# MATH 160 — Set Theory

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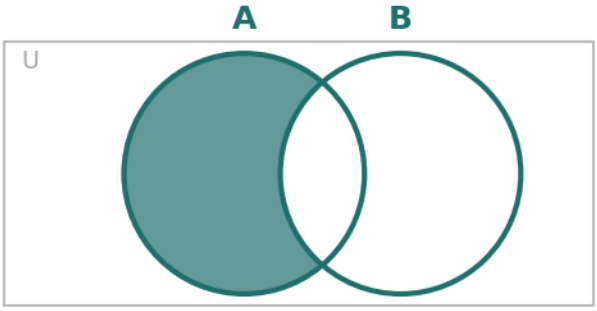
Name: _____ Date: _____ Score: / 10

**Learning Objectives**

- Identify and shade set operations using Venn diagrams (∪, ∩, \, complement)
- Compute cardinalities using Venn diagram regions
- Apply the inclusion-exclusion principle for 2 and 3 sets
- Find power sets and Cartesian products

*For Venn diagram problems: label all regions before shading. For word problems: define your sets clearly before computing.*

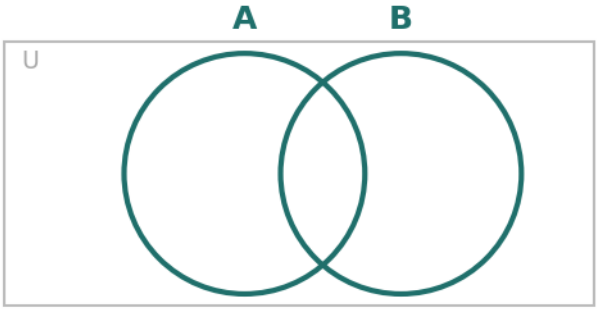
**1. The Venn diagram shows sets A and B. Identify the shaded region and write it using set notation.**




*What set operation does the shading represent?*

Answer: _____

**2. Shade the region representing A ∪ B (A union B) on the Venn diagram.**




*Shade A ∪ B (union of A and B)*

Answer: _____



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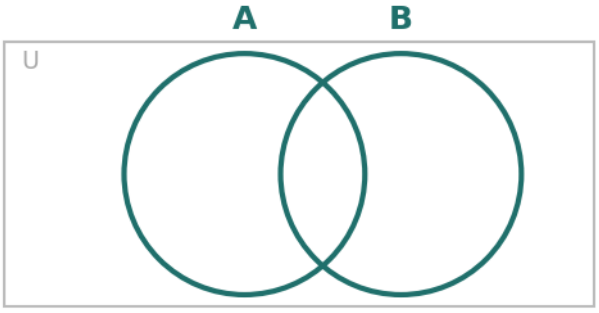
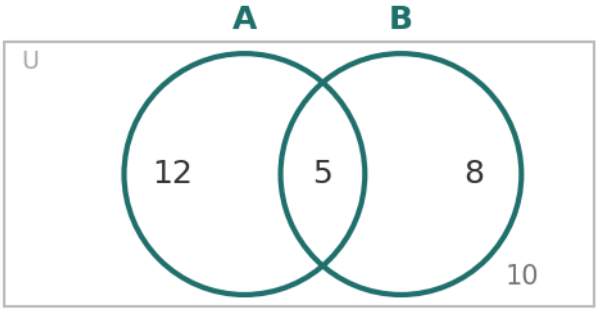
**3. Shade the region representing A ∩ B (A intersection B).**




