MATH155: Analytic Trigonometry

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Learning Objectives

- Apply the Law of Sines to find missing sides and angles
- Apply the Law of Cosines to find missing sides and angles
- Solve triangles using the appropriate law

Simplify each expression completely. Show all steps and circle your final answer.

Law of Cosines

1. In triangle ABC, a = 4, b = 3, and angle C = 60°. Use the Law of Cosines to find side c (round to 2 decimal places).

$$a = 4,\ b = 3,\ C = 60^\circ$$

Answer: _____

2. In triangle ABC, a = 4, b = 9, and angle C = 30°. Use the Law of Cosines to find side c (round to 2 decimal places).

$$a = 4,\ b = 9,\ C = 30^\circ$$

Answer: _____

3. In triangle ABC, a = 10, b = 7, and angle C = 60°. Use the Law of Cosines to find side c (round to 2 decimal places).

$$a = 10,\ b = 7,\ C = 60^\circ$$

Answer: _____

4. In triangle ABC, a = 5, b = 10, and angle C = 120°. Use the Law of Cosines to find side c (round to 2 decimal places).

$$a = 5,\ b = 10,\ C = 120^\circ$$

Answer: _____

5. In triangle ABC, a = 6, b = 6, and angle C = 45°. Use the Law of Cosines to find side c (round to 2 decimal places).

$$a = 6,\ b = 6,\ C = 45^\circ$$

Answer: _____

6. In triangle ABC, a = 5, b = 4, and angle C = 45°. Use the Law of Cosines to find side c (round to 2 decimal places).

$$a = 5,\ b = 4,\ C = 45^\circ$$

Answer: _____

7. In triangle ABC, a = 4, b = 7, and angle C = 90°. Use the Law of Cosines to find side c (round to 2 decimal places).

$$a = 4,\ b = 7,\ C = 90^\circ$$

Answer: _____

8. In triangle ABC, a = 4, b = 7, and angle C = 90°. Use the Law of Cosines to find side c (round to 2 decimal places).

$$a = 4,\ b = 7,\ C = 90^\circ$$

Answer: _____

Law of Cosines — find missing angle

9. In triangle ABC, a = 4, b = 6, c = 9. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$$a = 4,\ b = 6,\ c = 9$$

Answer: _____

10. In triangle ABC, a = 6, b = 4, c = 7. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$$a = 6,\ b = 4,\ c = 7$$

Answer: _____

11. In triangle ABC, a = 5, b = 8, c = 8. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$$a = 5,\ b = 8,\ c = 8$$

Answer: _____

12. In triangle ABC, a = 4, b = 5, c = 5. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$$a = 4,\ b = 5,\ c = 5$$

Answer: _____

13. In triangle ABC, a = 5, b = 4, c = 9. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$$a = 5,\ b = 4,\ c = 9$$

Answer: _____

14. In triangle ABC, a = 7, b = 7, c = 6. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$$a = 7,\ b = 7,\ c = 6$$

Answer: _____

15. In triangle ABC, a = 4, b = 4, c = 7. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$$a = 4,\ b = 4,\ c = 7$$

Answer: _____

16. In triangle ABC, a = 6, b = 8, c = 8. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$$a = 6,\ b = 8,\ c = 8$$

Answer: _____

Law of Sines

17. In triangle ABC, angle A = 45°, angle B = 45°, and side a = 6. Use the Law of Sines to find side b (round to 2 decimal places).

$$A = 45^\circ,\ B = 45^\circ,\ a = 6$$

Answer: _____

18. In triangle ABC, angle A = 60°, angle B = 75°, and side a = 6. Use the Law of Sines to find side b (round to 2 decimal places).

$$A = 60^\circ,\ B = 75^\circ,\ a = 6$$

Answer: _____

19. In triangle ABC, angle A = 45°, angle B = 45°, and side a = 12. Use the Law of Sines to find side b (round to 2 decimal places).

$$A = 45^\circ,\ B = 45^\circ,\ a = 12$$

Answer: _____

20. In triangle ABC, angle A = 60°, angle B = 45°, and side a = 11. Use the Law of Sines to find side b (round to 2 decimal places).

$$A = 60^\circ,\ B = 45^\circ,\ a = 11$$

Answer: _____

21. In triangle ABC, angle A = 45°, angle B = 75°, and side a = 6. Use the Law of Sines to find side b (round to 2 decimal places).

