MATH230: Applications of Double Integrals

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Name: _____ Date: _____ Score: / 30

Learning Objectives

- Calculate the mean, median, mode, and range of a data set
- Compute sample variance and standard deviation
- Construct quartiles, the IQR, and identify outliers
- Describe the shape, center, and spread of a distribution

Simplify each expression completely. Show all steps and circle your final answer.

Area and volume

1. Find the area of the rectangle R = [0,5] x [0,4] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [0,5]\times[0,4]$$

Answer: _____

2. Find the volume of the solid under z = 8 and above the rectangle [0,3] x [1,3] in the xy-plane.

$$\iint_R 8\,dA,\ [0,3]\times[1,3]$$

Answer: _____

3. Find the area of the rectangle R = [1,6] x [1,3] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [1,6]\times[1,3]$$

Answer: _____

4. Find the volume of the solid under z = 7 and above the rectangle [0,5] x [1,4] in the xy-plane.

$$\iint_R 7\,dA,\ [0,5]\times[1,4]$$

Answer: _____

5. Find the area of the rectangle R = [0,7] x [2,7] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [0,7]\times[2,7]$$

Answer: _____

6. Find the volume of the solid under z = 9 and above the rectangle [1,4] x [0,5] in the xy-plane.

$$\iint_R 9\,dA,\ [1,4]\times[0,5]$$

Answer: _____

7. Find the area of the rectangle R = [2,7] x [0,4] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [2,7]\times[0,4]$$

Answer: _____

8. Find the volume of the solid under z = 5 and above the rectangle [1,5] x [1,5] in the xy-plane.

$$\iint_R 5\,dA,\ [1,5]\times[1,5]$$

Answer: _____

9. Find the area of the rectangle R = [1,7] x [1,6] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [1,7]\times[1,6]$$

Answer: _____

10. Find the volume of the solid under z = 2 and above the rectangle [1,4] x [1,3] in the xy-plane.

$$\iint_R 2\,dA,\ [1,4]\times[1,3]$$

Answer: _____

11. Find the area of the rectangle R = [1,7] x [2,3] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [1,7]\times[2,3]$$

Answer: _____

12. Find the volume of the solid under z = 5 and above the rectangle [0,5] x [0,3] in the xy-plane.

$$\iint_R 5\,dA,\ [0,5]\times[0,3]$$

Answer: _____

13. Find the area of the rectangle R = [1,7] x [0,5] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [1,7]\times[0,5]$$

Answer: _____

14. Find the volume of the solid under z = 4 and above the rectangle [0,3] x [1,4] in the xy-plane.

$$\iint_R 4\,dA,\ [0,3]\times[1,4]$$

Answer: _____

15. Find the area of the rectangle R = [1,5] x [1,6] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [1,5]\times[1,6]$$

Answer: _____

16. Find the volume of the solid under z = 8 and above the rectangle [1,4] x [0,5] in the xy-plane.

$$\iint_R 8\,dA,\ [1,4]\times[0,5]$$

Answer: _____

17. Find the area of the rectangle R = [1,6] x [0,6] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [1,6]\times[0,6]$$

Answer: _____

18. Find the volume of the solid under z = 9 and above the rectangle [0,3] x [1,5] in the xy-plane.

$$\iint_R 9\,dA,\ [0,3]\times[1,5]$$

Answer: _____

19. Find the area of the rectangle R = [0,4] x [0,7] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [0,4]\times[0,7]$$

Answer: _____

20. Find the volume of the solid under z = 7 and above the rectangle [0,4] x [0,3] in the xy-plane.

$$\iint_R 7\,dA,\ [0,4]\times[0,3]$$

Answer: _____

Average value

21. A uniform temperature field has T = 3 degrees over rectangle [1,3] x [1,4]. Use a double integral to confirm the total heat content is 3·Area.

$$\iint_R 3\,dA,\ [1,3]\times[1,4]$$

Answer: _____

22. A uniform temperature field has T = 4 degrees over rectangle [1,2] x [0,3]. Use a double integral to confirm the total heat content is 4·Area.

$$\iint_R 4\,dA,\ [1,2]\times[0,3]$$

Answer: _____

23. A uniform temperature field has T = 4 degrees over rectangle [1,5] x [0,4]. Use a double integral to confirm the total heat content is 4·Area.

