MATH140: Differentiation Rules

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Name: _____ Date: _____ Score: / 30

Learning Objectives

- Calculate the mean, median, mode, and range of a data set
- Compute sample variance and standard deviation
- Construct quartiles, the IQR, and identify outliers
- Describe the shape, center, and spread of a distribution

Simplify each expression completely. Show all steps and circle your final answer.

Chain rule

1. A cost function is C(x) = (1x + 1)^3. Find C'(1) using the chain rule.

$$f(x) = (1x + 1)^3,\ x = 1$$

Answer: _____

2. A cost function is C(x) = (1x + 7)^2. Find C'(1) using the chain rule.

$$f(x) = (1x + 7)^2,\ x = 1$$

Answer: _____

3. A cost function is C(x) = (4x + 5)^3. Find C'(1) using the chain rule.

$$f(x) = (4x + 5)^3,\ x = 1$$

Answer: _____

4. A cost function is C(x) = (2x + 8)^3. Find C'(3) using the chain rule.

$$f(x) = (2x + 8)^3,\ x = 3$$

Answer: _____

5. A cost function is C(x) = (2x + 4)^2. Find C'(0) using the chain rule.

$$f(x) = (2x + 4)^2,\ x = 0$$

Answer: _____

6. A cost function is C(x) = (2x + 2)^2. Find C'(3) using the chain rule.

$$f(x) = (2x + 2)^2,\ x = 3$$

Answer: _____

7. A cost function is C(x) = (1x + 5)^3. Find C'(1) using the chain rule.

$$f(x) = (1x + 5)^3,\ x = 1$$

Answer: _____

8. A cost function is C(x) = (1x + 5)^3. Find C'(3) using the chain rule.

$$f(x) = (1x + 5)^3,\ x = 3$$

Answer: _____

Differentiating polynomials

9. Total revenue is R(x) = 1x^3 + -2x^2 + 14x + 59. Find the marginal revenue MR = R'(2).

$$f(x) = 1x^3 + -2x^2 + 14x + 59,\ x = 2$$

Answer: _____

10. Total revenue is R(x) = 2x^3 + -4x^2 + 23x + 70. Find the marginal revenue MR = R'(2).

$$f(x) = 2x^3 + -4x^2 + 23x + 70,\ x = 2$$

Answer: _____

11. Total revenue is R(x) = 1x^3 + 0x^2 + 27x + 20. Find the marginal revenue MR = R'(1).

$$f(x) = 1x^3 + 0x^2 + 27x + 20,\ x = 1$$

Answer: _____

12. Total revenue is R(x) = 1x^3 + -3x^2 + 14x + 94. Find the marginal revenue MR = R'(2).

$$f(x) = 1x^3 + -3x^2 + 14x + 94,\ x = 2$$

Answer: _____

13. Total revenue is R(x) = 1x^3 + -4x^2 + 30x + 58. Find the marginal revenue MR = R'(1).

$$f(x) = 1x^3 + -4x^2 + 30x + 58,\ x = 1$$

Answer: _____

14. Total revenue is R(x) = 2x^3 + -1x^2 + 19x + 32. Find the marginal revenue MR = R'(2).

$$f(x) = 2x^3 + -1x^2 + 19x + 32,\ x = 2$$

Answer: _____

15. Total revenue is R(x) = 1x^3 + -4x^2 + 23x + 59. Find the marginal revenue MR = R'(2).

$$f(x) = 1x^3 + -4x^2 + 23x + 59,\ x = 2$$

Answer: _____

16. Total revenue is R(x) = 2x^3 + 0x^2 + 29x + 1. Find the marginal revenue MR = R'(2).

$$f(x) = 2x^3 + 0x^2 + 29x + 1,\ x = 2$$

Answer: _____

Power rule

17. Find the derivative of f(x) = 7x^2 and evaluate f'(2).

