MATH140: Applications of Integration

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Name: ________________________________ Date: ________________ Score: / 30

Learning Objectives

- Find the area between two curves using integration
- Compute the average value of a function over an interval
- Set up and evaluate volumes of revolution (disk and washer methods)

Simplify each expression completely. Show all steps and circle your final answer.

Total accumulated change

1. A factory's output rate is O(t) = x^1 hundred units per hour. Find total output from hour 2 to hour 5.

$$\int_{2}^{5} x^{1}\,dx$$

Answer: ____

2. A factory's output rate is O(t) = x^2 hundred units per hour. Find total output from hour 1 to hour 5.

$$\int_{1}^{5} x^{2}\,dx$$

Answer: ____

3. A factory's output rate is O(t) = x^1 hundred units per hour. Find total output from hour 2 to hour 5.

$$\int_{2}^{5} x^{1}\,dx$$

Answer: ____

4. A factory's output rate is O(t) = x^1 hundred units per hour. Find total output from hour 0 to hour 4.

$$\int_{0}^{4} x^{1}\,dx$$

Answer: ____

5. A factory's output rate is O(t) = x^2 hundred units per hour. Find total output from hour 2 to hour 5.

$$\int_{2}^{5} x^{2}\,dx$$

Answer: ____

6. A factory's output rate is O(t) = x^2 hundred units per hour. Find total output from hour 2 to hour 3.

$$\int_{2}^{3} x^2 \, dx$$

Answer: ____

7. A factory's output rate is O(t) = x^2 hundred units per hour. Find total output from hour 2 to hour 4.

$$\int_{2}^{4} x^2 \, dx$$

Answer: ____

8. A factory's output rate is O(t) = x^2 hundred units per hour. Find total output from hour 1 to hour 5.

$$\int_{1}^{5} x^2 \, dx$$

Answer: ____

9. A factory's output rate is O(t) = x^2 hundred units per hour. Find total output from hour 1 to hour 4.

$$\int_{1}^{4} x^2 \, dx$$

Answer: ____

10. A factory's output rate is O(t) = x^1 hundred units per hour. Find total output from hour 0 to hour 3.

$$\int_{0}^{3} x^1 \, dx$$

Answer: ____

Area under a demand curve

11. Find the area under the demand curve f(x) = x^2 from x = 0 to x = 4. This represents the total willingness-to-pay over that output range.

$$\int_{0}^{4} x^2 \, dx$$

Answer: ____

12. Find the area under the demand curve f(x) = x^2 from x = 0 to x = 5. This represents the total willingness-to-pay over that output range.

$$\int_{0}^{5} x^2 \, dx$$

Answer: ____

13. Find the area under the demand curve f(x) = x^2 from x = 1 to x = 3. This represents the total willingness-to-pay over that output range.

$$\int_1^3 x^2 \, dx$$

Answer: ____

14. Find the area under the demand curve f(x) = x^1 from x = 1 to x = 4. This represents the total willingness-to-pay over that output range.

$$\int_1^4 x^1 \, dx$$

Answer: ____

15. Find the area under the demand curve f(x) = x^1 from x = 0 to x = 5. This represents the total willingness-to-pay over that output range.

$$\int_0^5 x^1 \, dx$$

Answer: ____

16. Find the area under the demand curve f(x) = x^1 from x = 0 to x = 3. This represents the total willingness-to-pay over that output range.

$$\int_0^3 x^1 \, dx$$

Answer: ____

17. Find the area under the demand curve f(x) = x^1 from x = 0 to x = 4. This represents the total willingness-to-pay over that output range.

$$\int_0^4 x^1 \, dx$$

Answer: ____

18. Find the area under the demand curve f(x) = x^1 from x = 1 to x = 5. This represents the total willingness-to-pay over that output range.

$$\int_1^5 x^1 \, dx$$

Answer: ____

19. Find the area under the demand curve f(x) = x^1 from x = 1 to x = 4. This represents the total willingness-to-pay over that output range.

