MATH 210 — Differentiation Rules

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Learning Objectives

- Apply the Product Rule to differentiate products of functions
- Apply the Quotient Rule to differentiate rational functions
- Use the Chain Rule for composite functions
- Differentiate implicitly and find second derivatives

Identify which rule applies before differentiating. For the Chain Rule, explicitly name the outer and inner functions.

1. Differentiate using the Product Rule.

$$f(x) = x^3 \cdot e^x$$

Answer: ______________________

2. Differentiate using the Product Rule.

$$f(x) = (2x^2 + 1)\sin x$$

Answer: ______________________

3. Differentiate using the Quotient Rule.

$$f(x) = \frac{x^2 + 1}{x - 3}$$

Answer: ______________________

4. Differentiate using the Quotient Rule.

$$f(x) = \frac{\sin x}{x^2}$$

Answer: ______________________

5. Differentiate using the Chain Rule.

$$f(x) = (3x^2 - 5)^6$$

Answer: ______________________

6. Differentiate using the Chain Rule.

$$f(x) = e^{x^2 + 2x}$$

Answer: ______________________

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7. Differentiate using the Chain Rule.

$f(x) = \sin(3x^2)$

Answer: ____________________________________

8. Differentiate using the Chain Rule with a natural log.

$f(x) = \ln(x^2 + 4)$

Answer: ____________________________________

9. Find f''(x). (Second derivative)

$f(x) = x^5 - 4x^3 + 2x$

Answer: ____________________________________

10. Use implicit differentiation to find dy/dx.

$x^2 + y^2 = 25$

Answer: ____________________________________

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Problems 5–8 cover Chain Rule in different contexts (power, exp, trig, log). Problem 10 (implicit differentiation) — remind students to treat y as a function of x.

Solutions

1. Differentiate using the Product Rule.

$f(x) = x^3 \cdot e^x$

→ *Product Rule: (uv)' = u'v + uv'.*
→ *$u = x^3$, $u' = 3x^2$. $v = e^x$, $v' = e^x$.*
→ *$f'(x) = 3x^2 \cdot e^x + x^3 \cdot e^x = e^x(x^3+3x^2)$.*

Answer: $f'(x) = x^3e^x + 3x^2e^x = e^x(x^3 + 3x^2)$

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2. Differentiate using the Product Rule.

$f(x) = (2x^2 + 1)\sin x$

→ *$u = 2x^2+1$, $u' = 4x$. $v = \sin x$, $v' = \cos x$.*
→ *$f'(x) = 4x \sin x + (2x^2+1) \cos x$.*

Answer: $f'(x) = 4x\sin x + (2x^2 + 1)\cos x$

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3. Differentiate using the Quotient Rule.

$f(x) = \frac{x^2 + 1}{x - 3}$

→ *Quotient Rule: $(u/v)' = (u'v – uv') / v^2$.*
→ *$u = x^2+1$, $u' = 2x$. $v = x–3$, $v' = 1$.*
→ *Numerator: $2x(x–3) – (x^2+1) = 2x^2–6x–x^2–1 = x^2–6x–1$.*
→ *$f'(x) = (x^2–6x–1) / (x–3)^2$.*

Answer: $f'(x) = \frac{(x - 3)(2x) - (x^2 + 1)}{(x - 3)^2} = \frac{x^2 - 6x - 1}{(x - 3)^2}$

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4. Differentiate using the Quotient Rule.

$f(x) = \frac{\sin x}{x^2}$

→ *$u = \sin x$, $u' = \cos x$. $v = x^2$, $v' = 2x$.*
→ *Numerator: $x^2 \cdot \cos x – \sin x \cdot 2x = x(x \cos x – 2 \sin x)$.*
→ *$f'(x) = (x \cos x – 2 \sin x) / x^3$.*

Answer: $f'(x) = \frac{x\cos x - 2\sin x}{x^3}$

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5. Differentiate using the Chain Rule.

$f(x) = (3x^2 - 5)^6$

→ *Chain Rule: $d/dx[g(x)^n] = n \cdot g(x)^{n-1} \cdot g'(x)$.*
→ *Outer: $6(3x^2–5)^5$. Inner derivative: $6x$.*
→ *$f'(x) = 36x(3x^2–5)^5$.*

Answer: $f'(x) = 6(3x^2 - 5)^5 \cdot 6x = 36x(3x^2 - 5)^5$

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6. Differentiate using the Chain Rule.

$f(x) = e^{x^2 + 2x}$

→ *$d/dx[e^u] = e^u \cdot u'$, where $u = x^2+2x$.*
→ *$u' = 2x + 2$.*
→ *$f'(x) = (2x+2)e^{x^2+2x}$.*

Answer: $f'(x) = (2x + 2)\,e^{x^2 + 2x}$

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7. Differentiate using the Chain Rule.

$f(x) = \sin(3x^2)$

→ *$d/dx[\sin(u)] = \cos(u) \cdot u'$, where $u = 3x^2$.*
→ *$u' = 6x$.*
→ *$f'(x) = 6x \cos(3x^2)$.*

Answer: $f'(x) = \cos(3x^2) \cdot 6x$

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8. Differentiate using the Chain Rule with a natural log.

$f(x) = \ln(x^2 + 4)$

→ *$d/dx[\ln u] = u'/u$, where $u = x^2+4$.*
→ *$u' = 2x$.*
→ *$f'(x) = 2x / (x^2+4)$.*

Answer: $f'(x) = \dfrac{2x}{x^2 + 4}$

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9. Find f''(x). (Second derivative)

$f(x) = x^5 - 4x^3 + 2x$

→ *$f'(x) = 5x^4 – 12x^2 + 2$.*
→ *$f''(x) = 20x^3 – 24x$.*

Answer: $f''(x) = 20x^3 - 24x$

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10. Use implicit differentiation to find dy/dx.

$x^2 + y^2 = 25$

→ *Differentiate both sides with respect to x.*
→ *$2x + 2y \cdot (dy/dx) = 0$.*
→ *Solve: $dy/dx = –x/y$.*

Answer: $\dfrac{dy}{dx} = -\dfrac{x}{y}$