$$f(x) = 7x^2, \quad x = 2$$

Answer: _____

18. Find the derivative of f(x) = 6x^3 and evaluate f'(3).

$$f(x) = 6x^3, \quad x = 3$$

Answer: _____

19. Find the derivative of f(x) = 5x^2 and evaluate f'(4).

$$f(x) = 5x^2, \quad x = 4$$

Answer: _____

20. Find the derivative of f(x) = 1x^5 and evaluate f'(3).

$$f(x) = 1x^5, \quad x = 3$$

Answer: _____

21. Find the derivative of f(x) = 5x^4 and evaluate f'(2).

$$f(x) = 5x^4, \quad x = 2$$

Answer: _____

22. Find the derivative of f(x) = 8x^2 and evaluate f'(4).

$$f(x) = 8x^2, \quad x = 4$$

Answer: _____

23. Find the derivative of f(x) = 8x^2 and evaluate f'(2).

$$f(x) = 8x^2, \quad x = 2$$

Answer: _____

Product rule

24. Revenue is R(x) = (3x + 7)(4x + 5). Use the product rule to find R'(0).

$$f(x) = (3x + 7)(4x + 5),\ x = 0$$

Answer: _____

25. Revenue is R(x) = (1x + 6)(4x + 2). Use the product rule to find R'(2).

$$f(x) = (1x + 6)(4x + 2),\ x = 2$$

Answer: _____

26. Revenue is R(x) = (5x + 4)(4x + 9). Use the product rule to find R'(3).

$$f(x) = (5x + 4)(4x + 9),\ x = 3$$

Answer: _____

27. Revenue is R(x) = (1x + 6)(5x + 10). Use the product rule to find R'(0).

$$f(x) = (1x + 6)(5x + 10),\ x = 0$$

Answer: _____

28. Revenue is R(x) = (4x + 3)(5x + 4). Use the product rule to find R'(2).

$$f(x) = (4x + 3)(5x + 4),\ x = 2$$

Answer: _____

29. Revenue is R(x) = (4x + 5)(3x + 9). Use the product rule to find R'(1).

$$f(x) = (4x + 5)(3x + 9),\ x = 1$$

Answer: _____

30. Revenue is R(x) = (1x + 2)(2x + 2). Use the product rule to find R'(4).

$$f(x) = (1x + 2)(2x + 2),\ x = 4$$

Answer: _____

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ANSWER KEY & SOLUTIONS

Topics: Power rule, Product rule, Chain rule, Differentiating polynomials. All answers verified by independent computation.

Solutions

Chain rule

1. A cost function is C(x) = (1x + 1)^3. Find C'(1) using the chain rule.

$$f(x) = (1x + 1)^3,\ x = 1$$

→ *Chain rule: d/dx[(ax+b)^n] = n·a·(ax+b)^(n-1).*
→ *C'(x) = {n}·{a}·({a}x+{b})^{new_n}.*
→ *C'({x_val}) = {n}·{a}·({inner})^{new_n} = {answer_deriv_chain210}.*

Answer: $f'(x) = 3 \cdot 1(1x + 1)^2, \quad f'(1) = 12$

2. A cost function is C(x) = (1x + 7)^2. Find C'(1) using the chain rule.

$$f(x) = (1x + 7)^2,\ x = 1$$

→ *Chain rule: d/dx[(ax+b)^n] = n·a·(ax+b)^(n-1).*
→ *C'(x) = {n}·{a}·({a}x+{b})^{new_n}.*
→ *C'({x_val}) = {n}·{a}·({inner})^{new_n} = {answer_deriv_chain210}.*

Answer: $f'(x) = 2 \cdot 1(1x + 7)^1, \quad f'(1) = 16$

3. A cost function is C(x) = (4x + 5)^3. Find C'(1) using the chain rule.

$$f(x) = (4x + 5)^3,\ x = 1$$

→ *Chain rule: d/dx[(ax+b)^n] = n·a·(ax+b)^(n-1).*
→ *C'(x) = {n}·{a}·({a}x+{b})^{new_n}.*
→ *C'({x_val}) = {n}·{a}·({inner})^{new_n} = {answer_deriv_chain210}.*