$$\int_{1}^{4} x^{1}\,dx$$

Answer: ____

20. Find the area under the demand curve f(x) = x^2 from x = 1 to x = 4. This represents the total willingness-to-pay over that output range.

$$\int_{1}^{4} x^{2}\,dx$$

Answer: ____

Total cost from marginal cost

21. Marginal cost is MC(x) = 5x^1. Find the total variable cost of producing from x = 0 to x = 6 units.

$$\int_{0}^{6} x^{1}\,dx$$

Answer: ____

22. Marginal cost is MC(x) = 4x^1. Find the total variable cost of producing from x = 1 to x = 7 units.

$$\int_{1}^{7} x^{1}\,dx$$

Answer: ____

23. Marginal cost is MC(x) = 5x^2. Find the total variable cost of producing from x = 1 to x = 4 units.

$$\int_{1}^{4} x^{2}\,dx$$

Answer: ____

24. Marginal cost is MC(x) = 6x^1. Find the total variable cost of producing from x = 1 to x = 7 units.

$$\int_{1}^{7} x^{1}\,dx$$

Answer: ____

25. Marginal cost is MC(x) = 8x^1. Find the total variable cost of producing from x = 1 to x = 7 units.

$$\int_{1}^{7} x^{1}\,dx$$

Answer: ____

26. Marginal cost is MC(x) = 3x^1. Find the total variable cost of producing from x = 1 to x = 3 units.

$$\int_1^3 x^1 \, dx$$

Answer: ____

27. Marginal cost is MC(x) = 6x^1. Find the total variable cost of producing from x = 0 to x = 4 units.

$$\int_0^4 x^1 \, dx$$

Answer: ____

28. Marginal cost is MC(x) = 5x^2. Find the total variable cost of producing from x = 1 to x = 7 units.

$$\int_1^7 x^2 \, dx$$

Answer: ____

29. Marginal cost is MC(x) = 7x^2. Find the total variable cost of producing from x = 0 to x = 7 units.

$$\int_0^7 x^2 \, dx$$

Answer: ____

30. Marginal cost is MC(x) = 4x^1. Find the total variable cost of producing from x = 1 to x = 4 units.

$$\int_1^4 x^1 \, dx$$

Answer: ____

ANSWER KEY & SOLUTIONS

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Topics: Total accumulated change, Area under a demand curve, Total cost from marginal cost. All answers verified by independent computation.

Solutions

Total accumulated change

1. A factory's output rate is O(t) = x^1 hundred units per hour. Find total output from hour 2 to hour 5.

$$\int_{2}^{5} x^{1}\,dx$$

→ *Total output = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210} hundred units.*

Answer: $\frac{25-4}{2} = 21/2$

2. A factory's output rate is O(t) = x^2 hundred units per hour. Find total output from hour 1 to hour 5.

$$\int_{1}^{5} x^{2}\,dx$$

→ *Total output = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210} hundred units.*

Answer: $\frac{125-1}{3} = 124/3$

3. A factory's output rate is O(t) = x^1 hundred units per hour. Find total output from hour 2 to hour 5.

$$\int_{2}^{5} x^{1}\,dx$$

→ *Total output = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210} hundred units.*

Answer: $\frac{25-4}{2} = 21/2$

4. A factory's output rate is O(t) = x^1 hundred units per hour. Find total output from hour 0 to hour 4.

$$\int_{0}^{4} x^{1}\,dx$$

→ *Total output = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210} hundred units.*

Answer: $\frac{16-0}{2} = 8$

5. A factory's output rate is O(t) = x^2 hundred units per hour. Find total output from hour 2 to hour 5.

$$\int_{2}^{5} x^{2}\,dx$$

→ *Total output = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210} hundred units.*

Answer: $\frac{125-8}{3} = 39$

6. A factory's output rate is O(t) = x^2 hundred units per hour. Find total output from hour 2 to hour 3.

$$\int_2^3 x^2\,dx$$

→ *Total output = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210} hundred units.*

