MATH120: Limits

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Learning Objectives

- Evaluate limits by direct substitution
- Use factoring to resolve 0/0 indeterminate forms
- Apply one-sided and two-sided limit definitions
- Determine where a function is continuous

Simplify each expression completely. Show all steps and circle your final answer.

Limits by factoring

1. Evaluate: lim as x approaches 1 of [1(x^2 - 1^2) / (x - 1)]. (0/0 form — factor first.)

$$\lim_{x \to 1} \frac{1(x^2 - 1^2)}{x - 1}$$

Answer: ____

2. Evaluate: lim as x approaches 4 of [1(x^2 - 4^2) / (x - 4)]. (0/0 form — factor first.)

$$\lim_{x \to 4} \frac{1(x^2 - 4^2)}{x - 4}$$

Answer: ____

3. Evaluate: lim as x approaches 3 of [4(x^2 - 3^2) / (x - 3)]. (0/0 form — factor first.)

$$\lim_{x \to 3} \frac{4(x^2 - 3^2)}{x - 3}$$

Answer: ____

4. Evaluate: lim as x approaches 4 of [2(x^2 - 4^2) / (x - 4)]. (0/0 form — factor first.)

$$\lim_{x \to 4} \frac{2(x^2 - 4^2)}{x - 4}$$

Answer: ____

5. Evaluate: lim as x approaches 2 of [2(x^2 - 2^2) / (x - 2)]. (0/0 form — factor first.)

$$\lim_{x \to 2} \frac{2(x^2 - 2^2)}{x - 2}$$

Answer: ____

6. Evaluate: lim as x approaches 1 of [2(x^2 - 1^2) / (x - 1)]. (0/0 form — factor first.)

$$\lim_{x \to 1} \frac{2(x^2 - 1^2)}{x - 1}$$

Answer: ____

7. Evaluate: lim as x approaches 3 of [1(x^2 - 3^2) / (x - 3)]. (0/0 form — factor first.)

$$\lim_{x \to 3} \frac{1(x^2 - 3^2)}{x - 3}$$

Answer: ____

8. Evaluate: lim as x approaches 3 of [1(x^2 - 3^2) / (x - 3)]. (0/0 form — factor first.)

$$\lim_{x \to 3} \frac{1(x^2 - 3^2)}{x - 3}$$

Answer: ____

Limits by direct substitution

9. Evaluate the limit by direct substitution: lim (x to 1) of (1x^2 + 0x + -3).

$$\lim_{x \to 1} (1x^2 + 0x - 3)$$

Answer: ____

10. A price-demand function is p(x) = -1x^2 + 13x + 119. Find the limit as x approaches 2 and interpret it as the price when demand approaches 2 units.

$$\lim_{x \to 2} (-1x^2 + 13x + 119)$$

Answer: ____

11. Evaluate: limit as x approaches 1 of (2x^2 + 0x + 7).

$$\lim_{x \to 1} (2x^2 + 0x + 7)$$

Answer: ____

12. Evaluate the limit by direct substitution: lim (x to 2) of (3x^2 + -3x + 1).

$$\lim_{x \to 2} (3x^2 - 3x + 1)$$

Answer: ____

13. A price-demand function is p(x) = -1x^2 + 7x + 121. Find the limit as x approaches 4 and interpret it as the price when demand approaches 4 units.

$$\lim_{x \to 4} (-1x^2 + 7x + 121)$$

Answer: ____

14. Evaluate: limit as x approaches -2 of (1x^2 + -1x + 5).

$$\lim_{x \to -2} (1x^2 - 1x + 5)$$

Answer: ____

15. Evaluate the limit by direct substitution: lim (x to -1) of (2x^2 + 4x + 3).

$$\lim_{x \to -1} (2x^2 + 4x + 3)$$

Answer: ____

16. A price-demand function is p(x) = -1x^2 + 5x + 142. Find the limit as x approaches 5 and interpret it as the price when demand approaches 5 units.

$$\lim_{x \to 5} (-1x^2 + 5x + 142)$$

Answer: ____

17. Evaluate: limit as x approaches 4 of (3x^2 + -3x + 6).

$$\lim_{x \to 4} (3x^2 - 3x + 6)$$

Answer: ____

18. Evaluate the limit by direct substitution: lim (x to 3) of (1x^2 + -1x + -3).

$$\lim_{x \to 3} (1x^2 - 1x - 3)$$

Answer: ____

19. A price-demand function is p(x) = -2x^2 + 12x + 133. Find the limit as x approaches 4 and interpret it as the price when demand approaches 4 units.

