_\_\_\_\_

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28. A factory's output rate is  $O(t) = x^1$  (hundreds of units/hour). Find the total output over hours 2 to 4.

$$\int_2^4 x^1 dx$$

Answer: \_\_\_\_\_

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29. The rate of sales is  $r(t) = 2t^2$  units/day. Find the total number of units sold from day 1 to day 6.

$$\int_1^6 x^2 dx$$

Answer: \_\_\_\_\_

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30. A factory's output rate is  $O(t) = t^1$  (hundreds of units/hour). Find the total output over hours 1 to 4.

$$\int_1^4 x^1 dx$$

Answer: \_\_\_\_\_

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# MATH120: Applications of Integration

Introduction to Statistics · C-ID MATH120 · numberbender.com

ANSWER KEY & SOLUTIONS

*Topics: Total accumulated change, Area under a curve. All answers verified by independent computation.*

## Solutions

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## Area under a curve

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1. Find the area of the region bounded by  $f(x) = x^2$ ,  $x = 1$ ,  $x = 2$ , and the x-axis.

$$\int_1^2 x^2 dx$$

→ Area = integral from 1 to 2 of  $x^2 dx$ .

→ =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$  =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  =  $\{answer\_defint210\}$  sq units.

**Answer:**  $\frac{8 - 1}{3} = 7/3$

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2. Find the area of the region bounded by  $f(x) = x^3$ ,  $x = 1$ ,  $x = 3$ , and the x-axis.

$$\int_1^3 x^3 dx$$

→ Area = integral from 1 to 3 of  $x^3 dx$ .

→ =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$  =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  =  $\{answer\_defint210\}$  sq units.

**Answer:**  $\frac{81 - 1}{4} = 20$

---

3. Find the area of the region bounded by  $f(x) = x^2$ ,  $x = 1$ ,  $x = 4$ , and the x-axis.

$$\int_1^4 x^2 dx$$

→ Area = integral from 1 to 4 of  $x^2 dx$ .

→ =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$  =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  =  $\{answer\_defint210\}$  sq units.

**Answer:**  $\frac{64 - 1}{3} = 21$

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4. Find the area of the region bounded by  $f(x) = x^2$ ,  $x = 0$ ,  $x = 2$ , and the x-axis.

$$\int_0^2 x^2 dx$$

→ Area = integral from 0 to 2 of  $x^2 dx$ .

→ =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$  =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  =  $\{answer\_defint210\}$  sq units.

**Answer:**  $\frac{8 - 0}{3} = 8/3$

---

5. Find the area of the region bounded by  $f(x) = x^3$ ,  $x = 1$ ,  $x = 3$ , and the x-axis.

$$\int_1^3 x^3 dx$$

→ Area = integral from 1 to 3 of  $x^3 dx$ .

→ =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$  =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  =  $\{answer\_defint210\}$  sq units.

**Answer:**  $\frac{81 - 1}{4} = 20$

---

6. Find the area of the region bounded by  $f(x) = x^3$ ,  $x = 0$ ,  $x = 2$ , and the x-axis.

$$\int_0^2 x^3 dx$$

→ Area = integral from 0 to 2 of  $x^3 dx$ .

→ =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$  =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  = {answer\_defint210} sq units.

**Answer:**  $\frac{16 - 0}{4} = 4$

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7. Find the area of the region bounded by  $f(x) = x^3$ ,  $x = 0$ ,  $x = 3$ , and the x-axis.

$$\int_0^3 x^3 dx$$

→ Area = integral from 0 to 3 of  $x^3 dx$ .

→ =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$  =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  = {answer\_defint210} sq units.

**Answer:**  $\frac{81 - 0}{4} = 81/4$

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8. Find the area of the region bounded by  $f(x) = x^3$ ,  $x = 1$ ,  $x = 3$ , and the x-axis.

$$\int_1^3 x^3 dx$$

→ Area = integral from 1 to 3 of  $x^3 dx$ .

