MATH140: Applications of Derivatives

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Learning Objectives

- Find critical points by setting f'(x) = 0
- Classify intervals where f is increasing or decreasing
- Apply the First and Second Derivative Tests
- Solve optimization problems using calculus

Simplify each expression completely. Show all steps and circle your final answer.

Profit maximization

1. Profit is P(x) = -5x^2 + 16x + 94. Find the production level x that maximizes profit and compute that maximum profit.

$$-5x^2 + 16x + 94$$

Answer: ______________________________

2. Revenue is R(x) = -4x^2 + 38x + 178. Find the output level that maximizes revenue.

$$-4x^2 + 38x + 178$$

Answer: ______________________________

3. A company's weekly profit (in thousands) is P(x) = -2x^2 + 28x + 34, where x is price in dollars. Find the optimal price and maximum weekly profit.

$$-2x^2 + 28x + 34$$

Answer: ______________________________

4. Profit is P(x) = -5x^2 + 40x + 5. Find the production level x that maximizes profit and compute that maximum profit.

$$-5x^2 + 40x + 5$$

Answer: ______________________________

5. Revenue is R(x) = -2x^2 + 24x + 110. Find the output level that maximizes revenue.

$$-2x^2 + 24x + 110$$

Answer: ______________________________

6. A company's weekly profit (in thousands) is P(x) = -1x^2 + 22x + 8, where x is price in dollars. Find the optimal price and maximum weekly profit.

$$-1x^2 + 22x + 8$$

Answer: ____

7. Profit is P(x) = -2x^2 + 32x + 46. Find the production level x that maximizes profit and compute that maximum profit.

$$-2x^2 + 32x + 46$$

Answer: ____

8. Revenue is R(x) = -3x^2 + 52x + 141. Find the output level that maximizes revenue.

$$-3x^2 + 52x + 141$$

Answer: ____

9. A company's weekly profit (in thousands) is P(x) = -2x^2 + 13x + 56, where x is price in dollars. Find the optimal price and maximum weekly profit.

$$-2x^2 + 13x + 56$$

Answer: ____

10. Profit is P(x) = -4x^2 + 43x + 71. Find the production level x that maximizes profit and compute that maximum profit.

$$-4x^2 + 43x + 71$$

Answer: ____

11. Revenue is R(x) = -4x^2 + 32x + 38. Find the output level that maximizes revenue.

$$-4x^2 + 32x + 38$$

Answer: ____

12. A company's weekly profit (in thousands) is P(x) = -1x^2 + 12x + 30, where x is price in dollars. Find the optimal price and maximum weekly profit.

$$-1x^2 + 12x + 30$$

Answer: ____

13. Profit is P(x) = -1x^2 + 27x + 26. Find the production level x that maximizes profit and compute that maximum profit.

$$-1x^2 + 27x + 26$$

Answer: ____

14. Revenue is R(x) = -3x^2 + 25x + 166. Find the output level that maximizes revenue.

$$-3x^2 + 25x + 166$$

Answer: ____

15. A company's weekly profit (in thousands) is P(x) = -2x^2 + 21x + 36, where x is price in dollars. Find the optimal price and maximum weekly profit.

$$-2x^2 + 21x + 36$$

Answer: ____

16. Profit is P(x) = -1x^2 + 26x + 8. Find the production level x that maximizes profit and compute that maximum profit.

$$-1x^2 + 26x + 8$$

Answer: ____

17. Revenue is R(x) = -1x^2 + 48x + 75. Find the output level that maximizes revenue.

$$-1x^2 + 48x + 75$$

Answer: ____

18. A company's weekly profit (in thousands) is P(x) = -2x^2 + 15x + 40, where x is price in dollars. Find the optimal price and maximum weekly profit.

$$-2x^2 + 15x + 40$$

Answer: ____

19. Profit is P(x) = -5x^2 + 34x + 55. Find the production level x that maximizes profit and compute that maximum profit.

