MATH 220 — Rates & Accumulation

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Learning Objectives

- Apply the Net Change Theorem to velocity, rate, and population problems
- Estimate definite integrals using Left, Right, Midpoint, and Trapezoidal Rules
- Evaluate accumulation functions F(x) = ∫■■ f(t) dt and find F'(x) by FTC
- Distinguish displacement from total distance using signed vs absolute area

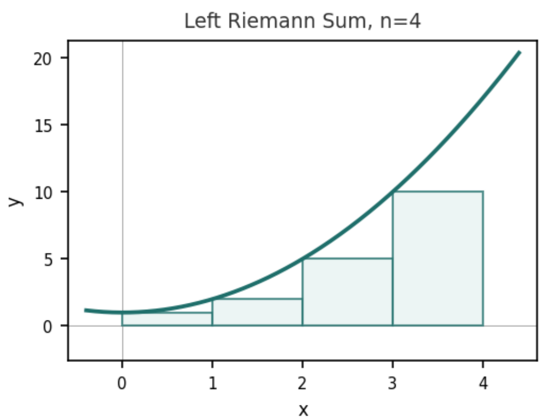
For Riemann sums, show your table of x-values and function values before summing. For total distance, find where v(t) = 0 and split the integral.

1. A particle moves with velocity v(t) = 3t^2 - 12t + 9 ft/s. Find the net displacement and total distance traveled on [0, 4].

$$\int_0^4 (3t^2 - 12t + 9)\,dt \quad \text{and} \quad \int_0^4 |v(t)|\,dt$$

Answer: ______________________________

2. Approximate the integral of f(x) = x^2 + 1 from x = 0 to x = 4 using a Left Riemann Sum with n = 4 rectangles.



Answer: ______________________________

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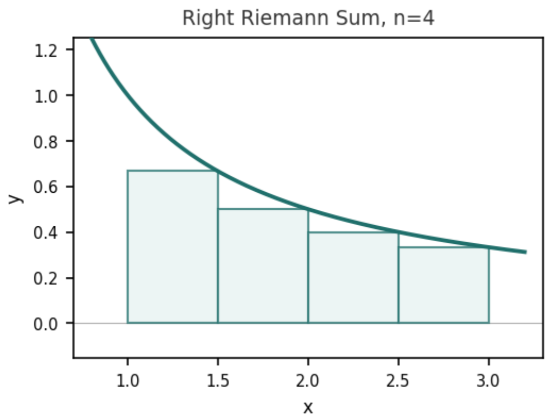
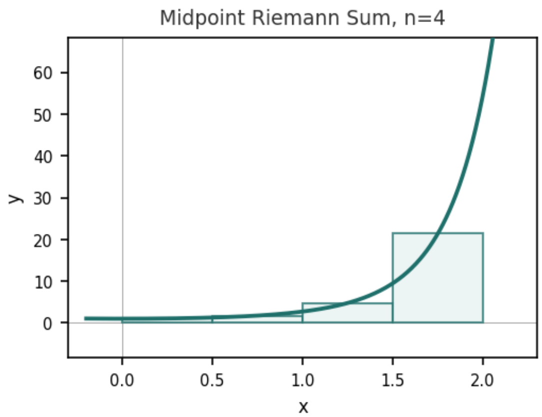
3. Approximate the integral of f(x) = 1/x from x = 1 to x = 3 using a Right Riemann Sum with n = 4 rectangles.



Answer: ______________________________

4. Approximate the integral of f(x) = e^(x^2) from x = 0 to x = 2 using the Midpoint Rule with n = 4.



Answer: ______________________________

5. Use the Trapezoidal Rule with n = 4 to approximate the integral of sin(x) from 0 to pi.

$$T_4 = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)]$$

Answer: ______________________________

6. Oil leaks from a pipe at rate r(t) = 4t - t^2 + 3 gallons/hour (0 <= t <= 3). Find the total oil leaked.

$$\int_0^3 (4t - t^2 + 3)\,dt$$

Answer: ______________________________

7. Let F(x) be defined by the integral below. Find F(3) and F'(x). Interpret each.

$$F(x) = \int_0^x (t^2 - t)\,dt$$

Answer: ______________________________

8. Evaluate using the Fundamental Theorem of Calculus.

$$\int_1^4 \left(\sqrt{x} - \frac{1}{x}\right) dx$$

Answer: ____

9. A city's population grows at the rate shown below. The population at t = 0 is 50,000. Use the Net Change Theorem to find P(10).

$$P'(t) = 400e^{0.02t}, \quad P(10) = P(0) + \int_0^{10} 400e^{0.02t}\, dt$$

Answer: ____

10. Compute L4, R4, and M4 with n = 4 for the integral below. Compare each approximation to the exact value.

$$\int_0^2 x^3\, dx, \quad \Delta x = 0.5$$

Answer: ____

Problems 2–4 include Riemann sum diagrams. Problem 1: displacement vs distance — the classic distinction. Problem 7 (accumulation function): connects FTC Part 1 and Part 2.

