MATH 160 — Logic & Propositions

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Name: ______________________________ Date: _______________ Score: / 10

Learning Objectives

- Build and interpret truth tables for ∧, ∨, ¬, →, and ↔
- Apply De Morgan's Laws and logical equivalences
- Evaluate compound propositions and identify tautologies
- Construct and validate logical arguments (Modus Ponens, contrapositive)

For truth tables: always list all 2■ rows for n variables. Use T and F consistently — not 1/0 or True/False.

1. Complete the truth table for p ∧ q (AND) and p ∨ q (OR).

p	q	p ∧ q	p ∨ q
T	T	____	____
T	F	____	____
F	T	____	____
F	F	____	____

Answer: ______________________________

2. Complete the truth table for ¬p, p → q (IF p THEN q), and ¬p ∨ q.

p	q	¬p	p → q	¬p ∨ q
T	T	____	____	____
T	F	____	____	____
F	T	____	____	____
F	F	____	____	____

Answer: ______________________________

3. Complete the truth table to verify De Morgan's Law: ¬(p ∧ q) ≡ ¬p ∨ ¬q.

p	q	p ∧ q	¬(p ∧ q)	¬p ∨ ¬q
T	T	____	____	____
T	F	____	____	____
F	T	____	____	____
F	F	____	____	____

Answer: ______________________________

4. Complete the truth table for p ∧ (q ∨ r).

p	q	r	q ∨ r	p ∧ (q ∨ r)
T	T	T	___	___
T	T	F	___	___
T	F	T	___	___
T	F	F	___	___
F	T	T	___	___
F	T	F	___	___
F	F	T	___	___
F	F	F	___	___

Answer: ______________________________________

5. Build a truth table for (p → q) ∧ (q → p). Is this a tautology, contradiction, or neither?

p	q	p → q	q → p	(p→q) ∧ (q→p)
T	T	___	___	___
T	F	___	___	___
F	T	___	___	___
F	F	___	___	___

Answer: ______________________________________

6. Evaluate the logical argument: "If it rains, the ground is wet. It is raining. Therefore, the ground is wet." Is this valid?

$$p \to q, \quad p \vdash q$$

Answer: ______________________________________

7. Write the converse, inverse, and contrapositive of: "If it snows, then school is cancelled."

$$p \to q$$

Answer: ______________________________________

8. Determine the truth value of (p ∨ ¬q) → (¬p ∧ q) when p = T and q = F.

$$p = T,\ q = F \Rightarrow (p \vee \neg q) \to (\neg p \wedge q)$$

Answer: ______________________________________

9. Complete the truth table for p ↔ q (biconditional, "p if and only if q").

p	q	p ↔ q	p → q	q → p
T	T	____	____	____
T	F	____	____	____
F	T	____	____	____
F	F	____	____	____

Answer: ____

10. Use a truth table to show whether p → q and ¬q → ¬p are logically equivalent.

p	q	¬p	¬q	p → q	¬q → ¬p
T	T	____	____	____	____
T	F	____	____	____	____
F	T	____	____	____	____
F	F	____	____	____	____

Answer: ____

Scan to watch

Problems 1–5 are full truth tables — key exam skill. Problem 3 verifies De Morgan's Law by table. Problem 7 introduces the contrapositive (only equivalent transform).

Solutions

1. Complete the truth table for $p \wedge q$ (AND) and $p \vee q$ (OR).

p	q	$p \wedge q$	$p \vee q$
T	T	___	___
T	F	___	___
F	T	___	___
F	F	___	___

→ $p \wedge q$ (AND) is T only when BOTH p and q are T.
→ $p \vee q$ (OR) is T when AT LEAST ONE of p, q is T.
→ Row 1: T∧T=T, T∨T=T. Row 2: T∧F=F, T∨F=T.
→ Row 3: F∧T=F, F∨T=T. Row 4: F∧F=F, F∨F=F.

Answer:

p	q	$p \wedge q$	$p \vee q$
T	T	T	**T**
T	F	F	**T**
F	T	F	**T**
F	F	F	**F**

2. Complete the truth table for $\neg p$, $p \rightarrow q$ (IF p THEN q), and $\neg p \vee q$.

p	q	$\neg p$	$p \rightarrow q$	$\neg p \vee q$
T	T	___	___	___
T	F	___	___	___
F	T	___	___	___
F	F	___	___	___

→ $\neg p$ flips the truth value of p.
→ $p \rightarrow q$ (conditional) is F only when p is T and q is F.
→ Key insight: $p \rightarrow q$ is logically equivalent to $\neg p \vee q$ (same column!).
→ This equivalence is used to prove theorems by contrapositive.

Answer:

p	q	$\neg p$	$p \rightarrow q$	$\neg p \vee q$
T	T	F	T	**T**
T	F	F	F	**F**
F	T	T	T	**T**
F	F	T	T	**T**

3. Complete the truth table to verify De Morgan's Law: $\neg(p \land q) \equiv \neg p \lor \neg q$.

p	q	$p \land q$	$\neg(p \land q)$	$\neg p \lor \neg q$
T	T	___	___	___
T	F	___	___	___
F	T	___	___	___
F	F	___	___	___

→ Columns 4 and 5 are identical → the two expressions are logically equivalent.
→ De Morgan's Law 1: $\neg(p \land q) \equiv \neg p \lor \neg q$.
→ De Morgan's Law 2: $\neg(p \lor q) \equiv \neg p \land \neg q$.
→ These laws let you distribute negation across AND/OR.

