



Name: _____

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Learning Objectives

- Apply the Multiplication Rule for sequential choices
- Compute permutations $P(n,r)$ when order matters
- Compute combinations $C(n,r)$ when order does not matter
- Use the Pigeonhole Principle to solve counting arguments

Always ask: does ORDER matter? If yes \rightarrow permutation. If no \rightarrow combination. For multi-step problems, multiply counts for each independent step.

1. A restaurant offers 4 appetizers, 6 entrees, and 3 desserts. How many different 3-course meals are possible? (Multiplication Rule)

$$4 \times 6 \times 3 = ?$$

Answer: _____

2. In how many ways can 6 students line up for a photo? (Permutation — all n objects)

$$6! = ?$$

Answer: _____

3. A club must elect a President and Vice-President from 10 members. How many ordered outcomes are possible? (Permutation $P(n, r)$)

$$P(10, 2) = \frac{10!}{(10 - 2)!}$$

Answer: _____

4. A committee of 4 is chosen from 9 people. How many committees are possible? (Combination $C(n, r)$)

$$C(9, 4) = \frac{9!}{4! 5!}$$

Answer: _____

5. A poker hand of 5 cards is dealt from a standard 52-card deck. How many hands contain exactly 3 Aces?

$$C(4, 3) \times C(48, 2) = ?$$

Answer: _____



6. How many 5-character passwords can be formed using lowercase letters (a–z) if repetition is allowed? If repetition is NOT allowed?

$$26^5 \text{ vs } P(26, 5)$$

Answer: _____

7. By the Pigeonhole Principle: If 13 socks are drawn from a drawer containing only red and blue socks, what is the minimum number guaranteed to have at least 7 socks of the same color?

Pigeonhole: n items, k holes $\Rightarrow \lceil n/k \rceil$

Answer: _____

8. How many ways can 8 books be arranged on a shelf if 3 specific books must stay together?

$$6! \times 3! = ?$$

Answer: _____

9. How many different 4-digit numbers can be formed using digits 1–9 with no digit repeated, and the first digit must be odd?

$$5 \times P(8, 3) = ?$$

Answer: _____

10. Expand $C(n, r) = C(n, n-r)$ to show that $C(10, 7) = C(10, 3)$. Compute both.

$$C(10, 7) = C(10, 3) = \frac{10!}{3! 7!}$$

Answer: _____





Problems 1–2: multiplication rule and factorial. Problems 3–4: $P(n,r)$ vs $C(n,r)$ contrast. Problem 7 (Pigeonhole): set up "worst case" scenario first.

Solutions

1. A restaurant offers 4 appetizers, 6 entrees, and 3 desserts. How many different 3-course meals are possible? (Multiplication Rule)

$$4 \times 6 \times 3 = ?$$

→ Multiplication Rule: choices for each independent step multiply.

→ $4 \times 6 \times 3 = 72$ different 3-course meals.

Answer: $4 \times 6 \times 3 = 72$ meals

2. In how many ways can 6 students line up for a photo? (Permutation — all n objects)

$$6! = ?$$

→ All 6 students are arranged in a row — order matters.

→ $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

Answer: $6! = 720$

3. A club must elect a President and Vice-President from 10 members. How many ordered outcomes are possible? (Permutation $P(n, r)$)

$$P(10, 2) = \frac{10!}{(10-2)!}$$

→ $P(n,r) = n!/(n-r)!$ — order matters (President \neq Vice-President).

→ $P(10,2) = 10!/8! = 10 \times 9 = 90$.

Answer: $P(10, 2) = 10 \times 9 = 90$

4. A committee of 4 is chosen from 9 people. How many committees are possible? (Combination $C(n, r)$)

$$C(9, 4) = \frac{9!}{4!5!}$$

→ $C(n,r) = n!/(r!(n-r)!)$ — order does NOT matter.

→ $C(9,4) = 9!/(4!5!) = (9 \times 8 \times 7 \times 6)/(4 \times 3 \times 2 \times 1) = 3024/24 = 126$.

Answer: $C(9, 4) = 126$

5. A poker hand of 5 cards is dealt from a standard 52-card deck. How many hands contain exactly 3 Aces?

$$C(4, 3) \times C(48, 2) = ?$$

→ Choose 3 Aces from 4: $C(4,3) = 4$.

→ Choose the remaining 2 cards from the 48 non-Aces: $C(48,2) = 1128$.

→ Total = $4 \times 1128 = 4512$ hands.

Answer: $C(4, 3) \cdot C(48, 2) = 4 \times 1128 = 4512$



6. How many 5-character passwords can be formed using lowercase letters (a–z) if repetition is allowed? If repetition is NOT allowed?

$$26^5 \text{ vs } P(26, 5)$$

→ With repetition: 26 choices for each of 5 slots = $26^5 = 11,881,376$.

→ Without repetition: $P(26, 5) = 26 \times 25 \times 24 \times 23 \times 22 = 7,893,600$.

Answer: $26^5 = 11,881,376$ vs $P(26, 5) = 7,893,600$

7. By the Pigeonhole Principle: If 13 socks are drawn from a drawer containing only red and blue socks, what is the minimum number guaranteed to have at least 7 socks of the same color?

Pigeonhole: n items, k holes $\Rightarrow \lceil n/k \rceil$

→ Worst case: 6 red and 6 blue = 12 socks with no color having 7.

→ The 13th sock must be one of the two colors → one color has ≥ 7 .

→ Pigeonhole: $\lceil 13/2 \rceil = 7$. Guaranteed after drawing 13.

Answer: 13 socks, 2 colors $\Rightarrow \geq 7$ of one color

8. How many ways can 8 books be arranged on a shelf if 3 specific books must stay together?

$$6! \times 3! = ?$$

→ Treat the 3 books as ONE block → 6 objects total (block + 5 other books).

→ Arrange 6 objects: $6! = 720$ ways.

→ Arrange the 3 books WITHIN the block: $3! = 6$ ways.

→ Total = $720 \times 6 = 4320$.

Answer: $6! \times 3! = 720 \times 6 = 4320$

9. How many different 4-digit numbers can be formed using digits 1–9 with no digit repeated, and the first digit must be odd?

$$5 \times P(8, 3) = ?$$

→ First digit: must be odd (1, 3, 5, 7, 9) → 5 choices.

→ Remaining 3 digits from 8 remaining (no repetition): $P(8, 3) = 8 \times 7 \times 6 = 336$.

→ Total = $5 \times 336 = 1680$.

Answer: $5 \times 8 \times 7 \times 6 = 1680$

10. Expand $C(n, r) = C(n, n-r)$ to show that $C(10, 7) = C(10, 3)$. Compute both.

$$C(10, 7) = C(10, 3) = \frac{10!}{3!7!}$$

→ $C(10, 7) = 10!/(7!3!) = C(10, 3)$ — same calculation.

→ $C(10, 3) = (10 \times 9 \times 8)/(3 \times 2 \times 1) = 720/6 = 120$.

→ Symmetry: choosing 7 to include = choosing 3 to exclude.

Answer: $C(10, 3) = 120$