→ =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$  =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  = {answer\_defint210} sq units.

**Answer:**  $\frac{81 - 1}{4} = 20$

---

9. Find the area of the region bounded by  $f(x) = x^3$ ,  $x = 1$ ,  $x = 2$ , and the x-axis.

$$\int_1^2 x^3 dx$$

→ Area = integral from 1 to 2 of  $x^3 dx$ .

→ =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$  =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  = {answer\_defint210} sq units.

**Answer:**  $\frac{16 - 1}{4} = 15/4$

---

10. Find the area of the region bounded by  $f(x) = x^2$ ,  $x = 0$ ,  $x = 2$ , and the x-axis.

$$\int_0^2 x^2 dx$$

→ Area = integral from 0 to 2 of  $x^2 dx$ .

→ =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$  =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  = {answer\_defint210} sq units.

**Answer:**  $\frac{8 - 0}{3} = 8/3$

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## Total accumulated change

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11. The rate of sales is  $r(t) = 2t^2$  units/day. Find the total number of units sold from day 0 to day 6.

$$\int_0^6 x^2 dx$$

→ Total units = integral from 0 to 6 of  $2t^2$  dt.

→  $\{a\} * [t^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\} = \{a\} * (\{hi\_pow\} - \{lo\_pow\}) / (n+1)$ .

→ Total = 72 units (Note: multiply by 2 if  $a > 1$ ).

**Answer:**  $\frac{216 - 0}{3} = 72$

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12. A factory's output rate is  $O(t) = x^1$  (hundreds of units/hour). Find the total output over hours 0 to 6.

$$\int_0^6 x^1 dx$$

→ Total = integral from  $\{lo\}$  to  $\{hi\}$  of  $x^n$  dx =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$ .

→  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1) = \{answer\_defint210\}$  hundred units.

**Answer:**  $\frac{36 - 0}{2} = 18$

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13. The rate of sales is  $r(t) = 2t^2$  units/day. Find the total number of units sold from day 0 to day 4.

$$\int_0^4 x^2 dx$$

→ Total units = integral from 0 to 4 of  $2t^2$  dt.

→  $\{a\} * [t^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\} = \{a\} * (\{hi\_pow\} - \{lo\_pow\}) / (n+1)$ .

→ Total =  $64/3$  units (Note: multiply by 2 if  $a > 1$ ).

**Answer:**  $\frac{64 - 0}{3} = 64/3$

---

14. A factory's output rate is  $O(t) = x^1$  (hundreds of units/hour). Find the total output over hours 1 to 6.

$$\int_1^6 x^1 dx$$

→ Total = integral from  $\{lo\}$  to  $\{hi\}$  of  $x^n$  dx =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$ .

→  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1) = \{answer\_defint210\}$  hundred units.

**Answer:**  $\frac{36 - 1}{2} = 35/2$

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15. The rate of sales is  $r(t) = 2t^2$  units/day. Find the total number of units sold from day 1 to day 3.

$$\int_1^3 x^2 dx$$

→ Total units = integral from 1 to 3 of  $2t^2$  dt.

→  $\{a\} * [t^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\} = \{a\} * (\{hi\_pow\} - \{lo\_pow\}) / (n+1)$ .

→ Total =  $26/3$  units (Note: multiply by 2 if  $a > 1$ ).

**Answer:**  $\frac{27 - 1}{3} = 26/3$

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16. A factory's output rate is  $O(t) = x^2$  (hundreds of units/hour). Find the total output over hours 1 to 4.

$$\int_1^4 x^2 dx$$

→ Total = integral from  $\{lo\}$  to  $\{hi\}$  of  $x^n$  dx =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$ .

→ =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  = {answer\_defint210} hundred units.