Answer:

p	q	$p \land q$	$\neg(p \land q)$	$\neg p \lor \neg q$
T	T	T	F	**F**
T	F	F	T	**T**
F	T	F	T	**T**
F	F	F	T	**T**

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4. Complete the truth table for $p \land (q \lor r)$.

p	q	r	$q \lor r$	$p \land (q \lor r)$
T	T	T	___	___
T	T	F	___	___
T	F	T	___	___
T	F	F	___	___
F	T	T	___	___
F	T	F	___	___
F	F	T	___	___
F	F	F	___	___

→ First evaluate $q \lor r$ (OR) for each row.
→ Then evaluate $p \land (q \lor r)$ — AND with result from column 4.
→ $p \land (q \lor r)$ is T only when p is T AND at least one of q, r is T.
→ Rows 1–3 are T; row 4 (T,F,F) gives F because $q \lor r$ = F.

Answer:

p	q	r	$q \lor r$	$p \land (q \lor r)$
T	T	T	T	**T**
T	T	F	T	**T**
T	F	T	T	**T**
T	F	F	F	**F**
F	T	T	T	**F**
F	T	F	T	**F**
F	F	T	T	**F**
F	F	F	F	**F**

Scan to watch

5. Build a truth table for (p → q) ∧ (q → p). Is this a tautology, contradiction, or neither?

p	q	p → q	q → p	(p→q) ∧ (q→p)
T	T	___	___	___
T	F	___	___	___
F	T	___	___	___
F	F	___	___	___

→ *(p→q)∧(q→p) = T only in rows 1 and 4 (when p and q have the same value).*
→ *This is the biconditional p ↔ q — neither always T (tautology) nor always F.*
→ *It is a contingency (sometimes T, sometimes F).*

Answer:

p	q	p → q	q → p	(p→q) ∧ (q→p)
T	T	T	T	**T**
T	F	F	T	**F**
F	T	T	F	**F**
F	F	T	T	**T**

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6. Evaluate the logical argument: "If it rains, the ground is wet. It is raining. Therefore, the ground is wet." Is this valid?

$$p \to q, \quad p \vdash q$$

→ *Let p = "It is raining", q = "The ground is wet".*
→ *Premise 1: p → q. Premise 2: p.*
→ *Modus Ponens rule: from p→q and p, we can conclude q.*
→ *The argument form is valid (always produces T when premises are T).*

Answer: Valid — Modus Ponens

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7. Write the converse, inverse, and contrapositive of: "If it snows, then school is cancelled."

$$p \to q$$

→ *Original (p→q): "If it snows, school is cancelled."*
→ *Converse (q→p): "If school is cancelled, then it snowed." (NOT equivalent)*
→ *Inverse (¬p→¬q): "If it does not snow, school is not cancelled." (NOT equivalent)*
→ *Contrapositive (¬q→¬p): "If school is not cancelled, it did not snow." (EQUIVALENT)*
→ *Only the contrapositive is logically equivalent to the original.*

Answer: Contrapositive: $\neg q \to \neg p \equiv p \to q$

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8. Determine the truth value of (p ∨ ¬q) → (¬p ∧ q) when p = T and q = F.

$$p = T,\ q = F \Rightarrow (p \lor \neg q) \to (\neg p \land q)$$

→ *¬q = ¬F = T. p ∨ ¬q = T ∨ T = T.*
→ *¬p = ¬T = F. ¬p ∧ q = F ∧ F = F.*
→ *(p ∨ ¬q) → (¬p ∧ q) = T → F = F.*
→ *A conditional is F only when the hypothesis is T and conclusion is F.*

Answer: $T \to F = F$

Scan to watch

9. Complete the truth table for p ↔ q (biconditional, "p if and only if q").

p	q	p ↔ q	p → q	q → p
T	T	___	___	___
T	F	___	___	___
F	T	___	___	___
F	F	___	___	___

→ p ↔ q is T when p and q have the SAME truth value.
→ Equivalently: p ↔ q = (p→q) ∧ (q→p).
→ Notice: p ↔ q column matches rows where p=q.

Answer:

p	q	p ↔ q	p → q	q → p
T	T	T	T	**T**
T	F	F	F	**T**
F	T	F	T	**F**
F	F	T	T	**T**

10. Use a truth table to show whether p → q and ¬q → ¬p are logically equivalent.

p	q	¬p	¬q	p → q	¬q → ¬p
T	T	___	___	___	___
T	F	___	___	___	___
F	T	___	___	___	___
F	F	___	___	___	___

→ Columns 5 and 6 are identical → p → q ≡ ¬q → ¬p.
→ ¬q → ¬p is the CONTRAPOSITIVE of p → q.
→ A conditional and its contrapositive are always logically equivalent.

Answer:

p	q	¬p	¬q	p → q	¬q → ¬p
T	T	F	F	T	**T**
T	F	F	T	F	**F**
F	T	T	F	T	**T**
F	F	T	T	T	**T**