Answer: $f'(x) = 3 \cdot 4(4x + 5)^2, \quad f'(1) = 972$

4. A cost function is C(x) = (2x + 8)^3. Find C'(3) using the chain rule.

$$f(x) = (2x + 8)^3,\ x = 3$$

→ *Chain rule: d/dx[(ax+b)^n] = n·a·(ax+b)^(n-1).*
→ *C'(x) = {n}·{a}·({a}x+{b})^{new_n}.*
→ *C'({x_val}) = {n}·{a}·({inner})^{new_n} = {answer_deriv_chain210}.*

Answer: $f'(x) = 3 \cdot 2(2x + 8)^2, \quad f'(3) = 1176$

5. A cost function is C(x) = (2x + 4)^2. Find C'(0) using the chain rule.

$$f(x) = (2x + 4)^2,\ x = 0$$

→ *Chain rule: d/dx[(ax+b)^n] = n·a·(ax+b)^(n-1).*
→ *C'(x) = {n}·{a}·({a}x+{b})^{new_n}.*
→ *C'({x_val}) = {n}·{a}·({inner})^{new_n} = {answer_deriv_chain210}.*

Answer: $f'(x) = 2 \cdot 2(2x + 4)^1, \quad f'(0) = 16$

6. A cost function is C(x) = (2x + 2)^2. Find C'(3) using the chain rule.

$$f(x) = (2x + 2)^2,\ x = 3$$

→ *Chain rule: d/dx[(ax+b)^n] = n·a·(ax+b)^(n-1).*
→ *C'(x) = {n}·{a}·({a}x+{b})^{new_n}.*
→ *C'({x_val}) = {n}·{a}·({inner})^{new_n} = {answer_deriv_chain210}.*

Answer: $f'(x) = 2 \cdot 2(2x + 2)^1, \quad f'(3) = 32$

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7. A cost function is C(x) = (1x + 5)^3. Find C'(1) using the chain rule.

$$f(x) = (1x + 5)^3,\ x = 1$$

→ *Chain rule: d/dx[(ax+b)^n] = n·a·(ax+b)^(n-1).*
→ *C'(x) = {n}·{a}·({a}x+{b})^{new_n}.*
→ *C'({x_val}) = {n}·{a}·({inner})^{new_n} = {answer_deriv_chain210}.*

Answer: $f'(x) = 3 \cdot 1(1x + 5)^2, \quad f'(1) = 108$

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8. A cost function is C(x) = (1x + 5)^3. Find C'(3) using the chain rule.

$$f(x) = (1x + 5)^3,\ x = 3$$

→ *Chain rule: d/dx[(ax+b)^n] = n·a·(ax+b)^(n-1).*
→ *C'(x) = {n}·{a}·({a}x+{b})^{new_n}.*
→ *C'({x_val}) = {n}·{a}·({inner})^{new_n} = {answer_deriv_chain210}.*