Answer: $\frac{27-8}{3} = 19/3$

7. A factory's output rate is O(t) = x^2 hundred units per hour. Find total output from hour 2 to hour 4.

$$\int_2^4 x^2\,dx$$

→ *Total output = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210} hundred units.*

Answer: $\frac{64-8}{3} = 56/3$

8. A factory's output rate is O(t) = x^2 hundred units per hour. Find total output from hour 1 to hour 5.

$$\int_1^5 x^2\,dx$$

→ *Total output = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210} hundred units.*

Answer: $\frac{125-1}{3} = 124/3$

9. A factory's output rate is O(t) = x^2 hundred units per hour. Find total output from hour 1 to hour 4.

$$\int_1^4 x^2\,dx$$

→ *Total output = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210} hundred units.*

Answer: $\frac{64-1}{3} = 21$

10. A factory's output rate is O(t) = x^1 hundred units per hour. Find total output from hour 0 to hour 3.

$$\int_0^3 x^1\,dx$$

→ *Total output = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210} hundred units.*

Answer: $\frac{9-0}{2} = 9/2$

Area under a demand curve

11. Find the area under the demand curve f(x) = x^2 from x = 0 to x = 4. This represents the total willingness-to-pay over that output range.

$$\int_0^4 x^2\,dx$$

→ *Area = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210}.*

Answer: $$\frac{64-0}{3} = 64/3$$

12. Find the area under the demand curve f(x) = x^2 from x = 0 to x = 5. This represents the total willingness-to-pay over that output range.

$$\int_0^5 x^2\,dx$$

→ *Area = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210}.*

Answer: $$\frac{125-0}{3} = 125/3$$

13. Find the area under the demand curve f(x) = x^2 from x = 1 to x = 3. This represents the total willingness-to-pay over that output range.

$$\int_1^3 x^2\,dx$$

→ *Area = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210}.*

Answer: $$\frac{27-1}{3} = 26/3$$

14. Find the area under the demand curve f(x) = x^1 from x = 1 to x = 4. This represents the total willingness-to-pay over that output range.

$$\int_1^4 x^1\,dx$$

→ *Area = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210}.*

Answer: $$\frac{16-1}{2} = 15/2$$

15. Find the area under the demand curve f(x) = x^1 from x = 0 to x = 5. This represents the total willingness-to-pay over that output range.

$$\int_0^5 x^1\,dx$$

→ *Area = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.*
→ *= ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210}.*

Answer: $$\frac{25-0}{2} = 25/2$$

16. Find the area under the demand curve f(x) = x^1 from x = 0 to x = 3. This represents the total willingness-to-pay over that output range.

$$\int_0^3 x^1 \, dx$$

→ Area = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.
→ = ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210}.

Answer: $\frac{9-0}{2} = 9/2$

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17. Find the area under the demand curve f(x) = x^1 from x = 0 to x = 4. This represents the total willingness-to-pay over that output range.

$$\int_0^4 x^1 \, dx$$

→ Area = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.
→ = ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210}.

Answer: $\frac{16-0}{2} = 8$

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18. Find the area under the demand curve f(x) = x^1 from x = 1 to x = 5. This represents the total willingness-to-pay over that output range.

$$\int_1^5 x^1 \, dx$$

→ Area = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.
→ = ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210}.

Answer: $\frac{25-1}{2} = 12$

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19. Find the area under the demand curve f(x) = x^1 from x = 1 to x = 4. This represents the total willingness-to-pay over that output range.

$$\int_1^4 x^1 \, dx$$

→ Area = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.
→ = ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210}.

Answer: $\frac{16-1}{2} = 15/2$

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20. Find the area under the demand curve f(x) = x^2 from x = 1 to x = 4. This represents the total willingness-to-pay over that output range.

$$\int_1^4 x^2 \, dx$$

→ Area = integral from {lo} to {hi} of x^{n} dx = [x^{n+1}/({n+1})] from {lo} to {hi}.
→ = ({hi_pow} - {lo_pow}) / ({n+1}) = {answer_defint210}.

