MATH 220: Sequences and Series

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Learning Objectives

- Find the nth term and partial sums of arithmetic sequences
- Find the nth term and partial sums of geometric sequences
- Determine convergence and find the sum of infinite geometric series

Simplify each expression completely. Show all steps and circle your final answer.

Arithmetic sequences

1. An arithmetic sequence has a_1 = 11 and d = 2. Find the 5th term.

$$a_1 = 11,\ d = 2,\ n = 5$$

Answer: _____

2. An arithmetic sequence has a_1 = 15 and d = 2. Find the 11th term.

$$a_1 = 15,\ d = 2,\ n = 11$$

Answer: _____

3. An arithmetic sequence has a_1 = 8 and d = 8. Find the 9th term.

$$a_1 = 8,\ d = 8,\ n = 9$$

Answer: _____

4. An arithmetic sequence has a_1 = 15 and d = 3. Find the 12th term.

$$a_1 = 15,\ d = 3,\ n = 12$$

Answer: _____

5. An arithmetic sequence has a_1 = 10 and d = 3. Find the 8th term.

$$a_1 = 10,\ d = 3,\ n = 8$$

Answer: _____

6. An arithmetic sequence has a_1 = 15 and d = 6. Find the 7th term.

$$a_1 = 15,\ d = 6,\ n = 7$$

Answer: _____

7. An arithmetic sequence has a_1 = 2 and d = 4. Find the 11th term.

$a_1 = 2,\ d = 4,\ n = 11$

Answer: _____

8. An arithmetic sequence has a_1 = 15 and d = 2. Find the 9th term.

$a_1 = 15,\ d = 2,\ n = 9$

Answer: _____

Arithmetic series — partial sum

9. Find the sum of the first 7 terms of the arithmetic sequence with a_1 = 1 and d = 4.

Answer: _____

10. Find the sum of the first 11 terms of the arithmetic sequence with a_1 = 6 and d = 2.

Answer: _____

11. Find the sum of the first 7 terms of the arithmetic sequence with a_1 = 4 and d = 6.

Answer: _____

12. Find the sum of the first 7 terms of the arithmetic sequence with a_1 = 2 and d = 3.

Answer: _____

13. Find the sum of the first 12 terms of the arithmetic sequence with a_1 = 4 and d = 2.

Answer: _____

14. Find the sum of the first 9 terms of the arithmetic sequence with a_1 = 8 and d = 5.

Answer: _____

15. Find the sum of the first 11 terms of the arithmetic sequence with a_1 = 2 and d = 2.

Answer: _____

16. Find the sum of the first 5 terms of the arithmetic sequence with a_1 = 6 and d = 6.

Answer: _____

Infinite geometric series

17. Find the sum of the infinite geometric series with a_1 = 7 and r = 1/2 (|r| < 1).

Answer: _____

18. Find the sum of the infinite geometric series with a_1 = 6 and r = 1/3 (|r| < 1).

Answer: _____

19. Find the sum of the infinite geometric series with $a_1 = 5$ and $r = 1/2$ ($|r| < 1$).

Answer: _____

20. Find the sum of the infinite geometric series with $a_1 = 1$ and $r = 1/5$ ($|r| < 1$).

Answer: _____

21. Find the sum of the infinite geometric series with $a_1 = 5$ and $r = 1/4$ ($|r| < 1$).

Answer: _____

22. Find the sum of the infinite geometric series with $a_1 = 8$ and $r = 1/2$ ($|r| < 1$).

Answer: _____

23. Find the sum of the infinite geometric series with $a_1 = 8$ and $r = 1/2$ ($|r| < 1$).

Answer: _____

Geometric series — partial sum

24. Find the sum of the first 6 terms of the geometric series with $a_1 = 3$ and $r = 3$.

Answer: _____

25. Find the sum of the first 6 terms of the geometric series with $a_1 = 1$ and $r = 3$.

Answer: _____

26. Find the sum of the first 6 terms of the geometric series with $a_1 = 2$ and $r = 3$.

Answer: _____

27. Find the sum of the first 3 terms of the geometric series with $a_1 = 1$ and $r = 3$.

Answer: _____

28. Find the sum of the first 4 terms of the geometric series with $a_1 = 4$ and $r = 2$.

Answer: _____

29. Find the sum of the first 5 terms of the geometric series with $a_1 = 4$ and $r = 3$.

Answer: _____

30. Find the sum of the first 4 terms of the geometric series with $a_1 = 1$ and $r = 2$.

Answer: _____

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ANSWER KEY & SOLUTIONS

Topics: Arithmetic series — partial sum, Geometric series — partial sum, Arithmetic sequences, Infinite geometric series. All answers verified by independent computation.