$$A = 45^\circ,\ B = 75^\circ,\ a = 6$$

Answer: _____

22. In triangle ABC, angle A = 45°, angle B = 45°, and side a = 10. Use the Law of Sines to find side b (round to 2 decimal places).

$$A = 45^\circ,\ B = 45^\circ,\ a = 10$$

Answer: _____

23. In triangle ABC, angle A = 60°, angle B = 90°, and side a = 12. Use the Law of Sines to find side b (round to 2 decimal places).

$$A = 60^\circ,\ B = 90^\circ,\ a = 12$$

Answer: _____

Law of Sines — find missing angle

24. In triangle ABC, angle A = 45°, side a = 11, and side b = 10. Use the Law of Sines to find angle B in degrees (round to 1 decimal place).

$$A = 45^\circ,\ a = 11,\ b = 10$$

Answer: _____

25. In triangle ABC, angle A = 30°, side a = 10, and side b = 9. Use the Law of Sines to find angle B in degrees (round to 1 decimal place).

$$A = 30^\circ,\ a = 10,\ b = 9$$

Answer: _____

26. In triangle ABC, angle A = 60°, side a = 8, and side b = 10. Use the Law of Sines to find angle B in degrees (round to 1 decimal place).

$$A = 60^\circ,\ a = 8,\ b = 10$$

Answer: _____

27. In triangle ABC, angle A = 30°, side a = 10, and side b = 3. Use the Law of Sines to find angle B in degrees (round to 1 decimal place).

$$A = 30^\circ,\ a = 10,\ b = 3$$

Answer: _____

28. In triangle ABC, angle A = 45°, side a = 7, and side b = 6. Use the Law of Sines to find angle B in degrees (round to 1 decimal place).

$$A = 45^\circ,\ a = 7,\ b = 6$$

Answer: _____

29. In triangle ABC, angle A = 45°, side a = 9, and side b = 7. Use the Law of Sines to find angle B in degrees (round to 1 decimal place).

$$A = 45^\circ,\ a = 9,\ b = 7$$

Answer: _____

30. In triangle ABC, angle A = 60°, side a = 5, and side b = 4. Use the Law of Sines to find angle B in degrees (round to 1 decimal place).

$$A = 60^\circ,\ a = 5,\ b = 4$$

Answer: _____

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ANSWER KEY & SOLUTIONS

Topics: Law of Cosines — find missing angle, Law of Sines, Law of Sines — find missing angle, Law of Cosines. All answers verified by independent computation.

Solutions

Law of Cosines

1. In triangle ABC, a = 4, b = 3, and angle C = 60°. Use the Law of Cosines to find side c (round to 2 decimal places).

$$a = 4,\ b = 3,\ C = 60^\circ$$

→ *Law of Cosines: c^2 = a^2 + b^2 - 2ab*cos(C).*
→ *c^2 = 4^2 + 3^2 - 2(4)(3)*cos(60°).*
→ *c^2 = 16 + 9 - 24*0.5 = 13.0.*
→ *c = sqrt(13.0) = 3.61.*

Answer: $c = 3.61$

2. In triangle ABC, a = 4, b = 9, and angle C = 30°. Use the Law of Cosines to find side c (round to 2 decimal places).

$$a = 4,\ b = 9,\ C = 30^\circ$$

→ *Law of Cosines: c^2 = a^2 + b^2 - 2ab*cos(C).*
→ *c^2 = 4^2 + 9^2 - 2(4)(9)*cos(30°).*
→ *c^2 = 16 + 81 - 72*0.866 = 34.6462.*
→ *c = sqrt(34.6462) = 5.89.*

Answer: $c = 5.89$

3. In triangle ABC, a = 10, b = 7, and angle C = 60°. Use the Law of Cosines to find side c (round to 2 decimal places).

$$a = 10,\ b = 7,\ C = 60^\circ$$

→ *Law of Cosines: c^2 = a^2 + b^2 - 2ab*cos(C).*
→ *c^2 = 10^2 + 7^2 - 2(10)(7)*cos(60°).*
→ *c^2 = 100 + 49 - 140*0.5 = 79.0.*
→ *c = sqrt(79.0) = 8.89.*

Answer: $c = 8.89$

4. In triangle ABC, a = 5, b = 10, and angle C = 120°. Use the Law of Cosines to find side c (round to 2 decimal places).

$$a = 5,\ b = 10,\ C = 120^\circ$$

→ *Law of Cosines: c^2 = a^2 + b^2 - 2ab*cos(C).*
→ *c^2 = 5^2 + 10^2 - 2(5)(10)*cos(120°).*
→ *c^2 = 25 + 100 - 100*-0.5 = 175.0.*
→ *c = sqrt(175.0) = 13.23.*