Solutions

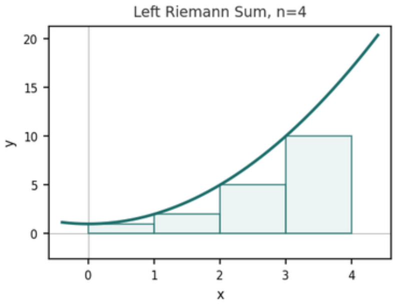
1. A particle moves with velocity v(t) = 3t^2 - 12t + 9 ft/s. Find the net displacement and total distance traveled on [0, 4].

$$\int_0^4 (3t^2 - 12t + 9)\,dt \quad \text{and} \quad \int_0^4 |v(t)|\,dt$$

→ *Antiderivative: t^3 - $6t^2$ + 9t.*
→ *Displacement: evaluate [t^3 - $6t^2$ + 9t] from 0 to 4 = 64 - 96 + 36 = 4 ft.*
→ *v(t) = 3(t-1)(t-3). Zeros at t=1 and t=3.*
→ *v > 0 on [0,1] and [3,4]; v < 0 on [1,3] — must split for total distance.*
→ *Total distance = |disp on [0,1]| + |disp on [1,3]| + |disp on [3,4]| = 4+4+4... evaluate each.*

Answer: $\text{Displacement} = 4\ \text{ft},\ \text{Distance} = 14\ \text{ft}$

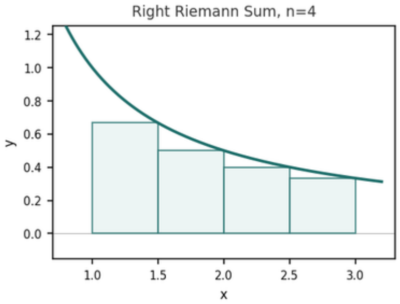
2. Approximate the integral of f(x) = x^2 + 1 from x = 0 to x = 4 using a Left Riemann Sum with n = 4 rectangles.



→ *dx = (4-0)/4 = 1. Left endpoints: x = 0, 1, 2, 3.*
→ *f(0)=1, f(1)=2, f(2)=5, f(3)=10.*
→ *L4 = 1*(1+2+5+10) = 18.*
→ *Exact value: [x^3/3 + x] from 0 to 4 = 64/3 + 4 = 76/3 ≈ 25.33.*
→ *L4 underestimates because f is increasing on [0, 4].*

Answer: $L_4 = 1 \cdot [1 + 2 + 5 + 10] = 18$

3. Approximate the integral of f(x) = 1/x from x = 1 to x = 3 using a Right Riemann Sum with n = 4 rectangles.



→ *dx = (3-1)/4 = 0.5. Right endpoints: x = 1.5, 2.0, 2.5, 3.0.*

→ *f(1.5)=2/3, f(2)=1/2, f(2.5)=2/5, f(3)=1/3.*

→ *R4 = 0.5*(2/3 + 1/2 + 2/5 + 1/3) = 0.5*(1.900) ≈ 0.950.*

→ *Exact: ln(3) ≈ 1.099. R4 underestimates (f is decreasing on [1,3]).*

Answer: $R_4 = \frac{1}{2}\left[\frac{2}{3} + \frac{1}{2} + \frac{2}{5} + \frac{1}{3}\right] \approx 0.95$

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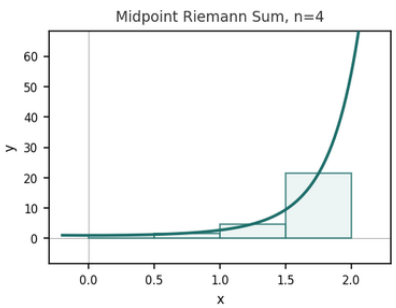
4. Approximate the integral of f(x) = e^(x^2) from x = 0 to x = 2 using the Midpoint Rule with n = 4.