$$-5x^2 + 34x + 55$$

Answer: ____

20. Revenue is R(x) = -4x^2 + 27x + 196. Find the output level that maximizes revenue.

$$-4x^2 + 27x + 196$$

Answer: ____

21. A company's weekly profit (in thousands) is P(x) = -1x^2 + 26x + 60, where x is price in dollars. Find the optimal price and maximum weekly profit.

$$-1x^2 + 26x + 60$$

Answer: ____

22. Profit is P(x) = -5x^2 + 33x + 89. Find the production level x that maximizes profit and compute that maximum profit.

$$-5x^2 + 33x + 89$$

Answer: ____

23. Revenue is R(x) = -2x^2 + 52x + 158. Find the output level that maximizes revenue.

$$-2x^2 + 52x + 158$$

Answer: ____

Marginal cost and revenue

24. Total cost is C(x) = 2x^3 + -2x^2 + 20x + 115. Find the marginal cost MC = C'(1).

$$f(x) = 2x^3 + \ -2x^2 + 20x + 115,\ x = 1$$

Answer: ____

25. Total cost is C(x) = 1x^3 + -3x^2 + 18x + 288. Find the marginal cost MC = C'(1).

$$f(x) = 1x^3 + \ -3x^2 + 18x + 288,\ x = 1$$

Answer: ____

26. Total cost is C(x) = 1x^3 + -2x^2 + 20x + 289. Find the marginal cost MC = C'(3).

$$f(x) = 1x^3 + \ -2x^2 + 20x + 289,\ x = 3$$

Answer: ____

27. Total cost is C(x) = 1x^3 + -3x^2 + 5x + 107. Find the marginal cost MC = C'(3).

$$f(x) = 1x^3 + \ -3x^2 + 5x + 107,\ x = 3$$

Answer: ____

28. Total cost is C(x) = 2x^3 + -4x^2 + 11x + 257. Find the marginal cost MC = C'(3).

$$f(x) = 2x^3 + \ -4x^2 + 11x + 257,\ x = 3$$

Answer: ____

29. Total cost is C(x) = 2x^3 + -3x^2 + 14x + 181. Find the marginal cost MC = C'(2).

$$f(x) = 2x^3 + \ -3x^2 + 14x + 181,\ x = 2$$

Answer: ____

30. Total cost is C(x) = 1x^3 + -5x^2 + 10x + 67. Find the marginal cost MC = C'(3).

$$f(x) = 1x^3 + \ -5x^2 + 10x + 67,\ x = 3$$

Answer: ____

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ANSWER KEY & SOLUTIONS

Topics: Marginal cost and revenue, Profit maximization. All answers verified by independent computation.

Solutions

Profit maximization

1. Profit is P(x) = -5x^2 + 16x + 94. Find the production level x that maximizes profit and compute that maximum profit.

$$-5x^2 + 16x + 94$$

→ *P'(x) = -10x + 16. Set P'(x) = 0.*
→ *x* = -16/-10 = 8/5.*
→ *Maximum profit = P(8/5) = 534/5.*

Answer: $x^* = -\frac{16}{-10} = \frac{8}{5},\ f(x^*) = \frac{534}{5}$

2. Revenue is R(x) = -4x^2 + 38x + 178. Find the output level that maximizes revenue.

$$-4x^2 + 38x + 178$$

→ *R'(x) = -8x + 38. Set equal to zero: x = 19/4.*
→ *Maximum revenue = 1073/4.*

Answer: $x^* = -\frac{38}{-8} = \frac{19}{4},\ f(x^*) = \frac{1073}{4}$

3. A company's weekly profit (in thousands) is P(x) = -2x^2 + 28x + 34, where x is price in dollars. Find the optimal price and maximum weekly profit.

$$-2x^2 + 28x + 34$$

→ *Set P'(x) = -4x + 28 = 0.*
→ *Optimal price: x = 7 dollars.*
→ *Maximum profit: P(7) = 132 thousand dollars.*