Answer: $c = 13.23$

5. In triangle ABC, a = 6, b = 6, and angle C = 45°. Use the Law of Cosines to find side c (round to 2 decimal places).

$a = 6,\ b = 6,\ C = 45^\circ$

→ *Law of Cosines: c^2 = a^2 + b^2 - 2ab*cos(C).*
→ *c^2 = 6^2 + 6^2 - 2(6)(6)*cos(45°).*
→ *c^2 = 36 + 36 - 72*0.7071 = 21.0883.*
→ *c = sqrt(21.0883) = 4.59.*

Answer: $c = 4.59$

6. In triangle ABC, a = 5, b = 4, and angle C = 45°. Use the Law of Cosines to find side c (round to 2 decimal places).

$a = 5,\ b = 4,\ C = 45^\circ$

→ *Law of Cosines: c^2 = a^2 + b^2 - 2ab*cos(C).*
→ *c^2 = 5^2 + 4^2 - 2(5)(4)*cos(45°).*
→ *c^2 = 25 + 16 - 40*0.7071 = 12.7157.*
→ *c = sqrt(12.7157) = 3.57.*

Answer: $c = 3.57$

7. In triangle ABC, a = 4, b = 7, and angle C = 90°. Use the Law of Cosines to find side c (round to 2 decimal places).

$a = 4,\ b = 7,\ C = 90^\circ$

→ *Law of Cosines: c^2 = a^2 + b^2 - 2ab*cos(C).*
→ *c^2 = 4^2 + 7^2 - 2(4)(7)*cos(90°).*
→ *c^2 = 16 + 49 - 56*0.0 = 65.0.*
→ *c = sqrt(65.0) = 8.06.*

Answer: $c = 8.06$

8. In triangle ABC, a = 4, b = 7, and angle C = 90°. Use the Law of Cosines to find side c (round to 2 decimal places).

$a = 4,\ b = 7,\ C = 90^\circ$

→ *Law of Cosines: c^2 = a^2 + b^2 - 2ab*cos(C).*
→ *c^2 = 4^2 + 7^2 - 2(4)(7)*cos(90°).*
→ *c^2 = 16 + 49 - 56*0.0 = 65.0.*
→ *c = sqrt(65.0) = 8.06.*

Answer: $c = 8.06$

Law of Cosines — find missing angle

9. In triangle ABC, a = 4, b = 6, c = 9. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$$a = 4,\ b = 6,\ c = 9$$

→ *Rearrange the Law of Cosines for angle C:*
→ *cos(C) = (a^2 + b^2 - c^2) / (2ab) = (4^2 + 6^2 - 9^2) / (2*4*6).*
→ *cos(C) = (16 + 36 - 81) / 48 = -0.6042.*
→ *C = arccos(-0.6042) = 127.2°.*

Answer: $C = 127.2^\circ$

10. In triangle ABC, a = 6, b = 4, c = 7. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$$a = 6,\ b = 4,\ c = 7$$

→ *Rearrange the Law of Cosines for angle C:*
→ *cos(C) = (a^2 + b^2 - c^2) / (2ab) = (6^2 + 4^2 - 7^2) / (2*6*4).*
→ *cos(C) = (36 + 16 - 49) / 48 = 0.0625.*
→ *C = arccos(0.0625) = 86.4°.*

Answer: $C = 86.4^\circ$

11. In triangle ABC, a = 5, b = 8, c = 8. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$$a = 5,\ b = 8,\ c = 8$$

→ *Rearrange the Law of Cosines for angle C:*
→ *cos(C) = (a^2 + b^2 - c^2) / (2ab) = (5^2 + 8^2 - 8^2) / (2*5*8).*
→ *cos(C) = (25 + 64 - 64) / 80 = 0.3125.*
→ *C = arccos(0.3125) = 71.8°.*

Answer: $C = 71.8^\circ$

12. In triangle ABC, a = 4, b = 5, c = 5. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$$a = 4,\ b = 5,\ c = 5$$

→ *Rearrange the Law of Cosines for angle C:*
→ *cos(C) = (a^2 + b^2 - c^2) / (2ab) = (4^2 + 5^2 - 5^2) / (2*4*5).*
→ *cos(C) = (16 + 25 - 25) / 40 = 0.4.*
→ *C = arccos(0.4) = 66.4°.*