Solutions

Arithmetic sequences

1. An arithmetic sequence has a_1 = 11 and d = 2. Find the 5th term.

$a_1 = 11,\ d = 2,\ n = 5$

→ *a_n = a_1 + (n-1)d = 11 + (5-1)*2 = 11 + 8 = 19.*

Answer: $a_5 = 19$

2. An arithmetic sequence has a_1 = 15 and d = 2. Find the 11th term.

$a_1 = 15,\ d = 2,\ n = 11$

→ *a_n = a_1 + (n-1)d = 15 + (11-1)*2 = 15 + 20 = 35.*

Answer: $a_{11} = 35$

3. An arithmetic sequence has a_1 = 8 and d = 8. Find the 9th term.

$a_1 = 8,\ d = 8,\ n = 9$

→ *a_n = a_1 + (n-1)d = 8 + (9-1)*8 = 8 + 64 = 72.*

Answer: $a_9 = 72$

4. An arithmetic sequence has a_1 = 15 and d = 3. Find the 12th term.

$a_1 = 15,\ d = 3,\ n = 12$

→ *a_n = a_1 + (n-1)d = 15 + (12-1)*3 = 15 + 33 = 48.*

Answer: $a_{12} = 48$

5. An arithmetic sequence has a_1 = 10 and d = 3. Find the 8th term.

$a_1 = 10,\ d = 3,\ n = 8$

→ *a_n = a_1 + (n-1)d = 10 + (8-1)*3 = 10 + 21 = 31.*

Answer: $a_8 = 31$

6. An arithmetic sequence has a_1 = 15 and d = 6. Find the 7th term.

$a_1 = 15,\ d = 6,\ n = 7$

→ *a_n = a_1 + (n-1)d = 15 + (7-1)*6 = 15 + 36 = 51.*

Answer: $a_7 = 51$

7. An arithmetic sequence has a_1 = 2 and d = 4. Find the 11th term.

$a_1 = 2,\ d = 4,\ n = 11$

→ *a_n = a_1 + (n-1)d = 2 + (11-1)*4 = 2 + 40 = 42.*

Answer: $a_{11} = 42$

8. An arithmetic sequence has a_1 = 15 and d = 2. Find the 9th term.

$a_1 = 15,\ d = 2,\ n = 9$

→ *a_n = a_1 + (n-1)d = 15 + (9-1)*2 = 15 + 16 = 31.*

Answer: $a_9 = 31$

Arithmetic series — partial sum

9. Find the sum of the first 7 terms of the arithmetic sequence with a_1 = 1 and d = 4.

→ *Find a_n: a_7 = 1 + (7-1)*4 = 25.*
→ *Sum formula: S_n = n/2 * (a_1 + a_n).*
→ *S_7 = 7/2 * (1 + 25) = 7/2 * 26 = 91.*

Answer:

10. Find the sum of the first 11 terms of the arithmetic sequence with a_1 = 6 and d = 2.

→ *Find a_n: a_11 = 6 + (11-1)*2 = 26.*
→ *Sum formula: S_n = n/2 * (a_1 + a_n).*
→ *S_11 = 11/2 * (6 + 26) = 11/2 * 32 = 176.*

Answer:

11. Find the sum of the first 7 terms of the arithmetic sequence with a_1 = 4 and d = 6.

→ *Find a_n: a_7 = 4 + (7-1)*6 = 40.*
→ *Sum formula: S_n = n/2 * (a_1 + a_n).*
→ *S_7 = 7/2 * (4 + 40) = 7/2 * 44 = 154.*

Answer:

12. Find the sum of the first 7 terms of the arithmetic sequence with a_1 = 2 and d = 3.

→ *Find a_n: a_7 = 2 + (7-1)*3 = 20.*
→ *Sum formula: S_n = n/2 * (a_1 + a_n).*
→ *S_7 = 7/2 * (2 + 20) = 7/2 * 22 = 77.*

Answer:

13. Find the sum of the first 12 terms of the arithmetic sequence with a_1 = 4 and d = 2.

→ *Find a_n: a_12 = 4 + (12-1)*2 = 26.*
→ *Sum formula: S_n = n/2 * (a_1 + a_n).*
→ *S_12 = 12/2 * (4 + 26) = 12/2 * 30 = 180.*

Answer:

14. Find the sum of the first 9 terms of the arithmetic sequence with a_1 = 8 and d = 5.

→ *Find a_n: a_9 = 8 + (9-1)*5 = 48.*
→ *Sum formula: S_n = n/2 * (a_1 + a_n).*
→ *S_9 = 9/2 * (8 + 48) = 9/2 * 56 = 252.*

Answer:

15. Find the sum of the first 11 terms of the arithmetic sequence with a_1 = 2 and d = 2.

→ *Find a_n: a_11 = 2 + (11-1)*2 = 22.*
→ *Sum formula: S_n = n/2 * (a_1 + a_n).*
→ *S_11 = 11/2 * (2 + 22) = 11/2 * 24 = 132.*

Answer:

16. Find the sum of the first 5 terms of the arithmetic sequence with a_1 = 6 and d = 6.

→ *Find a_n: a_5 = 6 + (5-1)*6 = 30.*
→ *Sum formula: S_n = n/2 * (a_1 + a_n).*
→ *S_5 = 5/2 * (6 + 30) = 5/2 * 36 = 90.*

