



Name: \_\_\_\_\_

Date: \_\_\_\_\_

Score: / 10

## Learning Objectives

- Evaluate and expand/condense logarithms using product, quotient, and power rules
- Solve exponential and logarithmic equations
- Apply the change of base formula
- Model exponential growth, decay, and compound interest

Always check for extraneous solutions in log equations — arguments must be positive. For exponential equations without common bases, take  $\ln$  of both sides.

### 1. Evaluate the logarithm.

$$\log_2 32$$

Answer: \_\_\_\_\_

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### 2. Expand using logarithm properties.

$$\log_b \left( \frac{x^3 \sqrt{y}}{z^2} \right)$$

Answer: \_\_\_\_\_

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### 3. Condense into a single logarithm.

$$2 \ln x + \ln(x + 1) - \ln 5$$

Answer: \_\_\_\_\_

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### 4. Solve the exponential equation.

$$3^{x+1} = 81$$

Answer: \_\_\_\_\_

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### 5. Solve the exponential equation using logarithms.

$$5^x = 20$$

Answer: \_\_\_\_\_

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### 6. Solve the logarithmic equation.

$$\log_3 (2x - 1) = 4$$

Answer: \_\_\_\_\_

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**7. Solve.**

$$\ln(x + 3) + \ln(x - 2) = \ln 14$$

Answer: \_\_\_\_\_

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**8. Use the change of base formula to evaluate.**

$$\log_7 50$$

Answer: \_\_\_\_\_

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**9. Compound interest: \$5000 invested at 4% compounded continuously. Find the amount after 10 years.**

$$A = Pe^{rt}, \quad P = 5000, \quad r = 0.04, \quad t = 10$$

Answer: \_\_\_\_\_

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**10. A population starts at 1000 and doubles every 5 years. Write the exponential model and find the population after 15 years.**

$$P(t) = 1000 \cdot 2^{t/5}$$

Answer: \_\_\_\_\_





Problems 2–3: expand vs condense — students confuse these directions. Problem 7 (log equation): extraneous root  $x=-5$  must be rejected. Problems 9–10: applied context.

## Solutions

1. Evaluate the logarithm.

$$\log_2 32$$

$$\rightarrow \log_2 32 = x \text{ means } 2^x = 32 = 2^5.$$

$$\rightarrow \text{Answer: } 5.$$

**Answer:**     5

2. Expand using logarithm properties.

$$\log_b \left( \frac{x^3 \sqrt{y}}{z^2} \right)$$

$$\rightarrow \log(A/B) = \log A - \log B. \log(AB) = \log A + \log B. \log(A^n) = n \log A.$$

$$\rightarrow = \log x^3 + \log y^{1/2} - \log z^2 = 3 \log x + \frac{1}{2} \log y - 2 \log z.$$

**Answer:**      $3\log_b x + \frac{1}{2}\log_b y - 2\log_b z$

3. Condense into a single logarithm.

$$2\ln x + \ln(x+1) - \ln 5$$

$$\rightarrow 2 \ln x = \ln x^2. \text{ Combine using product/quotient rules.}$$

$$\rightarrow \ln x^2 + \ln(x+1) - \ln 5 = \ln[x^2(x+1)/5].$$

**Answer:**      $\ln \left( \frac{x^2(x+1)}{5} \right)$

4. Solve the exponential equation.

$$3^{x+1} = 81$$

$$\rightarrow 81=3^4. 3^{(x+1)}=3^4 \rightarrow x+1=4 \rightarrow x=3.$$

**Answer:**      $x = 3$

5. Solve the exponential equation using logarithms.

$$5^x = 20$$

$$\rightarrow \text{Take } \ln \text{ of both sides: } x \ln 5 = \ln 20.$$

$$\rightarrow x = \ln 20 / \ln 5 = \log_5 20 \approx 1.861.$$

**Answer:**      $x = \frac{\ln 20}{\ln 5} \approx 1.861$



6. Solve the logarithmic equation.

$$\log_3(2x - 1) = 4$$

→ Convert to exponential:  $3^4 = 2x - 1$ .

→  $81 = 2x - 1 \rightarrow 2x = 82 \rightarrow x = 41$ .

**Answer:**  $x = 41$

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7. Solve.

$$\ln(x + 3) + \ln(x - 2) = \ln 14$$

→  $\ln[(x+3)(x-2)] = \ln 14 \rightarrow (x+3)(x-2) = 14$ .

→  $x^2 + x - 6 = 14 \rightarrow x^2 + x - 20 = 0 \rightarrow (x+5)(x-4) = 0$ .

→  $x = 4$  (reject  $x = -5$  since  $x - 2 = -7 < 0$  makes  $\ln$  undefined).

**Answer:**  $x = 4$

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8. Use the change of base formula to evaluate.

$$\log_7 50$$

→ Change of base:  $\log_7 50 = \frac{\ln 50}{\ln 7} = \frac{\log_{10}(50)}{\log_{10}(7)}$ .

→  $\approx 3.912/1.946 \approx 2.012$ .

**Answer:**  $\frac{\ln 50}{\ln 7} \approx 2.012$

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9. Compound interest: \$5000 invested at 4% compounded continuously. Find the amount after 10 years.

$$A = Pe^{rt}, \quad P = 5000, \quad r = 0.04, \quad t = 10$$

→  $A = 5000 \cdot e^{(0.04 \cdot 10)} = 5000 \cdot e^{0.4}$ .

→  $e^{0.4} \approx 1.4918$ .  $A \approx 5000 \cdot 1.4918 \approx \$7459$ .

**Answer:**  $A = 5000e^{0.4} \approx \$7459$

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10. A population starts at 1000 and doubles every 5 years. Write the exponential model and find the population after 15 years.

$$P(t) = 1000 \cdot 2^{t/5}$$

→ Doubling time 5 yrs →  $P(t) = 1000 \cdot 2^{(t/5)}$ .

→  $P(15) = 1000 \cdot 2^{(15/5)} = 1000 \cdot 2^3 = 1000 \cdot 8 = 8000$ .

**Answer:**  $P(15) = 1000 \cdot 2^3 = 8000$