$$\lim_{x \to 4} (-2x^2 + 12x + 133)$$

Answer: ____

20. Evaluate: limit as x approaches 0 of (1x^2 + -1x + -8).

$$\lim_{x \to 0} (1x^2 - 1x - 8)$$

Answer: ____

21. Evaluate the limit by direct substitution: lim (x to 1) of (2x^2 + -3x + 5).

$$\lim_{x \to 1} (2x^2 - 3x + 5)$$

Answer: ____

22. A price-demand function is p(x) = -1x^2 + 9x + 123. Find the limit as x approaches 3 and interpret it as the price when demand approaches 3 units.

$$\lim_{x \to 3} (-1x^2 + 9x + 123)$$

Answer: ____

23. Evaluate: limit as x approaches 2 of (2x^2 + -4x + -2).

$$\lim_{x \to 2} (2x^2 - 4x - 2)$$

Answer: ____

24. Evaluate the limit by direct substitution: lim (x to 0) of (4x^2 + 3x + -1).

$$\lim_{x \to 0} (4x^2 + 3x - 1)$$

Answer: ____

25. A price-demand function is p(x) = -1x^2 + 6x + 130. Find the limit as x approaches 5 and interpret it as the price when demand approaches 5 units.

$$\lim_{x \to 5} (-1x^2 + 6x + 130)$$

Answer: ____

26. Evaluate: limit as x approaches 0 of (2x^2 + -2x + 1).

$$\lim_{x \to 0} (2x^2 - 2x + 1)$$

Answer: ____

27. Evaluate the limit by direct substitution: lim (x to 1) of (1x^2 + -3x + 1).

$$\lim_{x \to 1} (1x^2 - 3x + 1)$$

Answer: ____

28. A price-demand function is p(x) = -1x^2 + 13x + 64. Find the limit as x approaches 3 and interpret it as the price when demand approaches 3 units.

$$\lim_{x \to 3} (-1x^2 + 13x + 64)$$

Answer: ____

29. Evaluate: limit as x approaches -1 of (3x^2 + -6x + -5).

$$\lim_{x \to -1} (3x^2 - 6x - 5)$$

Answer: ____

30. Evaluate the limit by direct substitution: lim (x to -2) of (3x^2 + 4x + 4).

$$\lim_{x \to -2} (3x^2 + 4x + 4)$$

Answer: ____

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Topics: Limits by direct substitution, Limits by factoring. All answers verified by independent computation.

Solutions

Limits by factoring

1. Evaluate: lim as x approaches 1 of [1(x^2 - 1^2) / (x - 1)]. (0/0 form — factor first.)

$$\lim_{x \to 1} \frac{1(x^2 - 1^2)}{x - 1}$$

→ *Factor the numerator: a*(x^2 - r^2) = a*(x-r)*(x+r).*
→ *Cancel (x-r) top and bottom. The limit simplifies to a*(x+r).*
→ *Substitute x = 1: 1*(1+1) = 1*2*1 = 2.*

Answer: $= (1)(x+1)|_{x=1} = 2$

2. Evaluate: lim as x approaches 4 of [1(x^2 - 4^2) / (x - 4)]. (0/0 form — factor first.)

$$\lim_{x \to 4} \frac{1(x^2 - 4^2)}{x - 4}$$

→ *Factor the numerator: a*(x^2 - r^2) = a*(x-r)*(x+r).*
→ *Cancel (x-r) top and bottom. The limit simplifies to a*(x+r).*
→ *Substitute x = 4: 1*(4+4) = 1*2*4 = 8.*

Answer: $= (1)(x+4)|_{x=4} = 8$

3. Evaluate: lim as x approaches 3 of [4(x^2 - 3^2) / (x - 3)]. (0/0 form — factor first.)

$$\lim_{x \to 3} \frac{4(x^2 - 3^2)}{x - 3}$$

→ *Factor the numerator: a*(x^2 - r^2) = a*(x-r)*(x+r).*
→ *Cancel (x-r) top and bottom. The limit simplifies to a*(x+r).*
→ *Substitute x = 3: 4*(3+3) = 4*2*3 = 24.*

