MATH 210 — Limits & Continuity

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Learning Objectives

- Evaluate limits by direct substitution, factoring, and rationalizing
- Apply the Squeeze Theorem and recognize one-sided limits
- Evaluate trigonometric and limit-at-infinity problems
- Understand the definition of the derivative as a limit

Show all steps. For 0/0 or ∞/∞ forms, state the technique used before simplifying.

1. Evaluate the limit by direct substitution.

$$\lim_{x \to 3} (2x^2 - 5x + 1)$$

Answer: ________________________________

2. Evaluate the limit by factoring.

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

Answer: ________________________________

3. Evaluate the limit by factoring.

$$\lim_{x \to -3} \frac{x^2 + 5x + 6}{x + 3}$$

Answer: ________________________________

4. Evaluate the limit by rationalizing the numerator.

$$\lim_{x \to 0} \frac{\sqrt{x+9} - 3}{x}$$

Answer: ________________________________

5. Evaluate the trigonometric limit.

$$\lim_{x \to 0} \frac{\sin(5x)}{x}$$

Answer: ________________________________

6. Evaluate the limit at infinity.

$$\lim_{x \to \infty} \frac{3x^2 - 1}{x^2 + 5}$$

Answer: ________________________________

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7. Evaluate the one-sided limit.

$$\lim_{x \to 2^-} \frac{x-2}{|x-2|}$$

Answer: ____

8. Determine if the limit exists. Evaluate both one-sided limits.

$$\lim_{x \to 1^\pm} f(x), \quad f(x) = x^2 \text{ if } x < 1, \quad f(x) = 2x - 1 \text{ if } x \geq 1$$

Answer: ____

9. Evaluate the limit involving a difference quotient.

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

Answer: ____

10. Apply the Squeeze Theorem. Given $-x^2 \leq f(x) \leq x^2$ near $x = 0$, find the limit.

$$\lim_{x \to 0} f(x) \text{ given } -x^2 \leq f(x) \leq x^2$$

Answer: ____

Problems 4 and 5 introduce rationalization and trig limits (sin x/x) — emphasize the standard result lim sin(u)/u = 1. Problem 9 connects limits to the derivative definition.

Solutions

1. Evaluate the limit by direct substitution.

$$\lim_{x\to 3}(2x^2 - 5x + 1)$$

→ *Substitute x = 3 directly: 2(9) – 5(3) + 1.*
→ *= 18 – 15 + 1 = 4.*

Answer: $2(3)^2 - 5(3) + 1 = 4$

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2. Evaluate the limit by factoring.

$$\lim_{x\to 2}\frac{x^2 - 4}{x - 2}$$

→ *Factor numerator: x^2 – 4 = (x–2)(x+2).*
→ *Cancel (x–2): limit becomes lim(x+2) as x→2.*
→ *= 2 + 2 = 4.*

Answer: $\lim_{x\to 2}(x+2)=4$

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3. Evaluate the limit by factoring.

$$\lim_{x\to -3}\frac{x^2 + 5x + 6}{x + 3}$$

→ *Factor numerator: x^2 + 5x + 6 = (x+3)(x+2).*
→ *Cancel (x+3): limit becomes lim(x+2) as x→–3.*
→ *= –3 + 2 = –1.*

Answer: $\lim_{x\to -3}(x+2)=-1$

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4. Evaluate the limit by rationalizing the numerator.

$$\lim_{x\to 0}\frac{\sqrt{x+9}-3}{x}$$

→ *Multiply by conjugate (√(x+9)+3)/(√(x+9)+3).*
→ *Numerator becomes (x+9)–9 = x.*
→ *Simplify: x / [x(√(x+9)+3)] = 1/(√(x+9)+3).*
→ *As x→0: 1/(√9+3) = 1/6.*

Answer: $\frac{1}{6}$

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5. Evaluate the trigonometric limit.

$$\lim_{x\to 0}\frac{\sin(5x)}{x}$$

→ *Rewrite: sin(5x)/x = 5·sin(5x)/(5x).*
→ *Let u = 5x. As x→0, u→0.*
→ *Use the standard limit: lim(sin u / u) = 1 as u→0.*
→ *Result: 5·1 = 5.*

Answer: $5 \cdot \lim_{u\to 0}\frac{\sin u}{u} = 5$

6. Evaluate the limit at infinity.

$$\lim_{x\to\infty}\frac{3x^2-1}{x^2+5}$$

→ *Divide numerator and denominator by x^2 (highest power).*
→ *= $(3 – 1/x^2) / (1 + 5/x^2)$.*
→ *As x→∞, $1/x^2$→0: result = 3/1 = 3.*

Answer: 3

7. Evaluate the one-sided limit.

$$\lim_{x\to 2^-}\frac{x-2}{|x-2|}$$

→ *For x < 2: |x–2| = –(x–2), so (x–2)/|x–2| = –1.*
→ *Left-hand limit = –1.*

Answer: -1

8. Determine if the limit exists. Evaluate both one-sided limits.

$$\lim_{x\to 1^\pm} f(x),\quad f(x) = x^2 \text{ if } x < 1,\quad f(x) = 2x-1 \text{ if } x \geq 1$$

→ *Left-hand limit (x < 1): lim x^2 = 1.*
→ *Right-hand limit (x ≥ 1): lim (2x–1) = 2(1)–1 = 1.*
→ *Both sides equal 1, so the limit exists and equals 1.*

Answer: $\lim_{x\to 1^-} x^2 = 1, \quad \lim_{x\to 1^+}(2x-1) = 1$

9. Evaluate the limit involving a difference quotient.

$$\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$$

→ *Expand $(x+h)^2 = x^2 + 2xh + h^2$.*
→ *Numerator: $x^2 + 2xh + h^2 – x^2 = 2xh + h^2$.*
→ *Factor h: h(2x + h)/h = 2x + h.*
→ *As h→0: limit = 2x.*

Answer: $2x$

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10. Apply the Squeeze Theorem. Given $-x^2 \le f(x) \le x^2$ near $x = 0$, find the limit.

$$\lim_{x \to 0} f(x) \text{ given } -x^2 \le f(x) \le x^2$$

→ *$\lim(-x^2) = 0$ and $\lim(x^2) = 0$ as $x \to 0$.*
→ *By the Squeeze Theorem: $\lim f(x) = 0$.*

Answer: 0

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