Answer: $C = 66.4^\circ$

13. In triangle ABC, a = 5, b = 4, c = 9. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$a = 5,\ b = 4,\ c = 9$

→ *Rearrange the Law of Cosines for angle C:*
→ *cos(C) = (a^2 + b^2 - c^2) / (2ab) = (5^2 + 4^2 - 9^2) / (2*5*4).*
→ *cos(C) = (25 + 16 - 81) / 40 = -1.0.*
→ *C = arccos(-1.0) = 180.0°.*

Answer: $C = 180.0\,^\circ$

14. In triangle ABC, a = 7, b = 7, c = 6. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$a = 7,\ b = 7,\ c = 6$

→ *Rearrange the Law of Cosines for angle C:*
→ *cos(C) = (a^2 + b^2 - c^2) / (2ab) = (7^2 + 7^2 - 6^2) / (2*7*7).*
→ *cos(C) = (49 + 49 - 36) / 98 = 0.6327.*
→ *C = arccos(0.6327) = 50.8°.*

Answer: $C = 50.8\,^\circ$

15. In triangle ABC, a = 4, b = 4, c = 7. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$a = 4,\ b = 4,\ c = 7$

→ *Rearrange the Law of Cosines for angle C:*
→ *cos(C) = (a^2 + b^2 - c^2) / (2ab) = (4^2 + 4^2 - 7^2) / (2*4*4).*
→ *cos(C) = (16 + 16 - 49) / 32 = -0.5312.*
→ *C = arccos(-0.5312) = 122.1°.*

Answer: $C = 122.1\,^\circ$

16. In triangle ABC, a = 6, b = 8, c = 8. Use the Law of Cosines to find angle C in degrees (round to 1 decimal place).

$a = 6,\ b = 8,\ c = 8$

→ *Rearrange the Law of Cosines for angle C:*
→ *cos(C) = (a^2 + b^2 - c^2) / (2ab) = (6^2 + 8^2 - 8^2) / (2*6*8).*
→ *cos(C) = (36 + 64 - 64) / 96 = 0.375.*
→ *C = arccos(0.375) = 68.0°.*

Answer: $C = 68.0\,^\circ$

Law of Sines

17. In triangle ABC, angle A = 45°, angle B = 45°, and side a = 6. Use the Law of Sines to find side b (round to 2 decimal places).

$A = 45^\circ,\ B = 45^\circ,\ a = 6$

→ *Law of Sines: a / sin(A) = b / sin(B).*
→ *b = a * sin(B) / sin(A) = 6 * sin(45°) / sin(45°).*
→ *b = 6 * 0.7071 / 0.7071 = 6.0.*

Answer: $b = 6.0$

18. In triangle ABC, angle A = 60°, angle B = 75°, and side a = 6. Use the Law of Sines to find side b (round to 2 decimal places).

$A = 60^\circ,\ B = 75^\circ,\ a = 6$

→ *Law of Sines: a / sin(A) = b / sin(B).*
→ *b = a * sin(B) / sin(A) = 6 * sin(75°) / sin(60°).*
→ *b = 6 * 0.9659 / 0.866 = 6.69.*

Answer: $b = 6.69$

19. In triangle ABC, angle A = 45°, angle B = 45°, and side a = 12. Use the Law of Sines to find side b (round to 2 decimal places).

$A = 45^\circ,\ B = 45^\circ,\ a = 12$

→ *Law of Sines: a / sin(A) = b / sin(B).*
→ *b = a * sin(B) / sin(A) = 12 * sin(45°) / sin(45°).*
→ *b = 12 * 0.7071 / 0.7071 = 12.0.*

Answer: $b = 12.0$

20. In triangle ABC, angle A = 60°, angle B = 45°, and side a = 11. Use the Law of Sines to find side b (round to 2 decimal places).

$A = 60^\circ,\ B = 45^\circ,\ a = 11$

→ *Law of Sines: a / sin(A) = b / sin(B).*
→ *b = a * sin(B) / sin(A) = 11 * sin(45°) / sin(60°).*
→ *b = 11 * 0.7071 / 0.866 = 8.98.*

Answer: $b = 8.98$

21. In triangle ABC, angle A = 45°, angle B = 75°, and side a = 6. Use the Law of Sines to find side b (round to 2 decimal places).

