MATH230: Double Integrals

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Name: _____ Date: _____ Score: / 30

Learning Objectives

- Calculate the mean, median, mode, and range of a data set
- Compute sample variance and standard deviation
- Construct quartiles, the IQR, and identify outliers
- Describe the shape, center, and spread of a distribution

Simplify each expression completely. Show all steps and circle your final answer.

Double integrals over rectangles

1. Evaluate the double integral: integral from 0 to 4 integral from 0 to 5 of 2 dx dy.

$$\iint_R 2\,dA,\ [0,5]\times[0,4]$$

Answer: _____

2. A density function rho = 2 kg/m^2 is uniform over the rectangle [1,3] x [1,4]. Find the total mass.

$$\iint_R 2\,dA,\ [1,3]\times[1,4]$$

Answer: _____

3. Evaluate: integral from 0 to 2 integral from 0 to 5 of xy dx dy.

$$\iint_R xy\,dA,\ [0,5]\times[0,2]$$

Answer: _____

4. Evaluate the iterated integral: integral_0^5 integral_0^4 xy dx dy. Use Fubini's theorem to evaluate as an iterated integral.

$$\iint_R xy\,dA,\ [0,4]\times[0,5]$$

Answer: _____

5. Evaluate the double integral: integral from 2 to 4 integral from 1 to 3 of 2 dx dy.

$$\iint_R 2\,dA,\ [1,3]\times[2,4]$$

Answer: _____

6. A density function rho = 7 kg/m^2 is uniform over the rectangle [0,5] x [1,4]. Find the total mass.

$$\iint_R 7\,dA,\ [0,5]\times[1,4]$$

Answer: _____

7. Evaluate: integral from 0 to 3 integral from 0 to 4 of xy dx dy.

$$\iint_R xy\,dA,\ [0,4]\times[0,3]$$

Answer: _____

8. Evaluate the iterated integral: integral_0^4 integral_0^2 xy dx dy. Use Fubini's theorem to evaluate as an iterated integral.

$$\iint_R xy\,dA,\ [0,2]\times[0,4]$$

Answer: _____

9. Evaluate the double integral: integral from 2 to 4 integral from 1 to 5 of 8 dx dy.

$$\iint_R 8\,dA,\ [1,5]\times[2,4]$$

Answer: _____

10. A density function rho = 5 kg/m^2 is uniform over the rectangle [0,3] x [0,4]. Find the total mass.

$$\iint_R 5\,dA,\ [0,3]\times[0,4]$$

Answer: _____

11. Evaluate: integral from 0 to 2 integral from 0 to 4 of xy dx dy.

$$\iint_R xy\,dA,\ [0,4]\times[0,2]$$

Answer: _____

12. Evaluate the iterated integral: integral_0^3 integral_0^6 xy dx dy. Use Fubini's theorem to evaluate as an iterated integral.

$$\iint_R xy\,dA,\ [0,6]\times[0,3]$$

Answer: _____

13. Evaluate the double integral: integral from 1 to 6 integral from 1 to 5 of 3 dx dy.

$$\iint_R 3\,dA,\ [1,5]\times[1,6]$$

Answer: _____

14. A density function rho = 3 kg/m^2 is uniform over the rectangle [0,3] x [1,2]. Find the total mass.

$$\iint_R 3\,dA,\ [0,3]\times[1,2]$$

Answer: _____

15. Evaluate: integral from 0 to 5 integral from 0 to 2 of xy dx dy.

$$\iint_R xy\,dA,\ [0,2]\times[0,5]$$

Answer: _____

16. Evaluate the iterated integral: integral_0^4 integral_0^2 xy dx dy. Use Fubini's theorem to evaluate as an iterated integral.

$$\iint_R xy\,dA,\ [0,2]\times[0,4]$$

Answer: _____

17. Evaluate the double integral: integral from 0 to 4 integral from 0 to 4 of 4 dx dy.

$$\iint_R 4\,dA,\ [0,4]\times[0,4]$$

Answer: _____

18. A density function rho = 5 kg/m^2 is uniform over the rectangle [0,5] x [0,3]. Find the total mass.

$$\iint_R 5\,dA,\ [0,5]\times[0,3]$$

Answer: _____

19. Evaluate: integral from 0 to 4 integral from 0 to 4 of xy dx dy.

$$\iint_R xy\,dA,\ [0,4]\times[0,4]$$

Answer: _____

20. Evaluate the iterated integral: integral_0^3 integral_0^5 xy dx dy. Use Fubini's theorem to evaluate as an iterated integral.

$$\iint_R xy\,dA,\ [0,5]\times[0,3]$$

Answer: _____

21. Evaluate the double integral: integral from 1 to 5 integral from 0 to 4 of 3 dx dy.

$$\iint_R 3\,dA,\ [0,4]\times[1,5]$$

Answer: _____

22. A density function rho = 9 kg/m^2 is uniform over the rectangle [1,4] x [1,5]. Find the total mass.

$$\iint_R 9\,dA,\ [1,4]\times[1,5]$$

Answer: _____

23. Evaluate: integral from 0 to 2 integral from 0 to 5 of xy dx dy.

$$\iint_R xy\,dA,\ [0,5]\times[0,2]$$

Answer: _____

24. Evaluate the iterated integral: integral_0^4 integral_0^5 xy dx dy. Use Fubini's theorem to evaluate as an iterated integral.

