MATH 210 — Introduction to Integrals

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Learning Objectives

- Find indefinite integrals using the Power Rule and basic rules
- Evaluate definite integrals using the Fundamental Theorem of Calculus
- Apply u-substitution to evaluate composite integrals
- Interpret definite integrals in applied contexts

Always include +C for indefinite integrals. For u-substitution, show your substitution clearly and convert the limits when evaluating definite integrals.

1. Find the indefinite integral.

$$\int (4x^3 - 6x + 5)dx$$

Answer: ________________________________

2. Evaluate the definite integral.

$$\int_1^3 (2x^2 - x + 4)dx$$

Answer: ________________________________

3. Find the indefinite integral of the exponential function.

$$\int \left(3e^x + \frac{2}{x}\right)dx$$

Answer: ________________________________

4. Evaluate using u-substitution. Let u = 2x + 3.

$$\int (2x+3)^5\, dx$$

Answer: ________________________________

5. Evaluate the definite integral.

$$\int_0^\pi \sin x\, dx$$

Answer: ________________________________

6. Use u-substitution to evaluate.

$$\int_0^1 x\, e^{x^2}\, dx$$

Answer: ________________________________

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7. Evaluate the definite integral using the Fundamental Theorem of Calculus.

$$\int_1^4 \frac{3}{\sqrt{x}}\,dx$$

Answer: ______________________________________

8. Find the indefinite integral by rewriting the integrand.

$$\int \frac{x^3 - 2x}{x}\,dx$$

Answer: ______________________________________

9. The rate of change of a population is P'(t) = 200 + 40t people/year. Find the total growth from t = 0 to t = 5.

$$\int_0^5 (200 + 40t)\,dt$$

Answer: ______________________________________

10. Evaluate using u-substitution.

$$\int \frac{4x}{x^2 + 1}\,dx$$

Answer: ______________________________________

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Problem 4 and 6 use u-sub with definite integrals — remind students to convert limits. Problem 8 requires simplifying the integrand algebraically first. Problem 9 is an applied rate/accumulation problem.

Solutions

1. Find the indefinite integral.

$$\int (4x^3 - 6x + 5)dx$$

→ *Apply the Power Rule: $\int x^n dx = x^{n+1}/(n+1) + C$.*
→ *$\int 4x^3\, dx = x^4$. $\int -6x\, dx = -3x^2$. $\int 5\, dx = 5x$.*
→ *Result: $x^4 - 3x^2 + 5x + C$.*

Answer: $x^4 - 3x^2 + 5x + C$

2. Evaluate the definite integral.

$$\int_1^3 (2x^2 - x + 4)dx$$

→ *Antiderivative: $2x^3/3 - x^2/2 + 4x$.*
→ *At x = 3: 18 – 9/2 + 12 = 30 – 9/2 = 51/2.*
→ *At x = 1: 2/3 – 1/2 + 4 = 25/6.*
→ *51/2 – 25/6 = 153/6 – 25/6 = 128/6 = 64/3.*

Answer: $\left[\frac{2x^3}{3} - \frac{x^2}{2} + 4x\right]_1^3 = \frac{92}{3}$

3. Find the indefinite integral of the exponential function.

$$\int \left(3e^x + \frac{2}{x}\right)dx$$

→ *$\int e^x\, dx = e^x$. $\int (1/x)\, dx = \ln|x|$.*
→ *Result: $3e^x + 2 \ln|x| + C$.*

Answer: $3e^x + 2\ln|x| + C$

4. Evaluate using u-substitution. Let u = 2x + 3.

$$\int (2x + 3)^5\, dx$$

→ *Let u = 2x+3, du = 2dx → dx = du/2.*
→ *$\int u^5 \cdot (du/2) = (1/2) \cdot u^6/6 + C$.*
→ *$= (2x+3)^6/12 + C$.*

Answer: $\frac{(2x + 3)^6}{12} + C$

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5. Evaluate the definite integral.

$$\int_0^{\pi} \sin x \, dx$$

→ ∫sin x dx = –cos x.
→ At x = π: –cos π = –(–1) = 1.
→ At x = 0: –cos 0 = –1.
→ Result: 1 – (–1) = 2.

Answer: $[-\cos x]_0^{\pi} = 2$

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6. Use u-substitution to evaluate.

$$\int_0^1 x\, e^{x^2} \, dx$$

→ Let u = x^2, du = 2x dx → x dx = du/2.
→ Limits: x=0→u=0; x=1→u=1.
→ = (1/2)∫■¹ e^u du = (1/2)[e^u]■¹.
→ = (1/2)(e – 1).

Answer: $\frac{1}{2}(e-1)$

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7. Evaluate the definite integral using the Fundamental Theorem of Calculus.

$$\int_1^4 \frac{3}{\sqrt{x}} \, dx$$

→ Rewrite: $3x^{-1/2}$. Antiderivative: 3·$x^{1/2}$/(1/2) = 6√x.
→ At x = 4: 6(2) = 12. At x = 1: 6(1) = 6.
→ Result: 12 – 6 = 6.

Answer: $[6\sqrt{x}]_1^4 = 6$

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8. Find the indefinite integral by rewriting the integrand.

$$\int \frac{x^3 - 2x}{x} \, dx$$

→ Divide each term: (x^3–2x)/x = x^2–2.
→ ∫(x^2–2) dx = x^3/3 – 2x + C.

Answer: $\frac{x^3}{3} - 2x + C$

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9. The rate of change of a population is P'(t) = 200 + 40t people/year. Find the total growth from t = 0 to t = 5.

$$\int_0^5 (200 + 40t) \, dt$$

→ Antiderivative: 200t + 20t^2.
→ At t = 5: 1000 + 500 = 1500.
→ At t = 0: 0.
→ Total growth = 1500 people.

Answer: $[200t + 20t^2]_0^5 = 1500$

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10. Evaluate using u-substitution.

$$\int \frac{4x}{x^2+1}\,dx$$

→ *Let u =* x^2*+1, du = 2x dx → 4x dx = 2 du.*
→ *∫(2/u) du = 2 ln|u| + C.*
→ *= 2 ln(*x^2*+1) + C. (*x^2*+1 > 0, no absolute value needed)*

Answer: $2\ln(x^2+1)+C$