Answer: $x^* = -\frac{28}{-4} = 7,\ f(x^*) = 132$

4. Profit is P(x) = -5x^2 + 40x + 5. Find the production level x that maximizes profit and compute that maximum profit.

$$-5x^2 + 40x + 5$$

→ *P'(x) = -10x + 40. Set P'(x) = 0.*
→ *x* = -40/-10 = 4.*
→ *Maximum profit = P(4) = 85.*

Answer: $x^* = -\frac{40}{-10} = 4,\ f(x^*) = 85$

5. Revenue is R(x) = -2x^2 + 24x + 110. Find the output level that maximizes revenue.

$$-2x^2 + 24x + 110$$

→ *R'(x) = -4x + 24. Set equal to zero: x = 6.*
→ *Maximum revenue = 182.*

Answer: $x^* = -\frac{24}{-4} = 6,\ f(x^*) = 182$

6. A company's weekly profit (in thousands) is P(x) = -1x^2 + 22x + 8, where x is price in dollars. Find the optimal price and maximum weekly profit.

$$-1x^2 + 22x + 8$$

→ *Set P'(x) = -2x + 22 = 0.*
→ *Optimal price: x = 11 dollars.*
→ *Maximum profit: P(11) = 129 thousand dollars.*

Answer: $x^* = -\frac{22}{-2} = 11,\ f(x^*) = 129$

7. Profit is P(x) = -2x^2 + 32x + 46. Find the production level x that maximizes profit and compute that maximum profit.

$$-2x^2 + 32x + 46$$

→ *P'(x) = -4x + 32. Set P'(x) = 0.*
→ *x* = -32/-4 = 8.*
→ *Maximum profit = P(8) = 174.*

Answer: $x^* = -\frac{32}{-4} = 8,\ f(x^*) = 174$

8. Revenue is R(x) = -3x^2 + 52x + 141. Find the output level that maximizes revenue.

$$-3x^2 + 52x + 141$$

→ *R'(x) = -6x + 52. Set equal to zero: x = 26/3.*
→ *Maximum revenue = 1099/3.*

Answer: $x^* = -\frac{52}{-6} = \frac{26}{3},\ f(x^*) = \frac{1099}{3}$

9. A company's weekly profit (in thousands) is P(x) = -2x^2 + 13x + 56, where x is price in dollars. Find the optimal price and maximum weekly profit.

$$-2x^2 + 13x + 56$$

→ *Set P'(x) = -4x + 13 = 0.*
→ *Optimal price: x = 13/4 dollars.*
→ *Maximum profit: P(13/4) = 617/8 thousand dollars.*

Answer: $x^* = -\frac{13}{-4} = \frac{13}{4},\ f(x^*) = \frac{617}{8}$

10. Profit is P(x) = -4x^2 + 43x + 71. Find the production level x that maximizes profit and compute that maximum profit.

$$-4x^2 + 43x + 71$$

→ *P'(x) = -8x + 43. Set P'(x) = 0.*
→ *x* = -43/-8 = 43/8.*
→ *Maximum profit = P(43/8) = 2985/16.*

Answer: $x^* = -\frac{43}{-8} = \frac{43}{8},\ f(x^*) = \frac{2985}{16}$

11. Revenue is R(x) = -4x^2 + 32x + 38. Find the output level that maximizes revenue.

$$-4x^2 + 32x + 38$$

→ R'(x) = -8x + 32. Set equal to zero: x = 4.
→ Maximum revenue = 102.

Answer: $x^* = -\frac{32}{-8} = 4,\ f(x^*) = 102$

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12. A company's weekly profit (in thousands) is P(x) = -1x^2 + 12x + 30, where x is price in dollars. Find the optimal price and maximum weekly profit.

$$-1x^2 + 12x + 30$$

→ Set P'(x) = -2x + 12 = 0.
→ Optimal price: x = 6 dollars.
→ Maximum profit: P(6) = 66 thousand dollars.