$$\iint_R 4\,dA,\ [1,5]\times[0,4]$$

Answer: _____

24. A uniform temperature field has T = 7 degrees over rectangle [0,5] x [1,5]. Use a double integral to confirm the total heat content is 7·Area.

$$\iint_R 7\,dA,\ [0,5]\times[1,5]$$

Answer: _____

25. A uniform temperature field has T = 4 degrees over rectangle [0,3] x [1,2]. Use a double integral to confirm the total heat content is 4·Area.

$$\iint_R 4\,dA,\ [0,3]\times[1,2]$$

Answer: _____

26. A uniform temperature field has T = 12 degrees over rectangle [0,3] x [0,2]. Use a double integral to confirm the total heat content is 12·Area.

$$\iint_R 12\,dA,\ [0,3]\times[0,2]$$

Answer: _____

27. A uniform temperature field has T = 10 degrees over rectangle [0,3] x [1,4]. Use a double integral to confirm the total heat content is 10·Area.

$$\iint_R 10\,dA,\ [0,3]\times[1,4]$$

Answer: _____

28. A uniform temperature field has T = 10 degrees over rectangle [0,5] x [0,3]. Use a double integral to confirm the total heat content is 10·Area.

$$\iint_R 10\, dA,\ [0,5] \times [0,3]$$

Answer: _____

29. A uniform temperature field has T = 4 degrees over rectangle [0,5] x [1,5]. Use a double integral to confirm the total heat content is 4·Area.

$$\iint_R 4\, dA,\ [0,5] \times [1,5]$$

Answer: _____

30. A uniform temperature field has T = 4 degrees over rectangle [1,5] x [1,3]. Use a double integral to confirm the total heat content is 4·Area.

$$\iint_R 4\, dA,\ [1,5] \times [1,3]$$

Answer: _____

MATH230: Applications of Double Integrals

ANSWER KEY & SOLUTIONS

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Topics: Area and volume, Average value. All answers verified by independent computation.

Solutions

Area and volume

1. Find the area of the rectangle R = [0,5] x [0,4] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [0,5]\times[0,4]$$

→ *Area = integral integral 1 dA = (5-0)·(4-0) = 5·4 = 20 square units.*

Answer: $= 1 \cdot 5 \cdot 4 = 20$

2. Find the volume of the solid under z = 8 and above the rectangle [0,3] x [1,3] in the xy-plane.

$$\iint_R 8\,dA,\ [0,3]\times[1,3]$$

→ *Volume = integral integral 8 dA over the rectangle.*
→ *= 8·(3-0)·(3-1) = 8·3·2 = 48 cubic units.*

Answer: $= 8 \cdot 3 \cdot 2 = 48$

3. Find the area of the rectangle R = [1,6] x [1,3] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [1,6]\times[1,3]$$

→ *Area = integral integral 1 dA = (6-1)·(3-1) = 5·2 = 10 square units.*

Answer: $= 1 \cdot 5 \cdot 2 = 10$

4. Find the volume of the solid under z = 7 and above the rectangle [0,5] x [1,4] in the xy-plane.

$$\iint_R 7\,dA,\ [0,5]\times[1,4]$$

→ *Volume = integral integral 7 dA over the rectangle.*
→ *= 7·(5-0)·(4-1) = 7·5·3 = 105 cubic units.*

Answer: $= 7 \cdot 5 \cdot 3 = 105$

5. Find the area of the rectangle R = [0,7] x [2,7] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [0,7]\times[2,7]$$

→ *Area = integral integral 1 dA = (7-0)·(7-2) = 7·5 = 35 square units.*

Answer: $= 1 \cdot 7 \cdot 5 = 35$

6. Find the volume of the solid under z = 9 and above the rectangle [1,4] x [0,5] in the xy-plane.

$$\iint_R 9\,dA,\ [1,4]\times[0,5]$$

→ *Volume = integral integral 9 dA over the rectangle.*
→ *= 9·(4-1)·(5-0) = 9·3·5 = 135 cubic units.*

Answer: $= 9 \cdot 3 \cdot 5 = 135$

7. Find the area of the rectangle R = [2,7] x [0,4] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [2,7]\times[0,4]$$

