MATH 210 — Derivative Fundamentals

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Learning Objectives

- Apply the Power Rule to polynomial and radical functions
- Differentiate e^x, ln x, sin x, and cos x
- Find equations of tangent lines using the derivative
- Use the limit definition of the derivative

Rewrite radicals and rational expressions using exponent notation before differentiating.

1. Differentiate using the Power Rule.

$$f(x) = 5x^4 - 3x^2 + 7x - 2$$

Answer: _____

2. Differentiate. Rewrite radicals as fractional exponents first.

$$f(x) = \sqrt{x} + \frac{1}{x^2}$$

Answer: _____

3. Find the slope of the tangent line to f(x) at x = 2.

$$f(x) = x^3 - 4x + 1$$

Answer: _____

4. Differentiate the exponential function.

$$f(x) = 4e^x - 3x^2$$

Answer: _____

5. Differentiate the trigonometric function.

$$f(x) = 3\sin x - 2\cos x$$

Answer: _____

6. Use the definition of the derivative to find f'(x).

$$f(x) = x^2 + 3x, \quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Answer: _____

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7. Find the equation of the tangent line to y = f(x) at x = 1.

$f(x) = 2x^3 - x + 5$

Answer: _____

8. Differentiate the natural log function.

$f(x) = \ln x + 5x^2$

Answer: _____

9. Find all x-values where f'(x) = 0.

$f(x) = x^3 - 6x^2 + 9x$

Answer: _____

10. Determine where f(x) is differentiable. State any points of non-differentiability.

$f(x) = |x - 3|$

Answer: _____

Problem 2 requires rewriting radicals — common student error. Problem 6 uses the limit definition; connect back to the limit unit. Problem 10 highlights that |x| is non-differentiable at x = 3.

Solutions

1. Differentiate using the Power Rule.

$f(x) = 5x^4 - 3x^2 + 7x - 2$

→ *Apply the Power Rule: d/dx[x^n] = nx^{n-1}.*
→ *d/dx[$5x^4$] = $20x^3$.*
→ *d/dx[$-3x^2$] = $-6x$.*
→ *d/dx[7x] = 7. d/dx[–2] = 0.*
→ *f′(x) = $20x^3 - 6x + 7$.*

Answer: $f'(x) = 20x^3 - 6x + 7$

2. Differentiate. Rewrite radicals as fractional exponents first.

$f(x) = \sqrt{x} + \frac{1}{x^2}$

→ *Rewrite: f(x) = $x^{1/2} + x^{-2}$.*
→ *f′(x) = $(1/2)x^{-1/2} + (-2)x^{-3}$.*
→ *= $1/(2\sqrt{x}) - 2/x^3$.*

Answer: $f'(x) = \frac{1}{2\sqrt{x}} - \frac{2}{x^3}$

3. Find the slope of the tangent line to f(x) at x = 2.

$f(x) = x^3 - 4x + 1$

→ *f′(x) = $3x^2 - 4$.*
→ *f′(2) = 3(4) – 4 = 12 – 4 = 8.*
→ *The slope of the tangent at x = 2 is 8.*

Answer: $f'(2) = 3(2)^2 - 4 = 8$

4. Differentiate the exponential function.

$f(x) = 4e^x - 3x^2$

→ *d/dx[e^x] = e^x.*
→ *f′(x) = $4e^x - 6x$.*

Answer: $f'(x) = 4e^x - 6x$

5. Differentiate the trigonometric function.

$f(x) = 3\sin x - 2\cos x$

→ *d/dx[sin x] = cos x. d/dx[cos x] = –sin x.*
→ *f′(x) = 3 cos x – 2(–sin x) = 3 cos x + 2 sin x.*

Answer: $f'(x) = 3\cos x + 2\sin x$

6. Use the definition of the derivative to find f'(x).

$f(x) = x^2 + 3x, \quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

→ *f(x+h) = (x+h)^2 + 3(x+h) = x^2 + 2xh + h^2 + 3x + 3h.*
→ *f(x+h) – f(x) = 2xh + h^2 + 3h = h(2x + h + 3).*
→ *Divide by h: 2x + h + 3.*
→ *As h→0: f'(x) = 2x + 3.*

Answer: $f'(x) = 2x + 3$

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7. Find the equation of the tangent line to y = f(x) at x = 1.

$f(x) = 2x^3 - x + 5$

→ *f'(x) = 6x^2 – 1. f'(1) = 6 – 1 = 5 (slope).*
→ *f(1) = 2 – 1 + 5 = 6. Point: (1, 6).*
→ *Tangent: y – 6 = 5(x – 1) → y = 5x + 1.*

Answer: $y = 5x + 1$

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8. Differentiate the natural log function.

$f(x) = \ln x + 5x^2$

→ *d/dx[ln x] = 1/x.*
→ *f'(x) = 1/x + 10x.*

Answer: $f'(x) = \frac{1}{x} + 10x$

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9. Find all x-values where f'(x) = 0.

$f(x) = x^3 - 6x^2 + 9x$

→ *f'(x) = 3x^2 – 12x + 9 = 3(x^2 – 4x + 3).*
→ *Factor: 3(x–1)(x–3).*
→ *f'(x) = 0 when x = 1 or x = 3.*

Answer: $x = 1$ and $x = 3$

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10. Determine where f(x) is differentiable. State any points of non-differentiability.

$f(x) = |x - 3|$

→ *f(x) = x–3 for x > 3 (derivative = 1).*
→ *f(x) = –(x–3) for x < 3 (derivative = –1).*
→ *Left and right derivatives differ at x = 3 — corner point.*
→ *f is differentiable everywhere except x = 3.*

Answer: Not differentiable at $x = 3$