Answer: $x^* = -\frac{12}{-2} = 6,\ f(x^*) = 66$

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13. Profit is P(x) = -1x^2 + 27x + 26. Find the production level x that maximizes profit and compute that maximum profit.

$$-1x^2 + 27x + 26$$

→ P'(x) = -2x + 27. Set P'(x) = 0.
→ x = -27/-2 = 27/2.*
→ Maximum profit = P(27/2) = 833/4.

Answer: $x^* = -\frac{27}{-2} = \frac{27}{2},\ f(x^*) = \frac{833}{4}$

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14. Revenue is R(x) = -3x^2 + 25x + 166. Find the output level that maximizes revenue.

$$-3x^2 + 25x + 166$$

→ R'(x) = -6x + 25. Set equal to zero: x = 25/6.
→ Maximum revenue = 2617/12.

Answer: $x^* = -\frac{25}{-6} = \frac{25}{6},\ f(x^*) = \frac{2617}{12}$

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15. A company's weekly profit (in thousands) is P(x) = -2x^2 + 21x + 36, where x is price in dollars. Find the optimal price and maximum weekly profit.

$$-2x^2 + 21x + 36$$

→ Set P'(x) = -4x + 21 = 0.
→ Optimal price: x = 21/4 dollars.
→ Maximum profit: P(21/4) = 729/8 thousand dollars.

Answer: $x^* = -\frac{21}{-4} = \frac{21}{4},\ f(x^*) = \frac{729}{8}$

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16. Profit is P(x) = -1x^2 + 26x + 8. Find the production level x that maximizes profit and compute that maximum profit.

$$-1x^2 + 26x + 8$$

→ P'(x) = -2x + 26. Set P'(x) = 0.
→ x = -26/-2 = 13.*
→ Maximum profit = P(13) = 177.

Answer: $x^* = -\frac{26}{-2} = 13,\ f(x^*) = 177$

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17. Revenue is R(x) = -1x^2 + 48x + 75. Find the output level that maximizes revenue.

$$-1x^2 + 48x + 75$$

→ R'(x) = -2x + 48. Set equal to zero: x = 24.
→ Maximum revenue = 651.

Answer: $x^* = -\frac{48}{-2} = 24,\ f(x^*) = 651$

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18. A company's weekly profit (in thousands) is P(x) = -2x^2 + 15x + 40, where x is price in dollars. Find the optimal price and maximum weekly profit.

$$-2x^2 + 15x + 40$$

→ Set P'(x) = -4x + 15 = 0.
→ Optimal price: x = 15/4 dollars.
→ Maximum profit: P(15/4) = 545/8 thousand dollars.

Answer: $x^* = -\frac{15}{-4} = \frac{15}{4},\ f(x^*) = \frac{545}{8}$

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19. Profit is P(x) = -5x^2 + 34x + 55. Find the production level x that maximizes profit and compute that maximum profit.

$$-5x^2 + 34x + 55$$

→ P'(x) = -10x + 34. Set P'(x) = 0.
→ x = -34/-10 = 17/5.*
→ Maximum profit = P(17/5) = 564/5.

Answer: $x^* = -\frac{34}{-10} = \frac{17}{5},\ f(x^*) = \frac{564}{5}$

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20. Revenue is R(x) = -4x^2 + 27x + 196. Find the output level that maximizes revenue.

$$-4x^2 + 27x + 196$$

→ R'(x) = -8x + 27. Set equal to zero: x = 27/8.
→ Maximum revenue = 3865/16.