Answer: $= (4)(x+3)|_{x=3} = 24$

4. Evaluate: lim as x approaches 4 of [2(x^2 - 4^2) / (x - 4)]. (0/0 form — factor first.)

$$\lim_{x \to 4} \frac{2(x^2 - 4^2)}{x - 4}$$

→ *Factor the numerator: a*(x^2 - r^2) = a*(x-r)*(x+r).*
→ *Cancel (x-r) top and bottom. The limit simplifies to a*(x+r).*
→ *Substitute x = 4: 2*(4+4) = 2*2*4 = 16.*

Answer: $= (2)(x+4)|_{x=4} = 16$

5. Evaluate: lim as x approaches 2 of [2(x^2 - 2^2) / (x - 2)]. (0/0 form — factor first.)

$$\lim_{x \to 2} \frac{2(x^2 - 2^2)}{x - 2}$$

→ *Factor the numerator: a*(x^2 - r^2) = a*(x-r)*(x+r).*
→ *Cancel (x-r) top and bottom. The limit simplifies to a*(x+r).*
→ *Substitute x = 2: 2*(2+2) = 2*2*2 = 8.*

Answer: $= (2)(x+2)|_{x=2} = 8$

6. Evaluate: lim as x approaches 1 of [2(x^2 - 1^2) / (x - 1)]. (0/0 form — factor first.)

$$\lim_{x \to 1} \frac{2(x^2 - 1^2)}{x - 1}$$

→ *Factor the numerator: a*(x^2 - r^2) = a*(x-r)*(x+r).*
→ *Cancel (x-r) top and bottom. The limit simplifies to a*(x+r).*
→ *Substitute x = 1: 2*(1+1) = 2*2*1 = 4.*

Answer: $= (2)(x + 1)|_{x = 1} = 4$

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7. Evaluate: lim as x approaches 3 of [1(x^2 - 3^2) / (x - 3)]. (0/0 form — factor first.)

$$\lim_{x \to 3} \frac{1(x^2 - 3^2)}{x - 3}$$

→ *Factor the numerator: a*(x^2 - r^2) = a*(x-r)*(x+r).*
→ *Cancel (x-r) top and bottom. The limit simplifies to a*(x+r).*
→ *Substitute x = 3: 1*(3+3) = 1*2*3 = 6.*

Answer: $= (1)(x + 3)|_{x = 3} = 6$

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8. Evaluate: lim as x approaches 3 of [1(x^2 - 3^2) / (x - 3)]. (0/0 form — factor first.)

$$\lim_{x \to 3} \frac{1(x^2 - 3^2)}{x - 3}$$

→ *Factor the numerator: a*(x^2 - r^2) = a*(x-r)*(x+r).*
→ *Cancel (x-r) top and bottom. The limit simplifies to a*(x+r).*
→ *Substitute x = 3: 1*(3+3) = 1*2*3 = 6.*

Answer: $= (1)(x + 3)|_{x = 3} = 6$

Limits by direct substitution

9. Evaluate the limit by direct substitution: lim (x to 1) of (1x^2 + 0x + -3).

$$\lim_{x \to 1} (1x^2 + 0x - 3)$$

→ *Since the function is a polynomial, substitute x = 1 directly.*

→ *= 1(1)^2 + 0(1) + -3 = 1 + 0 + -3 = -2.*

Answer: $\lim_{x \to 1} = 1 + 0 + -3 = -2$

10. A price-demand function is p(x) = -1x^2 + 13x + 119. Find the limit as x approaches 2 and interpret it as the price when demand approaches 2 units.