Answer: $f'(x) = 3 \cdot 1(1x + 5)^2, \quad f'(3) = 192$

Differentiating polynomials

9. Total revenue is R(x) = 1x^3 + -2x^2 + 14x + 59. Find the marginal revenue MR = R'(2).

$$f(x) = 1x^3 + -2x^2 + 14x + 59,\ x = 2$$

→ *MR = R'(x) = 3x^2 + -4x + 14.*
→ *R'(2) = 3(2)^2 + -4(2) + 14 = 18.*

Answer: $f'(x) = 3x^2 + -4x + 14, \quad f'(2) = 18$

10. Total revenue is R(x) = 2x^3 + -4x^2 + 23x + 70. Find the marginal revenue MR = R'(2).

$$f(x) = 2x^3 + -4x^2 + 23x + 70,\ x = 2$$

→ *MR = R'(x) = 6x^2 + -8x + 23.*
→ *R'(2) = 6(2)^2 + -8(2) + 23 = 31.*

Answer: $f'(x) = 6x^2 + -8x + 23, \quad f'(2) = 31$

11. Total revenue is R(x) = 1x^3 + 0x^2 + 27x + 20. Find the marginal revenue MR = R'(1).

$$f(x) = 1x^3 + 0x^2 + 27x + 20,\ x = 1$$

→ *MR = R'(x) = 3x^2 + 0x + 27.*
→ *R'(1) = 3(1)^2 + 0(1) + 27 = 30.*

Answer: $f'(x) = 3x^2 + 0x + 27, \quad f'(1) = 30$

12. Total revenue is R(x) = 1x^3 + -3x^2 + 14x + 94. Find the marginal revenue MR = R'(2).

$$f(x) = 1x^3 + -3x^2 + 14x + 94,\ x = 2$$

→ *MR = R'(x) = 3x^2 + -6x + 14.*
→ *R'(2) = 3(2)^2 + -6(2) + 14 = 14.*

Answer: $f'(x) = 3x^2 + -6x + 14, \quad f'(2) = 14$

13. Total revenue is R(x) = 1x^3 + -4x^2 + 30x + 58. Find the marginal revenue MR = R'(1).

$$f(x) = 1x^3 + -4x^2 + 30x + 58,\ x = 1$$

→ *MR = R'(x) = 3x^2 + -8x + 30.*
→ *R'(1) = 3(1)^2 + -8(1) + 30 = 25.*

Answer: $f'(x) = 3x^2 + -8x + 30, \quad f'(1) = 25$

14. Total revenue is R(x) = 2x^3 + -1x^2 + 19x + 32. Find the marginal revenue MR = R'(2).

$$f(x) = 2x^3 + -1x^2 + 19x + 32,\ x = 2$$

→ *MR = R'(x) = 6x^2 + -2x + 19.*
→ *R'(2) = 6(2)^2 + -2(2) + 19 = 39.*

Answer: $f'(x) = 6x^2 + -2x + 19, \quad f'(2) = 39$

15. Total revenue is R(x) = 1x^3 + -4x^2 + 23x + 59. Find the marginal revenue MR = R'(2).

$$f(x) = 1x^3 + -4x^2 + 23x + 59,\ x = 2$$

→ MR = R'(x) = 3x^2 + -8x + 23.
→ R'(2) = 3(2)^2 + -8(2) + 23 = 19.

Answer: $f'(x) = 3x^2 + -8x + 23, \quad f'(2) = 19$

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16. Total revenue is R(x) = 2x^3 + 0x^2 + 29x + 1. Find the marginal revenue MR = R'(2).

$$f(x) = 2x^3 + 0x^2 + 29x + 1,\ x = 2$$

→ MR = R'(x) = 6x^2 + 0x + 29.
→ R'(2) = 6(2)^2 + 0(2) + 29 = 53.