$A = 45^\circ,\ B = 75^\circ,\ a = 6$

→ *Law of Sines: a / sin(A) = b / sin(B).*
→ *b = a * sin(B) / sin(A) = 6 * sin(75°) / sin(45°).*
→ *b = 6 * 0.9659 / 0.7071 = 8.2.*

Answer: $b = 8.2$

22. In triangle ABC, angle A = 45°, angle B = 45°, and side a = 10. Use the Law of Sines to find side b (round to 2 decimal places).

$$A = 45^\circ,\ B = 45^\circ,\ a = 10$$

→ *Law of Sines: a / sin(A) = b / sin(B).*
→ *b = a * sin(B) / sin(A) = 10 * sin(45°) / sin(45°).*
→ *b = 10 * 0.7071 / 0.7071 = 10.0.*

Answer: $b = 10.0$

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23. In triangle ABC, angle A = 60°, angle B = 90°, and side a = 12. Use the Law of Sines to find side b (round to 2 decimal places).

$$A = 60^\circ,\ B = 90^\circ,\ a = 12$$

→ *Law of Sines: a / sin(A) = b / sin(B).*
→ *b = a * sin(B) / sin(A) = 12 * sin(90°) / sin(60°).*
→ *b = 12 * 1.0 / 0.866 = 13.86.*

Answer: $b = 13.86$

Law of Sines — find missing angle

24. In triangle ABC, angle A = 45°, side a = 11, and side b = 10. Use the Law of Sines to find angle B in degrees (round to 1 decimal place).

$A = 45^\circ,\ a = 11,\ b = 10$

→ *Law of Sines: sin(A)/a = sin(B)/b.*
→ *sin(B) = b*sin(A)/a = 10*sin(45°)/11 = 0.6428.*
→ *B = arcsin(0.6428) = 40.0°.*

Answer: $B = 40.0^\circ$

25. In triangle ABC, angle A = 30°, side a = 10, and side b = 9. Use the Law of Sines to find angle B in degrees (round to 1 decimal place).

$A = 30^\circ,\ a = 10,\ b = 9$

→ *Law of Sines: sin(A)/a = sin(B)/b.*
→ *sin(B) = b*sin(A)/a = 9*sin(30°)/10 = 0.45.*
→ *B = arcsin(0.45) = 26.7°.*

Answer: $B = 26.7^\circ$

26. In triangle ABC, angle A = 60°, side a = 8, and side b = 10. Use the Law of Sines to find angle B in degrees (round to 1 decimal place).

$A = 60^\circ,\ a = 8,\ b = 10$

→ *Law of Sines: sin(A)/a = sin(B)/b.*
→ *sin(B) = b*sin(A)/a = 10*sin(60°)/8 = 1.0.*
→ *B = arcsin(1.0) = 90.0°.*

Answer: $B = 90.0^\circ$

27. In triangle ABC, angle A = 30°, side a = 10, and side b = 3. Use the Law of Sines to find angle B in degrees (round to 1 decimal place).

$A = 30^\circ,\ a = 10,\ b = 3$

→ *Law of Sines: sin(A)/a = sin(B)/b.*
→ *sin(B) = b*sin(A)/a = 3*sin(30°)/10 = 0.15.*
→ *B = arcsin(0.15) = 8.6°.*

Answer: $B = 8.6^\circ$

28. In triangle ABC, angle A = 45°, side a = 7, and side b = 6. Use the Law of Sines to find angle B in degrees (round to 1 decimal place).

$A = 45^\circ,\ a = 7,\ b = 6$

→ *Law of Sines: sin(A)/a = sin(B)/b.*
→ *sin(B) = b*sin(A)/a = 6*sin(45°)/7 = 0.6061.*
→ *B = arcsin(0.6061) = 37.3°.*

Answer: $B = 37.3^\circ$

29. In triangle ABC, angle A = 45°, side a = 9, and side b = 7. Use the Law of Sines to find angle B in degrees (round to 1 decimal place).

$A = 45^\circ,\ a = 9,\ b = 7$

→ *Law of Sines: sin(A)/a = sin(B)/b.*
→ *sin(B) = b*sin(A)/a = 7*sin(45°)/9 = 0.55.*
→ *B = arcsin(0.55) = 33.4°.*

Answer: $B = 33.4^\circ$

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30. In triangle ABC, angle A = 60°, side a = 5, and side b = 4. Use the Law of Sines to find angle B in degrees (round to 1 decimal place).

$A = 60^\circ,\ a = 5,\ b = 4$

→ *Law of Sines: sin(A)/a = sin(B)/b.*
→ *sin(B) = b*sin(A)/a = 4*sin(60°)/5 = 0.6928.*
→ *B = arcsin(0.6928) = 43.9°.*

Answer: $B = 43.9^\circ$