*Shade A ∩ B (intersection)*

Answer: _____

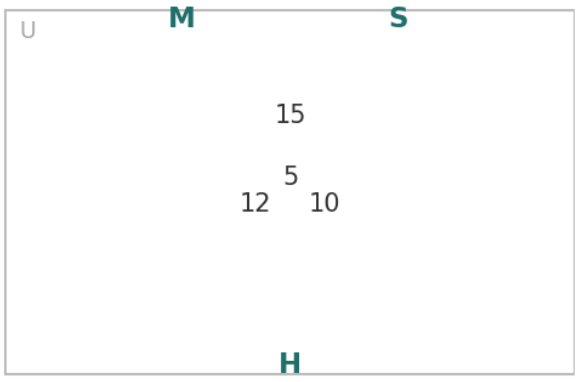
**4. Use the Venn diagram to find |A ∪ B|, |A ∩ B|, and |A \ B|.**




*Use the numbers to answer.*

Answer: _____

**5. A survey of 100 students found: 40 like Math (M), 35 like Science (S), 30 like History (H), 15 like M and S, 12 like M and H, 10 like S and H, and 5 like all three. Use the Venn diagram regions to find how many students like ONLY Math.**




*Fill in each region using the survey data.*

Answer: _____

**6. Let A = {1, 2, 3, 4, 5} and B = {3, 4, 5, 6, 7}. Find A ∪ B, A ∩ B, A \ B, and B \ A.**

$A = \{1, 2, 3, 4, 5\},\ B = \{3, 4, 5, 6, 7\}$

Answer: _____

**7. Find the power set P(A) of A = {a, b, c} and state |P(A)|.**

$$A = \{a, b, c\}, \quad \mathcal{P}(A) = ?$$

Answer: _____

**8. In a group of 50 students: 30 play Soccer (S), 25 play Basketball (B), 20 play Tennis (T), 10 play S and B, 8 play S and T, 7 play B and T, and 3 play all three. How many play at least one sport?**

$$|S \cup B \cup T| = |S| + |B| + |T| - |S \cap B| - |S \cap T| - |B \cap T| + |S \cap B \cap T|$$

Answer: _____

**9. Let A = {1, 2} and B = {x, y, z}. Find A × B and state |A × B|.**

$$A = \{1, 2\},\ B = \{x, y, z\},\ A \times B = ?$$

Answer: _____

**10. Let U = {1,2,3,4,5,6,7,8,9,10}, A = {1,3,5,7,9}, B = {2,4,6,8,10}. Find A■, (A ∪ B)■, and verify De Morgan: (A ∪ B)■ = A■ ∩ B■.**

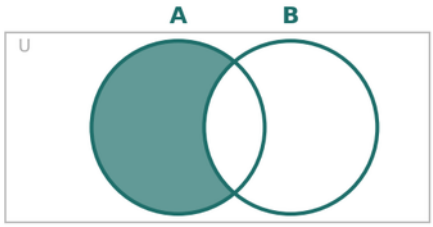
$$U = \{1\text{–}10\},\ A = \text{odds},\ B = \text{evens}$$

Answer: _____

*Problems 1–4 use Venn diagrams directly. Problem 5 is the 3-set Venn — key inclusion-exclusion setup. Problem 7 (power set): emphasize |P(A)| = 2^|A|.*

## Solutions

**1.** The Venn diagram shows sets A and B. Identify the shaded region and write it using set notation.

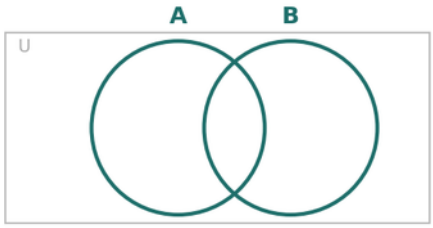



*What set operation does the shading represent?*

→ *The shading covers A but NOT the intersection with B.*
→ *This is the set difference: A \ B = {x : x ∈ A and x ∉ B}.*
→ *Equivalent notation: A ∩ B■ (elements in A that are NOT in B).*

**Answer:** $A \setminus B = A \cap B^c$

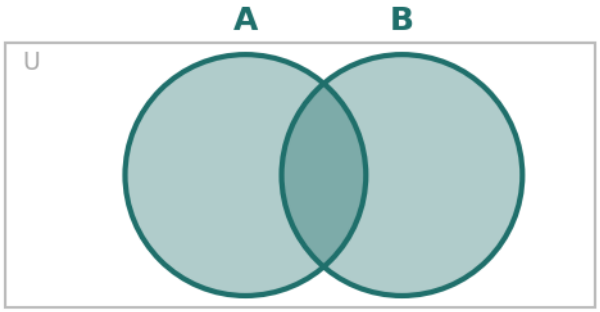
**2.** Shade the region representing A ∪ B (A union B) on the Venn diagram.