Answer: $x^* = -\frac{27}{-8} = \frac{27}{8},\ f(x^*) = \frac{3865}{16}$

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21. A company's weekly profit (in thousands) is P(x) = -1x^2 + 26x + 60, where x is price in dollars. Find the optimal price and maximum weekly profit.

$$-1x^2 + 26x + 60$$

→ *Set P'(x) = -2x + 26 = 0.*
→ *Optimal price: x = 13 dollars.*
→ *Maximum profit: P(13) = 229 thousand dollars.*

Answer: $x^* = -\frac{26}{-2} = 13,\ f(x^*) = 229$

22. Profit is P(x) = -5x^2 + 33x + 89. Find the production level x that maximizes profit and compute that maximum profit.

$$-5x^2 + 33x + 89$$

→ *P'(x) = -10x + 33. Set P'(x) = 0.*
→ *x* = -33/-10 = 33/10.*
→ *Maximum profit = P(33/10) = 2869/20.*

Answer: $x^* = -\frac{33}{-10} = \frac{33}{10},\ f(x^*) = \frac{2869}{20}$

23. Revenue is R(x) = -2x^2 + 52x + 158. Find the output level that maximizes revenue.

$$-2x^2 + 52x + 158$$

→ *R'(x) = -4x + 52. Set equal to zero: x = 13.*
→ *Maximum revenue = 496.*

Answer: $x^* = -\frac{52}{-4} = 13,\ f(x^*) = 496$

Marginal cost and revenue

24. Total cost is C(x) = 2x^3 + -2x^2 + 20x + 115. Find the marginal cost MC = C'(1).

$$f(x) = 2x^3 + -2x^2 + 20x + 115,\ x = 1$$

→ MC = C'(x) = 6x^2 + -4x + 20.
→ MC at x = 1: C'(1) = 22.

Answer: $f'(x) = 6x^2 + -4x + 20, \quad f'(1) = 22$

25. Total cost is C(x) = 1x^3 + -3x^2 + 18x + 288. Find the marginal cost MC = C'(1).

$$f(x) = 1x^3 + -3x^2 + 18x + 288,\ x = 1$$

→ MC = C'(x) = 3x^2 + -6x + 18.
→ MC at x = 1: C'(1) = 15.

Answer: $f'(x) = 3x^2 + -6x + 18, \quad f'(1) = 15$

26. Total cost is C(x) = 1x^3 + -2x^2 + 20x + 289. Find the marginal cost MC = C'(3).

$$f(x) = 1x^3 + -2x^2 + 20x + 289,\ x = 3$$

→ MC = C'(x) = 3x^2 + -4x + 20.
→ MC at x = 3: C'(3) = 35.

Answer: $f'(x) = 3x^2 + -4x + 20, \quad f'(3) = 35$

27. Total cost is C(x) = 1x^3 + -3x^2 + 5x + 107. Find the marginal cost MC = C'(3).

$$f(x) = 1x^3 + -3x^2 + 5x + 107,\ x = 3$$

→ MC = C'(x) = 3x^2 + -6x + 5.
→ MC at x = 3: C'(3) = 14.

Answer: $f'(x) = 3x^2 + -6x + 5, \quad f'(3) = 14$

28. Total cost is C(x) = 2x^3 + -4x^2 + 11x + 257. Find the marginal cost MC = C'(3).

$$f(x) = 2x^3 + -4x^2 + 11x + 257,\ x = 3$$

→ MC = C'(x) = 6x^2 + -8x + 11.
→ MC at x = 3: C'(3) = 41.

Answer: $f'(x) = 6x^2 + -8x + 11, \quad f'(3) = 41$

29. Total cost is C(x) = 2x^3 + -3x^2 + 14x + 181. Find the marginal cost MC = C'(2).

$$f(x) = 2x^3 + -3x^2 + 14x + 181,\ x = 2$$

→ MC = C'(x) = 6x^2 + -6x + 14.
→ MC at x = 2: C'(2) = 26.

Answer: $f'(x) = 6x^2 + -6x + 14, \quad f'(2) = 26$

30. Total cost is C(x) = 1x^3 + -5x^2 + 10x + 67. Find the marginal cost MC = C'(3).

$$f(x) = 1x^3 + -5x^2 + 10x + 67,\ x = 3$$

→ *MC = C'(x) = 3x^2 + -10x + 10.*
→ *MC at x = 3: C'(3) = 7.*

Answer: $f'(x) = 3x^2 + -10x + 10, \quad f'(3) = 7$