→ *Area = integral integral 1 dA = (7-2)·(4-0) = 5·4 = 20 square units.*

Answer: $= 1 \cdot 5 \cdot 4 = 20$

---

8. Find the volume of the solid under z = 5 and above the rectangle [1,5] x [1,5] in the xy-plane.

$$\iint_R 5\,dA,\ [1,5]\times[1,5]$$

→ *Volume = integral integral 5 dA over the rectangle.*
→ *= 5·(5-1)·(5-1) = 5·4·4 = 80 cubic units.*

Answer: $= 5 \cdot 4 \cdot 4 = 80$

---

9. Find the area of the rectangle R = [1,7] x [1,6] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [1,7]\times[1,6]$$

→ *Area = integral integral 1 dA = (7-1)·(6-1) = 6·5 = 30 square units.*

Answer: $= 1 \cdot 6 \cdot 5 = 30$

---

10. Find the volume of the solid under z = 2 and above the rectangle [1,4] x [1,3] in the xy-plane.

$$\iint_R 2\,dA,\ [1,4]\times[1,3]$$

→ *Volume = integral integral 2 dA over the rectangle.*
→ *= 2·(4-1)·(3-1) = 2·3·2 = 12 cubic units.*

Answer: $= 2 \cdot 3 \cdot 2 = 12$

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11. Find the area of the rectangle R = [1,7] x [2,3] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [1,7]\times[2,3]$$

→ *Area = integral integral 1 dA = (7-1)·(3-2) = 6·1 = 6 square units.*

Answer: $= 1 \cdot 6 \cdot 1 = 6$

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12. Find the volume of the solid under z = 5 and above the rectangle [0,5] x [0,3] in the xy-plane.

$$\iint_R 5\,dA,\ [0,5]\times[0,3]$$

→ *Volume = integral integral 5 dA over the rectangle.*
→ *= 5·(5-0)·(3-0) = 5·5·3 = 75 cubic units.*

Answer: $= 5 \cdot 5 \cdot 3 = 75$

---

13. Find the area of the rectangle R = [1,7] x [0,5] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [1,7]\times[0,5]$$

→ *Area = integral integral 1 dA = (7-1)·(5-0) = 6·5 = 30 square units.*

Answer: $= 1 \cdot 6 \cdot 5 = 30$

---

14. Find the volume of the solid under z = 4 and above the rectangle [0,3] x [1,4] in the xy-plane.

$$\iint_R 4\,dA,\ [0,3]\times[1,4]$$

→ *Volume = integral integral 4 dA over the rectangle.*
→ *= 4·(3-0)·(4-1) = 4·3·3 = 36 cubic units.*

Answer: $= 4 \cdot 3 \cdot 3 = 36$

---

15. Find the area of the rectangle R = [1,5] x [1,6] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [1,5]\times[1,6]$$

→ *Area = integral integral 1 dA = (5-1)·(6-1) = 4·5 = 20 square units.*

Answer: $= 1 \cdot 4 \cdot 5 = 20$

---

16. Find the volume of the solid under z = 8 and above the rectangle [1,4] x [0,5] in the xy-plane.

$$\iint_R 8\,dA,\ [1,4]\times[0,5]$$

→ *Volume = integral integral 8 dA over the rectangle.*
→ *= 8·(4-1)·(5-0) = 8·3·5 = 120 cubic units.*

Answer: $= 8 \cdot 3 \cdot 5 = 120$

---

17. Find the area of the rectangle R = [1,6] x [0,6] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [1,6]\times[0,6]$$

→ *Area = integral integral 1 dA = (6-1)·(6-0) = 5·6 = 30 square units.*

Answer: $= 1 \cdot 5 \cdot 6 = 30$

---

18. Find the volume of the solid under z = 9 and above the rectangle [0,3] x [1,5] in the xy-plane.

$$\iint_R 9\,dA,\ [0,3]\times[1,5]$$

→ *Volume = integral integral 9 dA over the rectangle.*
→ *= 9·(3-0)·(5-1) = 9·3·4 = 108 cubic units.*