Answer: $\frac{64-1}{3} = 21$

Total cost from marginal cost

21. Marginal cost is MC(x) = 5x^1. Find the total variable cost of producing from x = 0 to x = 6 units.

$$\int_0^6 x^1\,dx$$

→ *TVC = integral from 0 to 6 of 5x^1 dx.*
→ *Note: for a·x^n, multiply the standard result by 5: 5 × 18.*

Answer: $\frac{36-0}{2} = 18$

22. Marginal cost is MC(x) = 4x^1. Find the total variable cost of producing from x = 1 to x = 7 units.

$$\int_1^7 x^1\,dx$$

→ *TVC = integral from 1 to 7 of 4x^1 dx.*
→ *Note: for a·x^n, multiply the standard result by 4: 4 × 24.*

Answer: $\frac{49-1}{2} = 24$

23. Marginal cost is MC(x) = 5x^2. Find the total variable cost of producing from x = 1 to x = 4 units.

$$\int_1^4 x^2\,dx$$

→ *TVC = integral from 1 to 4 of 5x^2 dx.*
→ *Note: for a·x^n, multiply the standard result by 5: 5 × 21.*

Answer: $\frac{64-1}{3} = 21$

24. Marginal cost is MC(x) = 6x^1. Find the total variable cost of producing from x = 1 to x = 7 units.

$$\int_1^7 x^1\,dx$$

→ *TVC = integral from 1 to 7 of 6x^1 dx.*
→ *Note: for a·x^n, multiply the standard result by 6: 6 × 24.*

Answer: $\frac{49-1}{2} = 24$

25. Marginal cost is MC(x) = 8x^1. Find the total variable cost of producing from x = 1 to x = 7 units.

$$\int_1^7 x^1\,dx$$

→ *TVC = integral from 1 to 7 of 8x^1 dx.*
→ *Note: for a·x^n, multiply the standard result by 8: 8 × 24.*

Answer: $\frac{49-1}{2} = 24$

26. Marginal cost is MC(x) = 3x^1. Find the total variable cost of producing from x = 1 to x = 3 units.

$$\int_1^3 x^1\,dx$$

→ *TVC = integral from 1 to 3 of 3x^1 dx.*
→ *Note: for a·x^n, multiply the standard result by 3: 3 × 4.*

Answer: $\frac{9-1}{2} = 4$

27. Marginal cost is MC(x) = 6x^1. Find the total variable cost of producing from x = 0 to x = 4 units.

$$\int_0^4 x^1\,dx$$

→ *TVC = integral from 0 to 4 of 6x^1 dx.*
→ *Note: for a·x^n, multiply the standard result by 6: 6 × 8.*

Answer: $\frac{16-0}{2} = 8$

28. Marginal cost is MC(x) = 5x^2. Find the total variable cost of producing from x = 1 to x = 7 units.

$$\int_1^7 x^2\,dx$$

→ *TVC = integral from 1 to 7 of 5x^2 dx.*
→ *Note: for a·x^n, multiply the standard result by 5: 5 × 114.*

Answer: $\frac{343-1}{3} = 114$

29. Marginal cost is MC(x) = 7x^2. Find the total variable cost of producing from x = 0 to x = 7 units.

$$\int_0^7 x^2\,dx$$

→ *TVC = integral from 0 to 7 of 7x^2 dx.*
→ *Note: for a·x^n, multiply the standard result by 7: 7 × 343/3.*

Answer: $\frac{343-0}{3} = 343/3$

30. Marginal cost is MC(x) = 4x^1. Find the total variable cost of producing from x = 1 to x = 4 units.

$$\int_1^4 x^1\,dx$$

→ *TVC = integral from 1 to 4 of 4x^1 dx.*
→ *Note: for a·x^n, multiply the standard result by 4: 4 × 15/2.*

Answer: $\frac{16-1}{2} = 15/2$