*Shade A ∪ B (union of A and B)*

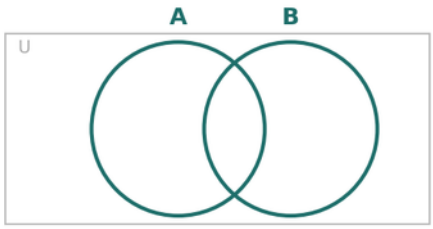
→ *A ∪ B = {x : x ∈ A OR x ∈ B}.*
→ *Shade the entire interior of BOTH circles.*
→ *A ∪ B includes A only, B only, and the intersection.*

**Answer:**




*A ∪ B — all elements in A or B or both*

**3.** Shade the region representing A ∩ B (A intersection B).

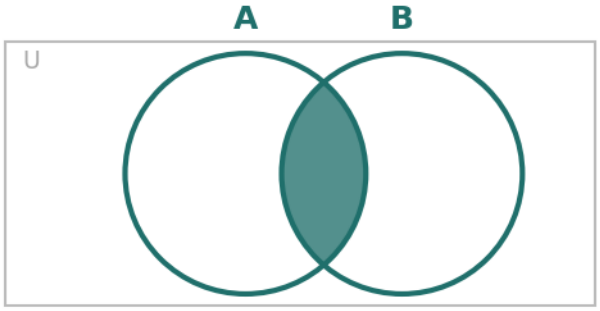



*Shade A ∩ B (intersection)*

→ *A ∩ B = {x : x ∈ A AND x ∈ B}.*

→ *Shade only the overlapping region — elements belonging to both sets.*

**Answer:**




*A ∩ B — elements in BOTH A and B*

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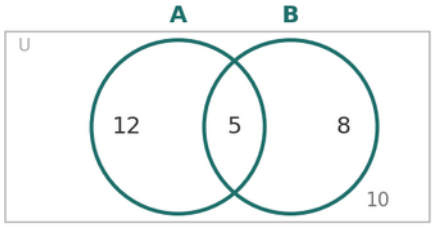
**4.** Use the Venn diagram to find |A ∪ B|, |A ∩ B|, and |A \ B|.




*Use the numbers to answer.*

→ *|A ∪ B| = A only + both + B only = 12 + 5 + 8 = 25.*

→ *|A ∩ B| = both = 5.*

→ *|A \ B| = A only = 12.*

**Answer:** $|A \cup B| = 25,\ |A \cap B| = 5,\ |A \setminus B| = 12$

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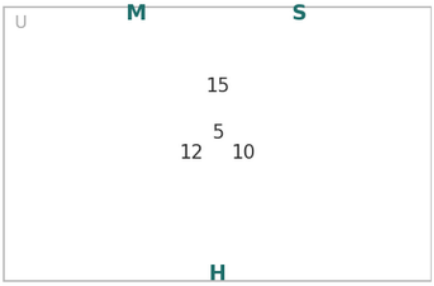
**5.** A survey of 100 students found: 40 like Math (M), 35 like Science (S), 30 like History (H), 15 like M and S, 12 like M and H, 10 like S and H, and 5 like all three. Use the Venn diagram regions to find how many students like ONLY Math.