**Answer:**  $\frac{64 - 1}{3} = 21$

---

17. The rate of sales is  $r(t) = 4t^1$  units/day. Find the total number of units sold from day 1 to day 5.

$$\int_1^5 x^1 dx$$

→ Total units = integral from 1 to 5 of  $4t^1$  dt.

→ =  $\{a\} * [t^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$  =  $\{a\} * (\{hi\_pow\} - \{lo\_pow\}) / (n+1)$ .

→ Total = 12 units (Note: multiply by 4 if  $a > 1$ ).

**Answer:**  $\frac{25 - 1}{2} = 12$

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18. A factory's output rate is  $O(t) = x^2$  (hundreds of units/hour). Find the total output over hours 2 to 6.

$$\int_2^6 x^2 dx$$

→ Total = integral from  $\{lo\}$  to  $\{hi\}$  of  $x^n$  dx =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$ .

→ =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  = {answer\_defint210} hundred units.

**Answer:**  $\frac{216 - 8}{3} = 208/3$

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19. The rate of sales is  $r(t) = 2t^1$  units/day. Find the total number of units sold from day 0 to day 5.

$$\int_0^5 x^1 dx$$

→ Total units = integral from 0 to 5 of  $2t^1$  dt.

→ =  $\{a\} * [t^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$  =  $\{a\} * (\{hi\_pow\} - \{lo\_pow\}) / (n+1)$ .

→ Total = 25/2 units (Note: multiply by 2 if  $a > 1$ ).

**Answer:**  $\frac{25 - 0}{2} = 25/2$

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20. A factory's output rate is  $O(t) = x^2$  (hundreds of units/hour). Find the total output over hours 2 to 5.

$$\int_2^5 x^2 dx$$

→ Total = integral from  $\{lo\}$  to  $\{hi\}$  of  $x^n$  dx =  $[x^{n+1}/(n+1)]$  from  $\{lo\}$  to  $\{hi\}$ .

→ =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  = {answer\_defint210} hundred units.

**Answer:**  $\frac{125 - 8}{3} = 39$

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21. The rate of sales is  $r(t) = 6t^1$  units/day. Find the total number of units sold from day 0 to day 4.

$$\int_0^4 x^1 dx$$

→ Total units = integral from 0 to 4 of  $6t^1$  dt.

→ =  $\{a\} * [t^{\{n+1\}} / \{\{n+1\}\}]$  from  $\{lo\}$  to  $\{hi\}$  =  $\{a\} * (\{hi\_pow\} - \{lo\_pow\}) / (\{n+1\})$ .

→ Total = 8 units (Note: multiply by 6 if  $a > 1$ ).

**Answer:**  $\frac{16 - 0}{2} = 8$

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22. A factory's output rate is  $O(t) = x^1$  (hundreds of units/hour). Find the total output over hours 2 to 6.

$$\int_2^6 x^1 dx$$

→ Total = integral from  $\{lo\}$  to  $\{hi\}$  of  $x^{\{n\}}$  dx =  $[x^{\{n+1\}} / \{\{n+1\}\}]$  from  $\{lo\}$  to  $\{hi\}$ .

→ =  $(\{hi\_pow\} - \{lo\_pow\}) / (\{n+1\})$  = {answer\_defint210} hundred units.

**Answer:**  $\frac{36 - 4}{2} = 16$

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23. The rate of sales is  $r(t) = 5t^1$  units/day. Find the total number of units sold from day 0 to day 5.

$$\int_0^5 x^1 dx$$

→ Total units = integral from 0 to 5 of  $5t^1$  dt.

→ =  $\{a\} * [t^{\{n+1\}} / \{\{n+1\}\}]$  from  $\{lo\}$  to  $\{hi\}$  =  $\{a\} * (\{hi\_pow\} - \{lo\_pow\}) / (\{n+1\})$ .

→ Total = 25/2 units (Note: multiply by 5 if  $a > 1$ ).