Answer:

Infinite geometric series

17. Find the sum of the infinite geometric series with a_1 = 7 and r = 1/2 (|r| < 1).

→ *Since |r| = 1/2 < 1, the series converges.*
→ *Sum = a_1 / (1 - r) = 7 / (1 - 1/2) = 7 / (1/2).*
→ *= 7 * 2/1 = 14.*

Answer:

18. Find the sum of the infinite geometric series with a_1 = 6 and r = 1/3 (|r| < 1).

→ *Since |r| = 1/3 < 1, the series converges.*
→ *Sum = a_1 / (1 - r) = 6 / (1 - 1/3) = 6 / (2/3).*
→ *= 6 * 3/2 = 9.*

Answer:

19. Find the sum of the infinite geometric series with a_1 = 5 and r = 1/2 (|r| < 1).

→ *Since |r| = 1/2 < 1, the series converges.*
→ *Sum = a_1 / (1 - r) = 5 / (1 - 1/2) = 5 / (1/2).*
→ *= 5 * 2/1 = 10.*

Answer:

20. Find the sum of the infinite geometric series with a_1 = 1 and r = 1/5 (|r| < 1).

→ *Since |r| = 1/5 < 1, the series converges.*
→ *Sum = a_1 / (1 - r) = 1 / (1 - 1/5) = 1 / (4/5).*
→ *= 1 * 5/4 = 5/4.*

Answer:

21. Find the sum of the infinite geometric series with a_1 = 5 and r = 1/4 (|r| < 1).

→ *Since |r| = 1/4 < 1, the series converges.*
→ *Sum = a_1 / (1 - r) = 5 / (1 - 1/4) = 5 / (3/4).*
→ *= 5 * 4/3 = 20/3.*

Answer:

22. Find the sum of the infinite geometric series with a_1 = 8 and r = 1/2 (|r| < 1).

→ *Since |r| = 1/2 < 1, the series converges.*
→ *Sum = a_1 / (1 - r) = 8 / (1 - 1/2) = 8 / (1/2).*
→ *= 8 * 2/1 = 16.*

Answer:

23. Find the sum of the infinite geometric series with a_1 = 8 and r = 1/2 (|r| < 1).

→ *Since |r| = 1/2 < 1, the series converges.*
→ *Sum = a_1 / (1 - r) = 8 / (1 - 1/2) = 8 / (1/2).*
→ *= 8 * 2/1 = 16.*

Answer:

Geometric series — partial sum

24. Find the sum of the first 6 terms of the geometric series with a_1 = 3 and r = 3.

→ *Use S_n = a_1 * (1 - r^n) / (1 - r).*
→ *S_6 = 3 * (1 - 3^6) / (1 - 3).*
→ *= 3 * (1 - 729) / -2 = 3 * -728 / -2 = 1092.*

Answer:

25. Find the sum of the first 6 terms of the geometric series with a_1 = 1 and r = 3.

→ *Use S_n = a_1 * (1 - r^n) / (1 - r).*
→ *S_6 = 1 * (1 - 3^6) / (1 - 3).*
→ *= 1 * (1 - 729) / -2 = 1 * -728 / -2 = 364.*

Answer:

26. Find the sum of the first 6 terms of the geometric series with a_1 = 2 and r = 3.

→ *Use S_n = a_1 * (1 - r^n) / (1 - r).*
→ *S_6 = 2 * (1 - 3^6) / (1 - 3).*
→ *= 2 * (1 - 729) / -2 = 2 * -728 / -2 = 728.*

Answer:

27. Find the sum of the first 3 terms of the geometric series with a_1 = 1 and r = 3.

→ *Use S_n = a_1 * (1 - r^n) / (1 - r).*
→ *S_3 = 1 * (1 - 3^3) / (1 - 3).*
→ *= 1 * (1 - 27) / -2 = 1 * -26 / -2 = 13.*

Answer:

28. Find the sum of the first 4 terms of the geometric series with a_1 = 4 and r = 2.

→ *Use S_n = a_1 * (1 - r^n) / (1 - r).*
→ *S_4 = 4 * (1 - 2^4) / (1 - 2).*
→ *= 4 * (1 - 16) / -1 = 4 * -15 / -1 = 60.*

Answer:

29. Find the sum of the first 5 terms of the geometric series with a_1 = 4 and r = 3.

→ *Use S_n = a_1 * (1 - r^n) / (1 - r).*
→ *S_5 = 4 * (1 - 3^5) / (1 - 3).*
→ *= 4 * (1 - 243) / -2 = 4 * -242 / -2 = 484.*

Answer:

30. Find the sum of the first 4 terms of the geometric series with a_1 = 1 and r = 2.

→ *Use S_n = a_1 * (1 - r^n) / (1 - r).*
→ *S_4 = 1 * (1 - 2^4) / (1 - 2).*
→ *= 1 * (1 - 16) / -1 = 1 * -15 / -1 = 15.*

Answer: