MATH 220 — Integration Techniques

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Learning Objectives

- Apply the Power Rule to evaluate definite integrals
- Use u-substitution to simplify and evaluate integrals
- Find antiderivatives of e^x and 1/x (natural log rule)
- Apply the Fundamental Theorem of Calculus (Parts 1 & 2)

Show all work. For u-substitution problems, state your choice of u and du explicitly.

1. Evaluate the definite integral using the Power Rule.

$$\int_0^3 x^4 \, dx$$

Answer: ________________________________

2. Evaluate the definite integral.

$$\int_1^4 (3x^2 - 2x + 5)dx$$

Answer: ________________________________

3. Find the antiderivative of the exponential function.

$$\int e^x \, dx$$

Answer: ________________________________

4. Evaluate using u-substitution. Let $u = x^2 + 1$.

$$\int_0^2 2x\, e^{x^2+1} \, dx$$

Answer: ________________________________

5. Evaluate using u-substitution.

$$\int_0^1 \frac{4x^3}{x^4 + 1} dx$$

Answer: ________________________________

6. Evaluate the integral involving a natural log antiderivative.

$$\int_1^e \frac{3}{x} dx$$

Answer: ________________________________

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7. Apply the Fundamental Theorem of Calculus Part 1. Find F'(x).

$$F(x) = \int_1^x t^3 \, dt$$

Answer: ___________________________________

8. Apply FTC Part 1. Find G'(x).

$$G(x) = \int_0^x \sqrt{t^3 + 1} \, dt$$

Answer: ___________________________________

9. A particle has velocity $v(t) = 3t^2 - 2t$ ft/s. Find net displacement from $t = 0$ to $t = 3$.

$$\int_0^3 (3t^2 - 2t) \, dt$$

Answer: ___________________________________

10. Evaluate using u-substitution. Let $u = 3x + 2$.

$$\int_0^2 (3x + 2)^4 \, dx$$

Answer: ___________________________________

FTC Part 1 (problems 7–8): students often forget to substitute x for t. Net change (problem 9): displacement = ∫v dt, remind students this is signed.

Solutions

1. Evaluate the definite integral using the Power Rule.

$$\int_0^3 x^4\,dx$$

→ *Apply the Power Rule: antiderivative of x^4 is $x^5/5$.*
→ *Evaluate at upper bound: $(3)^5/5 = 243/5$.*
→ *Evaluate at lower bound: $(0)^5/5 = 0$.*
→ *Result: 243/5 – 0 = 243/5.*

Answer: $\left[\frac{x^5}{5}\right]_0^3 = \frac{243}{5}$

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2. Evaluate the definite integral.

$$\int_1^4 (3x^2 - 2x + 5)dx$$

→ *Antiderivative: $x^3 - x^2 + 5x$.*
→ *At x = 4: 64 – 16 + 20 = 68.*
→ *At x = 1: 1 – 1 + 5 = 5.*
→ *Result: 68 – 5 = 63. (Corrected: 68–5 = 63)*

Answer: $[x^3 - x^2 + 5x]_1^4 = 66$

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3. Find the antiderivative of the exponential function.

$$\int e^x\,dx$$

→ *The antiderivative of e^x is itself.*
→ *Result: $e^x + C$.*

Answer: $e^x + C$

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4. Evaluate using u-substitution. Let u = x² + 1.

$$\int_0^2 2x e^{x^2+1}\,dx$$

→ *Let $u = x^2 + 1$, so du = 2x dx.*
→ *When x = 0: u = 1. When x = 2: u = 5.*
→ *Integral becomes: ∫■■ e^u du = [e^u]■■.*
→ *Result: $e^5 - e^1 = e(e^4 - 1)$.*

Answer: $e^5 - e^1 = e(e^4 - 1)$

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5. Evaluate using u-substitution.

$$\int_0^1 \frac{4x^3}{x^4+1}\,dx$$

→ *Let $u = x^4 + 1$, so $du = 4x^3\,dx$.*
→ *When x = 0: u = 1. When x = 1: u = 2.*
→ *Integral becomes: ∫■² (1/u) du = [ln|u|]■².*
→ *Result: ln 2 – ln 1 = ln 2.*

Answer: $\ln 2$

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6. Evaluate the integral involving a natural log antiderivative.

$$\int_1^e \frac{3}{x}\,dx$$

→ *Antiderivative of 3/x is 3 ln|x|.*
→ *At x = e: 3 ln(e) = 3(1) = 3.*
→ *At x = 1: 3 ln(1) = 3(0) = 0.*
→ *Result: 3 – 0 = 3.*

Answer: $[3\ln x]_1^e = 3$

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7. Apply the Fundamental Theorem of Calculus Part 1. Find F'(x).

$$F(x) = \int_1^x t^3\,dt$$

→ *By FTC Part 1: if F(x) = ∫■■ f(t) dt, then F'(x) = f(x).*
→ *Here $f(t) = t^3$, so $F'(x) = x^3$.*

Answer: $F'(x) = x^3$

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8. Apply FTC Part 1. Find G'(x).

$$G(x) = \int_0^x \sqrt{t^3+1}\,dt$$

→ *By FTC Part 1: G'(x) = f(x) where $f(t) = \sqrt{(t^3+1)}$.*
→ *Result: $G'(x) = \sqrt{(x^3+1)}$.*

Answer: $G'(x) = \sqrt{x^3+1}$

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9. A particle has velocity $v(t) = 3t^2 – 2t$ ft/s. Find net displacement from t = 0 to t = 3.

$$\int_0^3 (3t^2 - 2t)\,dt$$

→ *Antiderivative: $t^3 – t^2$.*
→ *At t = 3: 27 – 9 = 18.*
→ *At t = 0: 0 – 0 = 0.*
→ *Net displacement: 18 – 0 = 18 ft.*

Answer: $[t^3 - t^2]_0^3 = 18$

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10. Evaluate using u-substitution. Let u = 3x + 2.

$$\int_0^2 (3x + 2)^4 \, dx$$

→ *Let u = 3x + 2, so du = 3 dx, meaning dx = du/3.*
→ *When x = 0: u = 2. When x = 2: u = 8.*
→ *Integral becomes: (1/3) ∫■■ u^4 du = (1/3)[u^5/5]■■.*
→ *= (1/15)[8^5 – 2^5] = (1/15)[32768 – 32] = 32736/15.*

Answer: $$\left[\frac{(3x+2)^5}{15}\right]_0^2 = \frac{32736}{15}$$

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