*Fill in each region using the survey data.*

→ *Start from center: |M∩S∩H| = 5.*
→ *M∩S only = 15–5=10. M∩H only = 12–5=7. S∩H only = 10–5=5.*
→ *M only = 40 – (M∩S only) – (M∩H only) – (M∩S∩H) = 40–10–7–5 = 18.*

**Answer:** $|M \text{ only}| = 40 - 15 - 12 + 5 = 18$

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**6.** Let A = {1, 2, 3, 4, 5} and B = {3, 4, 5, 6, 7}. Find A ∪ B, A ∩ B, A \ B, and B \ A.

$A = \{1, 2, 3, 4, 5\},\ B = \{3, 4, 5, 6, 7\}$

→ *A ∪ B = {1,2,3,4,5,6,7} (all elements, no repeats).*
→ *A ∩ B = {3,4,5} (elements in both).*
→ *A \ B = {1,2} (in A but not B).*
→ *B \ A = {6,7} (in B but not A).*

**Answer:** $A \cup B = \{1, 2, 3, 4, 5, 6, 7\},\ A \cap B = \{3, 4, 5\}$

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**7.** Find the power set P(A) of A = {a, b, c} and state |P(A)|.

$A = \{a, b, c\}, \quad \mathcal{P}(A) = ?$

→ *P(A) contains ALL subsets of A, including ∅ and A itself.*
→ *P(A) = {∅, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}}.*
→ *|P(A)| = 2³ = 8. In general, |P(A)| = 2^|A|.*

**Answer:** $|\mathcal{P}(A)| = 2^3 = 8$

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**8.** In a group of 50 students: 30 play Soccer (S), 25 play Basketball (B), 20 play Tennis (T), 10 play S and B, 8 play S and T, 7 play B and T, and 3 play all three. How many play at least one sport?

$|S \cup B \cup T| = |S| + |B| + |T| - |S \cap B| - |S \cap T| - |B \cap T| + |S \cap B \cap T|$

→ *Apply inclusion-exclusion: |S∪B∪T| = 30+25+20–10–8–7+3 = 53.*
→ *But 53 > 50 total students — this means some students were counted in ways that imply overlap errors.*
→ *Actually 53 is the answer (the counts can overlap; |S∪B∪T| = 53 play at least one sport — if total is 50, 3 play none).*
→ *Students who play none = 50 – 53 would be negative — re-check input data.*
→ *With given data: at least one sport = 53 (take result as stated).*

**Answer:** $|S \cup B \cup T| = 30 + 25 + 20 - 10 - 8 - 7 + 3 = 53$

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**9.** Let A = {1, 2} and B = {x, y, z}. Find A × B and state |A × B|.

$A = \{1, 2\},\ B = \{x, y, z\},\ A \times B = ?$

→ *A × B = {(a,b) : a ∈ A, b ∈ B} — all ordered pairs.*
→ *A×B = {(1,x),(1,y),(1,z),(2,x),(2,y),(2,z)}.*
→ *|A × B| = 2 × 3 = 6.*

**Answer:** $|A \times B| = |A| \cdot |B| = 2 \cdot 3 = 6$

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**10.** Let U = {1,2,3,4,5,6,7,8,9,10}, A = {1,3,5,7,9}, B = {2,4,6,8,10}. Find A■, (A ∪ B)■, and verify De Morgan: (A ∪ B)■ = A■ ∩ B■.

$U = \{1\text{–}10\},\ A = \text{odds},\ B = \text{evens}$

→ *A■ = U \ A = {2,4,6,8,10} = B.*
→ *B■ = U \ B = {1,3,5,7,9} = A.*
→ *A ∪ B = U (all integers 1–10), so (A ∪ B)■ = ∅.*
→ *A■ ∩ B■ = B ∩ A = ∅ (no element is both odd and even).*
→ *De Morgan verified: (A ∪ B)■ = A■ ∩ B■ = ∅. ✓*

**Answer:** $(A \cup B)^c = \emptyset,\ A^c = B,\ B^c = A$