→ *dx = 0.5. Midpoints: x = 0.25, 0.75, 1.25, 1.75.*

→ *f(0.25) =* $e^{0.0625}$ *≈ 1.064, f(0.75) =* $e^{0.5625}$ *≈ 1.755.*

→ *f(1.25) =* $e^{1.5625}$ *≈ 4.772, f(1.75) =* $e^{3.0625}$ *≈ 21.38.*

→ *M4 = 0.5*(1.064 + 1.755 + 4.772 + 21.38) = 0.5 * 28.97 ≈ 14.49.*

Answer: $M_4 \approx 0.5[e^{0.0625} + e^{0.5625} + e^{1.5625} + e^{3.0625}] \approx 14.5$

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5. Use the Trapezoidal Rule with n = 4 to approximate the integral of sin(x) from 0 to pi.

$$T_4 = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)]$$

→ *dx = pi/4. x-values: 0, pi/4, pi/2, 3pi/4, pi.*

→ *sin(0)=0, sin(pi/4)=sqrt(2)/2, sin(pi/2)=1, sin(3pi/4)=sqrt(2)/2, sin(pi)=0.*

→ *T4 = (pi/8)[0 + 2(sqrt(2)/2) + 2(1) + 2(sqrt(2)/2) + 0] = (pi/8)(2sqrt(2)+2) ≈ 1.896.*

→ *Exact: integral of sin(x) from 0 to pi = 2. Error ≈ 0.104.*

Answer: $T_4 \approx \frac{\pi}{8}\left[0 + 2\cdot\frac{\sqrt{2}}{2} + 2 + 2\cdot\frac{\sqrt{2}}{2} + 0\right] \approx 1.896$

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6. Oil leaks from a pipe at rate r(t) = 4t - t^2 + 3 gallons/hour (0 <= t <= 3). Find the total oil leaked.

$$\int_0^3 (4t - t^2 + 3)\,dt$$

→ Antiderivative: $2t^2 – t^3/3 + 3t$.
→ At t=3: 2(9)–27/3+9 = 18–9+9 = 18.
→ At t=0: 0.
→ Total oil leaked = 18 gallons.

Answer: $$\left[2t^2 - \frac{t^3}{3} + 3t\right]_0^3 = 18$$

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7. Let F(x) be defined by the integral below. Find F(3) and F'(x). Interpret each.

$$F(x) = \int_0^x (t^2 - t)\,dt$$

→ Antiderivative: $t^3/3 - t^2/2$.
→ F(3) = $[t^3/3 - t^2/2]$ from 0 to 3 = 27/3 - 9/2 = 9 - 4.5 = 9/2.
→ F'(x) = x^2 - x by the Fundamental Theorem of Calculus (Part 1).
→ F(3) = net signed area under t^2 - t from 0 to 3.

Answer: $$F(3) = \frac{9}{2}, \quad F'(x) = x^2 - x$$

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8. Evaluate using the Fundamental Theorem of Calculus.

$$\int_1^4 \left(\sqrt{x} - \frac{1}{x}\right) dx$$

→ Antiderivative of √x = $x^{1/2}$ is $(2/3)x^{3/2}$.
→ Antiderivative of 1/x is ln x.
→ At x=4: (2/3)(8)–ln4 = 16/3–ln4.
→ At x=1: (2/3)(1)–ln1 = 2/3.
→ Result: (16/3–ln4)–2/3 = 14/3–ln4.
→ Note: 14/3 ≈ 4.667 and ln4 ≈ 1.386, so result ≈ 3.28.

Answer: $$\left[\frac{2}{3}x^{3/2} - \ln x\right]_1^4 = \frac{28}{3} - \ln 4$$

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9. A city's population grows at the rate shown below. The population at t = 0 is 50,000. Use the Net Change Theorem to find P(10).

$$P'(t) = 400e^{0.02t}, \quad P(10) = P(0) + \int_0^{10} 400e^{0.02t}\,dt$$

→ Net Change Theorem: P(10) = P(0) + integral from 0 to 10 of P'(t) dt.
→ Antiderivative of $400e^{0.02t} = 20000e^{0.02t}$.
→ Evaluate: $20000e^{0.2} - 20000 = 20000(e^{0.2} - 1) \approx 4392$.
→ P(10) ≈ 50000 + 4392 = 54,392 people.

Answer: $$P(10) = 50000 + [20000e^{0.02t}]_0^{10} \approx 54392$$

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10. Compute L4, R4, and M4 with n = 4 for the integral below. Compare each approximation to the exact value.

$$\int_0^2 x^3\,dx, \quad \Delta x = 0.5$$

→ *Exact: [x^4/4] from 0 to 2 = 16/4 = 4.*

→ *dx=0.5. Left pts: 0, 0.5, 1, 1.5 -> f: 0, 0.125, 1, 3.375. L4=0.5(4.5+...)=2.25.*

→ *Right pts: 0.5, 1, 1.5, 2 -> f: 0.125, 1, 3.375, 8. R4=0.5(12.5)=6.25.*

→ *Midpts: 0.25, 0.75, 1.25, 1.75 -> f: 0.0156, 0.422, 1.953, 5.359. M4 ≈ 3.875.*

→ *M4 is closest to the exact value of 4. L4 underestimates, R4 overestimates (f is increasing).*

Answer: $\text{Exact} = 4; \quad L_4 = 2.75,\ R_4 = 6.75,\ M_4 = 4.0$

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