Answer: $f'(x) = 6x^2 + 0x + 29, \quad f'(2) = 53$

Power rule

17. Find the derivative of f(x) = 7x^2 and evaluate f'(2).

$$f(x) = 7x^2, \quad x = 2$$

→ *f'(x) = 14x^1 (power rule).*
→ *f'(2) = 14(2)^1 = 28.*

Answer: $f'(x) = 14x^1, \quad f'(2) = 28$

18. Find the derivative of f(x) = 6x^3 and evaluate f'(3).

$$f(x) = 6x^3, \quad x = 3$$

→ *f'(x) = 18x^2 (power rule).*
→ *f'(3) = 18(3)^2 = 162.*

Answer: $f'(x) = 18x^2, \quad f'(3) = 162$

19. Find the derivative of f(x) = 5x^2 and evaluate f'(4).

$$f(x) = 5x^2, \quad x = 4$$

→ *f'(x) = 10x^1 (power rule).*
→ *f'(4) = 10(4)^1 = 40.*

Answer: $f'(x) = 10x^1, \quad f'(4) = 40$

20. Find the derivative of f(x) = 1x^5 and evaluate f'(3).

$$f(x) = 1x^5, \quad x = 3$$

→ *f'(x) = 5x^4 (power rule).*
→ *f'(3) = 5(3)^4 = 405.*

Answer: $f'(x) = 5x^4, \quad f'(3) = 405$

21. Find the derivative of f(x) = 5x^4 and evaluate f'(2).

$$f(x) = 5x^4, \quad x = 2$$

→ *f'(x) = 20x^3 (power rule).*
→ *f'(2) = 20(2)^3 = 160.*

Answer: $f'(x) = 20x^3, \quad f'(2) = 160$

22. Find the derivative of f(x) = 8x^2 and evaluate f'(4).

$$f(x) = 8x^2, \quad x = 4$$

→ *f'(x) = 16x^1 (power rule).*
→ *f'(4) = 16(4)^1 = 64.*

Answer: $f'(x) = 16x^1, \quad f'(4) = 64$

23. Find the derivative of f(x) = 8x^2 and evaluate f'(2).

$$f(x) = 8x^2, \quad x = 2$$

→ *f'(x) = 16x^1 (power rule).*
→ *f'(2) = 16(2)^1 = 32.*

Answer: $f'(x) = 16x^1, \quad f'(2) = 32$

Product rule

24. Revenue is R(x) = (3x + 7)(4x + 5). Use the product rule to find R'(0).

$$f(x) = (3x + 7)(4x + 5),\ x = 0$$

→ *u = 3x+7, v = 4x+5. u' = 3, v' = 4.*
→ *R'(x) = u'v + uv' = 3(4x+5) + 4(3x+7).*
→ *R'(0) = 3(5) + 4(7) = 43.*

Answer: $f'(0) = 3 \cdot 5 + 4 \cdot 7 = 43$

25. Revenue is R(x) = (1x + 6)(4x + 2). Use the product rule to find R'(2).

$$f(x) = (1x + 6)(4x + 2),\ x = 2$$

→ *u = 1x+6, v = 4x+2. u' = 1, v' = 4.*
→ *R'(x) = u'v + uv' = 1(4x+2) + 4(1x+6).*
→ *R'(2) = 1(10) + 4(8) = 42.*

Answer: $f'(2) = 1 \cdot 10 + 4 \cdot 8 = 42$

26. Revenue is R(x) = (5x + 4)(4x + 9). Use the product rule to find R'(3).

$$f(x) = (5x + 4)(4x + 9),\ x = 3$$

→ *u = 5x+4, v = 4x+9. u' = 5, v' = 4.*
→ *R'(x) = u'v + uv' = 5(4x+9) + 4(5x+4).*
→ *R'(3) = 5(21) + 4(19) = 181.*

Answer: $f'(3) = 5 \cdot 21 + 4 \cdot 19 = 181$

27. Revenue is R(x) = (1x + 6)(5x + 10). Use the product rule to find R'(0).

$$f(x) = (1x + 6)(5x + 10),\ x = 0$$

→ *u = 1x+6, v = 5x+10. u' = 1, v' = 5.*
→ *R'(x) = u'v + uv' = 1(5x+10) + 5(1x+6).*
→ *R'(0) = 1(10) + 5(6) = 40.*

Answer: $f'(0) = 1 \cdot 10 + 5 \cdot 6 = 40$

28. Revenue is R(x) = (4x + 3)(5x + 4). Use the product rule to find R'(2).

$$f(x) = (4x + 3)(5x + 4),\ x = 2$$

→ *u = 4x+3, v = 5x+4. u' = 4, v' = 5.*
→ *R'(x) = u'v + uv' = 4(5x+4) + 5(4x+3).*
→ *R'(2) = 4(14) + 5(11) = 111.*

Answer: $f'(2) = 4 \cdot 14 + 5 \cdot 11 = 111$

29. Revenue is R(x) = (4x + 5)(3x + 9). Use the product rule to find R'(1).

$$f(x) = (4x + 5)(3x + 9),\ x = 1$$

→ *u = 4x+5, v = 3x+9. u' = 4, v' = 3.*
→ *R'(x) = u'v + uv' = 4(3x+9) + 3(4x+5).*
→ *R'(1) = 4(12) + 3(9) = 75.*

Answer: $f'(1) = 4 \cdot 12 + 3 \cdot 9 = 75$

30. Revenue is R(x) = (1x + 2)(2x + 2). Use the product rule to find R'(4).

$$f(x) = (1x + 2)(2x + 2),\ x = 4$$

→ *u = 1x+2, v = 2x+2. u' = 1, v' = 2.*
→ *R'(x) = u'v + uv' = 1(2x+2) + 2(1x+2).*
→ *R'(4) = 1(10) + 2(6) = 22.*

Answer: $f'(4) = 1 \cdot 10 + 2 \cdot 6 = 22$