$$\lim_{x \to 2} (-1x^2 + 13x + 119)$$

→ *Substitute x = 2: lim = -1(2)^2 + 13(2) + 119.*

→ *= -4 + 26 + 119 = 141.*

→ *The price approaches $141 as demand approaches 2 units.*

Answer: $\lim_{x \to 2} = -4 + 26 + 119 = 141$

11. Evaluate: limit as x approaches 1 of (2x^2 + 0x + 7).

$$\lim_{x \to 1} (2x^2 + 0x + 7)$$

→ *Polynomial limit — plug in x = 1 directly.*

→ *= 2(1)^2 + 0(1) + 7 = 9.*

Answer: $\lim_{x \to 1} = 2 + 0 + 7 = 9$

12. Evaluate the limit by direct substitution: lim (x to 2) of (3x^2 + -3x + 1).

$$\lim_{x \to 2} (3x^2 - 3x + 1)$$

→ *Since the function is a polynomial, substitute x = 2 directly.*

→ *= 3(2)^2 + -3(2) + 1 = 12 + -6 + 1 = 7.*

Answer: $\lim_{x \to 2} = 12 + -6 + 1 = 7$

13. A price-demand function is p(x) = -1x^2 + 7x + 121. Find the limit as x approaches 4 and interpret it as the price when demand approaches 4 units.

$$\lim_{x \to 4} (-1x^2 + 7x + 121)$$

→ *Substitute x = 4: lim = -1(4)^2 + 7(4) + 121.*

→ *= -16 + 28 + 121 = 133.*

→ *The price approaches $133 as demand approaches 4 units.*

Answer: $\lim_{x \to 4} = -16 + 28 + 121 = 133$

14. Evaluate: limit as x approaches -2 of (1x^2 + -1x + 5).

$$\lim_{x \to -2} (1x^2 - 1x + 5)$$

→ *Polynomial limit — plug in x = -2 directly.*
→ *= 1(-2)^2 + -1(-2) + 5 = 11.*

Answer: $\lim_{x \to -2} = 4 + 2 + 5 = 11$

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15. Evaluate the limit by direct substitution: lim (x to -1) of (2x^2 + 4x + 3).

$$\lim_{x \to -1} (2x^2 + 4x + 3)$$

→ *Since the function is a polynomial, substitute x = -1 directly.*
→ *= 2(-1)^2 + 4(-1) + 3 = 2 + -4 + 3 = 1.*

Answer: $\lim_{x \to -1} = 2 + -4 + 3 = 1$

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16. A price-demand function is p(x) = -1x^2 + 5x + 142. Find the limit as x approaches 5 and interpret it as the price when demand approaches 5 units.

$$\lim_{x \to 5} (-1x^2 + 5x + 142)$$

→ *Substitute x = 5: lim = -1(5)^2 + 5(5) + 142.*
→ *= -25 + 25 + 142 = 142.*
→ *The price approaches $142 as demand approaches 5 units.*

Answer: $\lim_{x \to 5} = -25 + 25 + 142 = 142$

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17. Evaluate: limit as x approaches 4 of (3x^2 + -3x + 6).

$$\lim_{x \to 4} (3x^2 - 3x + 6)$$

→ *Polynomial limit — plug in x = 4 directly.*
→ *= 3(4)^2 + -3(4) + 6 = 42.*

Answer: $\lim_{x \to 4} = 48 + -12 + 6 = 42$

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18. Evaluate the limit by direct substitution: lim (x to 3) of (1x^2 + -1x + -3).

$$\lim_{x \to 3} (1x^2 - 1x - 3)$$

→ *Since the function is a polynomial, substitute x = 3 directly.*
→ *= 1(3)^2 + -1(3) + -3 = 9 + -3 + -3 = 3.*

Answer: $\lim_{x \to 3} = 9 + -3 + -3 = 3$

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19. A price-demand function is p(x) = -2x^2 + 12x + 133. Find the limit as x approaches 4 and interpret it as the price when demand approaches 4 units.

$$\lim_{x\to 4}(-2x^2 + 12x + 133)$$

→ *Substitute x = 4: lim = -2(4)^2 + 12(4) + 133.*

→ *= -32 + 48 + 133 = 149.*

→ *The price approaches $149 as demand approaches 4 units.*

Answer: $\lim_{x\to 4} = -32 + 48 + 133 = 149$

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20. Evaluate: limit as x approaches 0 of (1x^2 + -1x + -8).

$$\lim_{x\to 0}(1x^2 - 1x - 8)$$

→ *Polynomial limit — plug in x = 0 directly.*

→ *= 1(0)^2 + -1(0) + -8 = -8.*

Answer: $\lim_{x\to 0} = 0 + 0 + -8 = -8$

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21. Evaluate the limit by direct substitution: lim (x to 1) of (2x^2 + -3x + 5).

