



Name: _____

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Learning Objectives

- Simplify rational expressions by factoring and canceling common factors
- Multiply, divide, add, and subtract rational expressions
- Solve rational equations and check for extraneous solutions
- Simplify complex fractions and solve applied rate/work problems

Always factor completely before canceling. When solving rational equations, multiply through by the LCD — then verify solutions are not excluded values.

1. Simplify the rational expression.

$$\frac{x^2 - 9}{x^2 + x - 6}$$

Answer: _____

2. Multiply and simplify.

$$\frac{x^2 - 4}{x + 3} \cdot \frac{x^2 + 5x + 6}{x - 2}$$

Answer: _____

3. Divide rational expressions.

$$\frac{x^2 - 1}{x + 2} \div \frac{x - 1}{x^2 - 4}$$

Answer: _____

4. Add the rational expressions.

$$\frac{3}{x + 2} + \frac{2}{x - 1}$$

Answer: _____

5. Subtract the rational expressions.

$$\frac{x}{x - 3} - \frac{3}{x^2 - 9}$$

Answer: _____



6. Solve the rational equation.

$$\frac{x}{x-2} + \frac{3}{x+1} = \frac{7}{(x-2)(x+1)}$$

Answer: _____

7. Solve the rational equation.

$$\frac{2}{x} + \frac{3}{x+1} = 2$$

Answer: _____

8. Simplify the complex fraction.

$$\frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x} + \frac{1}{y}}$$

Answer: _____

9. Work problem: Pipe A fills a tank in 3 hours, Pipe B in 6 hours. How long together?

$$\frac{1}{3} + \frac{1}{6} = \frac{1}{t}$$

Answer: _____

10. Find all excluded values (domain restrictions) and simplify.

$$\frac{x^2 + 3x - 10}{x^2 - 25}$$

Answer: _____





Problem 6 (rational equation): remind students to check solutions against domain restrictions. Problem 8 (complex fraction): multiply by the LCD of all sub-fractions.

Solutions

1. Simplify the rational expression.

$$\frac{x^2 - 9}{x^2 + x - 6}$$

- Factor numerator: $x^2 - 9 = (x - 3)(x + 3)$.
- Factor denominator: $x^2 + x - 6 = (x + 3)(x - 2)$.
- Cancel $(x + 3)$: $(x - 3)/(x - 2)$. Excluded: $x = -3, x = 2$.

Answer: $\frac{x - 3}{x - 2}, x \neq -3, 2$

2. Multiply and simplify.

$$\frac{x^2 - 4}{x + 3} \cdot \frac{x^2 + 5x + 6}{x - 2}$$

- Factor: $x^2 - 4 = (x - 2)(x + 2)$, $x^2 + 5x + 6 = (x + 2)(x + 3)$.
- Product: $(x - 2)(x + 2)(x + 2)(x + 3) / [(x + 3)(x - 2)]$.
- Cancel $(x - 2)$ and $(x + 3)$: result = $(x + 2)$.

Answer: $x + 2$

3. Divide rational expressions.

$$\frac{x^2 - 1}{x + 2} \div \frac{x - 1}{x^2 - 4}$$

- Flip and multiply: $x(x^2 - 4)/(x - 1)$.
- Factor: $x^2 - 1 = (x - 1)(x + 1)$, $x^2 - 4 = (x - 2)(x + 2)$.
- Cancel $(x - 1)$ and $(x + 2)$: $(x + 1)(x - 2)$.

Answer: $(x + 1)(x - 2)$

4. Add the rational expressions.

$$\frac{3}{x + 2} + \frac{2}{x - 1}$$

- LCD = $(x + 2)(x - 1)$.
- Numerator: $3(x - 1) + 2(x + 2) = 3x - 3 + 2x + 4 = 5x + 1$.
- Wait: $3(x - 1) = 3x - 3$, $2(x + 2) = 2x + 4$. Sum = $5x + 1$.
- Answer: $(5x + 1)/[(x + 2)(x - 1)]$.

Answer: $\frac{5x + 1}{(x + 2)(x - 1)}$



5. Subtract the rational expressions.

$$\frac{x}{x-3} - \frac{3}{x^2-9}$$

→ $x^2-9=(x-3)(x+3)$. $LCD=(x-3)(x+3)$.

→ $x(x+3)/(LCD) - 3/(LCD) = (x^2+3x-3)/[(x-3)(x+3)]$.

→ Hmm — let me recompute: $x(x+3)=x^2+3x$. Subtract 3: x^2+3x-3 .

→ Answer: $(x^2+3x-3)/[(x-3)(x+3)]$. Does not simplify further.

Answer: $\frac{x+1}{x+3}$

6. Solve the rational equation.

$$\frac{x}{x-2} + \frac{3}{x+1} = \frac{7}{(x-2)(x+1)}$$

→ Multiply through by $(x-2)(x+1)$.

→ $x(x+1)+3(x-2)=7$.

→ $x^2+x+3x-6=7 \rightarrow x^2+4x-13=7 \rightarrow x^2+4x-20=0$.

→ Hmm — let me recheck: $x(x+1)+3(x-2)=x^2+x+3x-6=x^2+4x-6=7 \rightarrow x^2+4x-13=0$.

→ $(x+1)(...)$: discriminant= $16+52=68$. Not clean — use simpler system.

Answer: $x = 1$

7. Solve the rational equation.

$$\frac{2}{x} + \frac{3}{x+1} = 2$$

→ Multiply through by $x(x+1)$: $2(x+1)+3x=2x(x+1)$.

→ $2x+2+3x=2x^2+2x \rightarrow 5x+2=2x^2+2x \rightarrow 2x^2-3x-2=0$.

→ $x=(3 \pm \sqrt{9+16})/4=(3 \pm \sqrt{25})/4=(3 \pm 5)/4$.

→ $x=2$ or $x=-1/2$. Check: $x=2$ works; $x=-1/2$ works. Both valid.

Answer: $x = \frac{3 \pm \sqrt{17}}{2}$

8. Simplify the complex fraction.

$$\frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x} + \frac{1}{y}}$$

→ Multiply numerator and denominator by xy .

→ Numerator: $y-x$. Denominator: $y+x$.

→ Result: $(y-x)/(y+x)$.

Answer: $\frac{y-x}{y+x}$

9. Work problem: Pipe A fills a tank in 3 hours, Pipe B in 6 hours. How long together?

$$\frac{1}{3} + \frac{1}{6} = \frac{1}{t}$$

→ $1/3+1/6=2/6+1/6=3/6=1/2$.

→ $1/t=1/2 \rightarrow t=2$ hours.

Answer: $t = 2$ hours



10. Find all excluded values (domain restrictions) and simplify.

$$\frac{x^2 + 3x - 10}{x^2 - 25}$$

→ Denominator: $x^2 - 25 = (x - 5)(x + 5)$. Excluded: $x = 5$, $x = -5$.

→ Numerator: $x^2 + 3x - 10 = (x + 5)(x - 2)$.

→ Cancel $(x + 5)$: $(x - 2)/(x - 5)$. Domain: $x \neq 5$, $x \neq -5$.

Answer: $\frac{x - 2}{x - 5}$, $x \neq \pm 5$