**Answer:**  $\frac{25 - 0}{2} = 25/2$

---

24. A factory's output rate is  $O(t) = x^1$  (hundreds of units/hour). Find the total output over hours 0 to 6.

$$\int_0^6 x^1 dx$$

→ Total = integral from  $\{lo\}$  to  $\{hi\}$  of  $x^{\{n\}}$  dx =  $[x^{\{n+1\}} / \{\{n+1\}\}]$  from  $\{lo\}$  to  $\{hi\}$ .

→ =  $(\{hi\_pow\} - \{lo\_pow\}) / (\{n+1\})$  = {answer\_defint210} hundred units.

**Answer:**  $\frac{36 - 0}{2} = 18$

---

25. The rate of sales is  $r(t) = 5t^1$  units/day. Find the total number of units sold from day 1 to day 3.

$$\int_1^3 x^1 dx$$

→ Total units = integral from 1 to 3 of  $5t^1$  dt.

→ =  $\{a\} * [t^{\{n+1\}} / \{\{n+1\}\}]$  from  $\{lo\}$  to  $\{hi\}$  =  $\{a\} * (\{hi\_pow\} - \{lo\_pow\}) / (\{n+1\})$ .

→ Total = 4 units (Note: multiply by 5 if  $a > 1$ ).

**Answer:**  $\frac{9 - 1}{2} = 4$

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26. A factory's output rate is  $O(t) = x^2$  (hundreds of units/hour). Find the total output over hours 1 to 4.

$$\int_1^4 x^2 dx$$

→ Total = integral from {lo} to {hi} of  $x^n$  dx =  $[x^{n+1}/(n+1)]$  from {lo} to {hi}.

→ =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  = {answer\_defint210} hundred units.

**Answer:**  $\frac{64 - 1}{3} = 21$

---

27. The rate of sales is  $r(t) = 2t^1$  units/day. Find the total number of units sold from day 1 to day 6.

$$\int_1^6 x^1 dx$$

→ Total units = integral from 1 to 6 of  $2t^1$  dt.

→ =  $\{a\} * [t^{n+1}/(n+1)]$  from {lo} to {hi} =  $\{a\} * (\{hi\_pow\} - \{lo\_pow\}) / (n+1)$ .

→ Total = 35/2 units (Note: multiply by 2 if  $a > 1$ ).

**Answer:**  $\frac{36 - 1}{2} = 35/2$

---

28. A factory's output rate is  $O(t) = x^1$  (hundreds of units/hour). Find the total output over hours 2 to 4.

$$\int_2^4 x^1 dx$$

→ Total = integral from {lo} to {hi} of  $x^n$  dx =  $[x^{n+1}/(n+1)]$  from {lo} to {hi}.

→ =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  = {answer\_defint210} hundred units.

**Answer:**  $\frac{16 - 4}{2} = 6$

---

29. The rate of sales is  $r(t) = 2t^2$  units/day. Find the total number of units sold from day 1 to day 6.

$$\int_1^6 x^2 dx$$

→ Total units = integral from 1 to 6 of  $2t^2$  dt.

→ =  $\{a\} * [t^{n+1}/(n+1)]$  from {lo} to {hi} =  $\{a\} * (\{hi\_pow\} - \{lo\_pow\}) / (n+1)$ .

→ Total = 215/3 units (Note: multiply by 2 if  $a > 1$ ).

**Answer:**  $\frac{216 - 1}{3} = 215/3$

---

30. A factory's output rate is  $O(t) = x^1$  (hundreds of units/hour). Find the total output over hours 1 to 4.

$$\int_1^4 x^1 dx$$

→ Total = integral from {lo} to {hi} of  $x^n$  dx =  $[x^{n+1}/(n+1)]$  from {lo} to {hi}.

→ =  $(\{hi\_pow\} - \{lo\_pow\}) / (n+1)$  = {answer\_defint210} hundred units.

**Answer:**  $\frac{16 - 1}{2} = 15/2$

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