$$\lim_{x\to 1}(2x^2 - 3x + 5)$$

→ *Since the function is a polynomial, substitute x = 1 directly.*

→ *= 2(1)^2 + -3(1) + 5 = 2 + -3 + 5 = 4.*

Answer: $\lim_{x\to 1} = 2 + -3 + 5 = 4$

---

22. A price-demand function is p(x) = -1x^2 + 9x + 123. Find the limit as x approaches 3 and interpret it as the price when demand approaches 3 units.

$$\lim_{x\to 3}(-1x^2 + 9x + 123)$$

→ *Substitute x = 3: lim = -1(3)^2 + 9(3) + 123.*

→ *= -9 + 27 + 123 = 141.*

→ *The price approaches $141 as demand approaches 3 units.*

Answer: $\lim_{x\to 3} = -9 + 27 + 123 = 141$

---

23. Evaluate: limit as x approaches 2 of (2x^2 + -4x + -2).

$$\lim_{x\to 2}(2x^2 - 4x - 2)$$

→ *Polynomial limit — plug in x = 2 directly.*

→ *= 2(2)^2 + -4(2) + -2 = -2.*

Answer: $\lim_{x\to 2} = 8 + -8 + -2 = -2$

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24. Evaluate the limit by direct substitution: lim (x to 0) of (4x^2 + 3x + -1).

$$\lim_{x \to 0} (4x^2 + 3x - 1)$$

→ *Since the function is a polynomial, substitute x = 0 directly.*
→ *= 4(0)^2 + 3(0) + -1 = 0 + 0 + -1 = -1.*

Answer: $$\lim_{x \to 0} = 0 + 0 + -1 = -1$$

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25. A price-demand function is p(x) = -1x^2 + 6x + 130. Find the limit as x approaches 5 and interpret it as the price when demand approaches 5 units.

$$\lim_{x \to 5} (-1x^2 + 6x + 130)$$

→ *Substitute x = 5: lim = -1(5)^2 + 6(5) + 130.*
→ *= -25 + 30 + 130 = 135.*
→ *The price approaches $135 as demand approaches 5 units.*

Answer: $$\lim_{x \to 5} = -25 + 30 + 130 = 135$$

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26. Evaluate: limit as x approaches 0 of (2x^2 + -2x + 1).

$$\lim_{x \to 0} (2x^2 - 2x + 1)$$

→ *Polynomial limit — plug in x = 0 directly.*
→ *= 2(0)^2 + -2(0) + 1 = 1.*

Answer: $$\lim_{x \to 0} = 0 + 0 + 1 = 1$$

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27. Evaluate the limit by direct substitution: lim (x to 1) of (1x^2 + -3x + 1).

$$\lim_{x \to 1} (1x^2 - 3x + 1)$$

→ *Since the function is a polynomial, substitute x = 1 directly.*
→ *= 1(1)^2 + -3(1) + 1 = 1 + -3 + 1 = -1.*

Answer: $$\lim_{x \to 1} = 1 + -3 + 1 = -1$$

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28. A price-demand function is p(x) = -1x^2 + 13x + 64. Find the limit as x approaches 3 and interpret it as the price when demand approaches 3 units.

$$\lim_{x \to 3} (-1x^2 + 13x + 64)$$

→ *Substitute x = 3: lim = -1(3)^2 + 13(3) + 64.*
→ *= -9 + 39 + 64 = 94.*
→ *The price approaches $94 as demand approaches 3 units.*

Answer: $$\lim_{x \to 3} = -9 + 39 + 64 = 94$$

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29. Evaluate: limit as x approaches -1 of (3x^2 + -6x + -5).

$$\lim_{x \to -1} (3x^2 - 6x - 5)$$

→ *Polynomial limit — plug in x = -1 directly.*
→ *= 3(-1)^2 + -6(-1) + -5 = 4.*

Answer: $\lim_{x \to -1} = 3 + 6 + -5 = 4$

30. Evaluate the limit by direct substitution: lim (x to -2) of (3x^2 + 4x + 4).

$$\lim_{x \to -2} (3x^2 + 4x + 4)$$

→ *Since the function is a polynomial, substitute x = -2 directly.*
→ *= 3(-2)^2 + 4(-2) + 4 = 12 + -8 + 4 = 8.*

Answer: $\lim_{x \to -2} = 12 + -8 + 4 = 8$