MATH 150 — Polynomials & Factoring

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Learning Objectives

- Factor using GCF, difference of squares, and perfect square trinomials
- Factor trinomials of the form $ax^2 + bx + c$ by grouping
- Factor sums and differences of cubes
- Perform polynomial long division

Always check for a GCF first before applying other factoring techniques. For trinomials, use the ac-method (factor by grouping) when the leading coefficient ≠ 1.

1. Factor out the Greatest Common Factor (GCF).

$$12x^3 - 8x^2 + 4x$$

Answer: _____

2. Factor the difference of squares.

$$25x^2 - 49$$

Answer: _____

3. Factor the perfect square trinomial.

$$x^2 + 10x + 25$$

Answer: _____

4. Factor the trinomial.

$$x^2 - 3x - 18$$

Answer: _____

5. Factor by grouping.

$$3x^3 + x^2 - 12x - 4$$

Answer: _____

6. Factor the trinomial with leading coefficient ≠ 1.

$$2x^2 + 7x + 3$$

Answer: _____

7. Factor the sum of cubes.

$$x^3 + 27$$

Answer: _____

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8. Factor completely.

$3x^4 - 75x^2$

Answer: _____

9. Factor the difference of cubes.

$8x^3 - 125$

Answer: _____

10. Perform polynomial long division.

$(x^3 - 2x^2 + 4x - 8) \div (x - 2)$

Answer: _____

Problems 1–4: build from simplest to more complex factoring. Problem 5 (grouping): connect to synthetic division. Problem 10 (long division): preview of rational root theorem.

Solutions

1. Factor out the Greatest Common Factor (GCF).

$12x^3 - 8x^2 + 4x$

→ *GCF of 12, 8, 4 = 4. GCF of x^3, x^2, x = x. GCF = $4x$.*
→ *$4x(3x^2–2x+1)$.*

Answer: $4x(3x^2 - 2x + 1)$

2. Factor the difference of squares.

$25x^2 - 49$

→ *Pattern: $a^2–b^2 = (a–b)(a+b)$. Here $a=5x$, $b=7$.*
→ *$(5x–7)(5x+7)$.*

Answer: $(5x - 7)(5x + 7)$

3. Factor the perfect square trinomial.

$x^2 + 10x + 25$

→ *Recognize: $a^2+2ab+b^2 = (a+b)^2$ with $a=x$, $b=5$.*
→ *Check: $2(x)(5)=10x$ ✓. Answer: $(x+5)^2$.*

Answer: $(x + 5)^2$

4. Factor the trinomial.

$x^2 - 3x - 18$

→ *Find two numbers: multiply to –18, add to –3 → –6 and +3.*
→ *$(x–6)(x+3)$.*

Answer: $(x - 6)(x + 3)$

5. Factor by grouping.

$3x^3 + x^2 - 12x - 4$

→ *Group: $(3x^3+x^2)+(–12x–4)$.*
→ *$x^2(3x+1) – 4(3x+1) = (3x+1)(x^2–4)$.*
→ *Factor further: $(3x+1)(x–2)(x+2)$.*

Answer: $(3x + 1)(x^2 - 4) = (3x + 1)(x - 2)(x + 2)$

6. Factor the trinomial with leading coefficient ≠ 1.

$2x^2 + 7x + 3$

→ *a·c = 6. Find factors of 6 that add to 7: 1 and 6.*
→ *Split: 2x²+x+6x+3 = x(2x+1)+3(2x+1) = (2x+1)(x+3).*

Answer: $(2x + 1)(x + 3)$

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7. Factor the sum of cubes.

$x^3 + 27$

→ *Pattern: a³+b³=(a+b)(a²–ab+b²). a=x, b=3.*
→ *(x+3)(x²–3x+9).*

Answer: $(x + 3)(x^2 - 3x + 9)$

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8. Factor completely.

$3x^4 - 75x^2$

→ *GCF = 3x²: 3x²(x²–25).*
→ *x²–25 is difference of squares: (x–5)(x+5).*
→ *Complete factorization: 3x²(x–5)(x+5).*

Answer: $3x^2(x - 5)(x + 5)$

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9. Factor the difference of cubes.

$8x^3 - 125$

→ *Pattern: a³–b³=(a–b)(a²+ab+b²). a=2x, b=5.*
→ *(2x–5)((2x)²+(2x)(5)+5²) = (2x–5)(4x²+10x+25).*

Answer: $(2x - 5)(4x^2 + 10x + 25)$

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10. Perform polynomial long division.

$(x^3 - 2x^2 + 4x - 8) \div (x - 2)$

→ *x³÷x=x². x²·(x–2)=x³–2x². Subtract: –2x²+2x²=0.*
→ *Bring down: 4x–8. 4x÷x=4. 4·(x–2)=4x–8. Remainder=0.*
→ *Quotient: x²+4.*

Answer: $x^2 + 4,\ \text{remainder } 0$