Answer: $= 9 \cdot 3 \cdot 4 = 108$

---

19. Find the area of the rectangle R = [0,4] x [0,7] using a double integral of f(x,y) = 1 over R.

$$\iint_R 1\,dA,\ [0,4]\times[0,7]$$

→ *Area = integral integral 1 dA = (4-0)·(7-0) = 4·7 = 28 square units.*

Answer: $= 1 \cdot 4 \cdot 7 = 28$

---

20. Find the volume of the solid under z = 7 and above the rectangle [0,4] x [0,3] in the xy-plane.

$$\iint_R 7\,dA,\ [0,4]\times[0,3]$$

→ *Volume = integral integral 7 dA over the rectangle.*
→ *= 7·(4-0)·(3-0) = 7·4·3 = 84 cubic units.*

Answer: $= 7 \cdot 4 \cdot 3 = 84$

Average value

21. A uniform temperature field has T = 3 degrees over rectangle [1,3] x [1,4]. Use a double integral to confirm the total heat content is 3·Area.

$$\iint_R 3\,dA,\ [1,3]\times[1,4]$$

→ *Total = integral integral 3 dA = 3 · 2 · 3 = 18.*

Answer: $= 3 \cdot 2 \cdot 3 = 18$

22. A uniform temperature field has T = 4 degrees over rectangle [1,2] x [0,3]. Use a double integral to confirm the total heat content is 4·Area.

$$\iint_R 4\,dA,\ [1,2]\times[0,3]$$

→ *Total = integral integral 4 dA = 4 · 1 · 3 = 12.*

Answer: $= 4 \cdot 1 \cdot 3 = 12$

23. A uniform temperature field has T = 4 degrees over rectangle [1,5] x [0,4]. Use a double integral to confirm the total heat content is 4·Area.

$$\iint_R 4\,dA,\ [1,5]\times[0,4]$$

→ *Total = integral integral 4 dA = 4 · 4 · 4 = 64.*

Answer: $= 4 \cdot 4 \cdot 4 = 64$

24. A uniform temperature field has T = 7 degrees over rectangle [0,5] x [1,5]. Use a double integral to confirm the total heat content is 7·Area.

$$\iint_R 7\,dA,\ [0,5]\times[1,5]$$

→ *Total = integral integral 7 dA = 7 · 5 · 4 = 140.*

Answer: $= 7 \cdot 5 \cdot 4 = 140$

25. A uniform temperature field has T = 4 degrees over rectangle [0,3] x [1,2]. Use a double integral to confirm the total heat content is 4·Area.

$$\iint_R 4\,dA,\ [0,3]\times[1,2]$$

→ *Total = integral integral 4 dA = 4 · 3 · 1 = 12.*

Answer: $= 4 \cdot 3 \cdot 1 = 12$

26. A uniform temperature field has T = 12 degrees over rectangle [0,3] x [0,2]. Use a double integral to confirm the total heat content is 12·Area.

$$\iint_R 12\,dA,\ [0,3]\times[0,2]$$

→ *Total = integral integral 12 dA = 12 · 3 · 2 = 72.*

Answer: $= 12 \cdot 3 \cdot 2 = 72$

27. A uniform temperature field has T = 10 degrees over rectangle [0,3] x [1,4]. Use a double integral to confirm the total heat content is 10·Area.

$$\iint_R 10\,dA,\ [0,3]\times[1,4]$$

→ *Total = integral integral 10 dA = 10 · 3 · 3 = 90.*

Answer: $= 10 \cdot 3 \cdot 3 = 90$

28. A uniform temperature field has T = 10 degrees over rectangle [0,5] x [0,3]. Use a double integral to confirm the total heat content is 10·Area.

$$\iint_R 10\,dA,\ [0,5]\times[0,3]$$

→ *Total = integral integral 10 dA = 10 · 5 · 3 = 150.*

Answer: $= 10 \cdot 5 \cdot 3 = 150$

29. A uniform temperature field has T = 4 degrees over rectangle [0,5] x [1,5]. Use a double integral to confirm the total heat content is 4·Area.

$$\iint_R 4\,dA,\ [0,5]\times[1,5]$$

→ *Total = integral integral 4 dA = 4 · 5 · 4 = 80.*

Answer: $= 4 \cdot 5 \cdot 4 = 80$

30. A uniform temperature field has T = 4 degrees over rectangle [1,5] x [1,3]. Use a double integral to confirm the total heat content is 4·Area.

$$\iint_R 4\,dA,\ [1,5]\times[1,3]$$

→ *Total = integral integral 4 dA = 4 · 4 · 2 = 32.*

Answer: $= 4 \cdot 4 \cdot 2 = 32$