$$\iint_R xy\,dA,\ [0,5]\times[0,4]$$

Answer: _____

25. Evaluate the double integral: integral from 0 to 6 integral from 1 to 6 of 2 dx dy.

$$\iint_R 2\,dA,\ [1,6]\times[0,6]$$

Answer: _____

26. A density function rho = 3 kg/m^2 is uniform over the rectangle [0,5] x [1,5]. Find the total mass.

$$\iint_R 3\,dA,\ [0,5]\times[1,5]$$

Answer: _____

27. Evaluate: integral from 0 to 2 integral from 0 to 5 of xy dx dy.

$$\iint_R xy\,dA,\ [0,5]\times[0,2]$$

Answer: _____

28. Evaluate the iterated integral: integral_0^2 integral_0^2 xy dx dy. Use Fubini's theorem to evaluate as an iterated integral.

$$\iint_R xy\,dA,\ [0,2]\times[0,2]$$

Answer: _____

29. Evaluate the double integral: integral from 1 to 4 integral from 1 to 6 of 2 dx dy.

$$\iint_R 2\,dA,\ [1,6]\times[1,4]$$

Answer: _____

30. A density function rho = 7 kg/m^2 is uniform over the rectangle [0,4] x [0,3]. Find the total mass.

$$\iint_R 7\,dA,\ [0,4]\times[0,3]$$

Answer: _____

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ANSWER KEY & SOLUTIONS

Topics: Double integrals over rectangles. All answers verified by independent computation.

Solutions

Double integrals over rectangles

1. Evaluate the double integral: integral from 0 to 4 integral from 0 to 5 of 2 dx dy.

$$\iint_R 2\,dA,\ [0,5]\times[0,4]$$

→ *Inner integral: integral from 0 to 5 of 2 dx = 2·(5-0) = 2·5.*
→ *Outer integral: integral from 0 to 4 of 2·5 dy = 2·5·(4-0) = 40.*

Answer: $= 2 \cdot 5 \cdot 4 = 40$

2. A density function rho = 2 kg/m^2 is uniform over the rectangle [1,3] x [1,4]. Find the total mass.

$$\iint_R 2\,dA,\ [1,3]\times[1,4]$$

→ *Mass = integral integral rho dA = 2 · (width) · (height).*
→ *= 2 · (3-1) · (4-1) = 2·2·3 = 12 kg.*

Answer: $= 2 \cdot 2 \cdot 3 = 12$

3. Evaluate: integral from 0 to 2 integral from 0 to 5 of xy dx dy.

$$\iint_R xy\,dA,\ [0,5]\times[0,2]$$

→ *Inner: integral_0^5 xy dx = x^2 y/2 from 0 to 5 = 5^2 y/2.*
→ *Outer: integral_0^2 (5^2 y/2) dy = 5^2/2 · y^2/2 from 0 to 2 = 5^2 · 2^2 / 4 = 25.*

Answer: $= \frac{25}{2} \cdot \frac{4}{2} = 25$

4. Evaluate the iterated integral: integral_0^5 integral_0^4 xy dx dy. Use Fubini's theorem to evaluate as an iterated integral.

$$\iint_R xy\,dA,\ [0,4]\times[0,5]$$

→ *Integrate with respect to x first: integral_0^4 xy dx = 4^2 y/2.*
→ *Then with respect to y: integral_0^5 4^2 y/2 dy = 1625/4 = 100.*

Answer: $= \frac{16}{2} \cdot \frac{25}{2} = 100$

5. Evaluate the double integral: integral from 2 to 4 integral from 1 to 3 of 2 dx dy.

$$\iint_R 2\,dA,\ [1,3]\times[2,4]$$

→ *Inner integral: integral from 1 to 3 of 2 dx = 2·(3-1) = 2·2.*
→ *Outer integral: integral from 2 to 4 of 2·2 dy = 2·2·(4-2) = 8.*

Answer: $= 2 \cdot 2 \cdot 2 = 8$

6. A density function rho = 7 kg/m^2 is uniform over the rectangle [0,5] x [1,4]. Find the total mass.

$$\iint_R 7\,dA,\ [0,5]\times[1,4]$$

→ *Mass = integral integral rho dA = 7 · (width) · (height).*
→ *= 7 · (5-0) · (4-1) = 7·5·3 = 105 kg.*

Answer: $= 7 \cdot 5 \cdot 3 = 105$

7. Evaluate: integral from 0 to 3 integral from 0 to 4 of xy dx dy.

$$\iint_R xy\,dA,\ [0,4]\times[0,3]$$

→ *Inner: integral_0^4 xy dx = x^2 y/2 from 0 to 4 = 4^2 y/2.*
→ *Outer: integral_0^3 (4^2 y/2) dy = 4^2/2 · y^2/2 from 0 to 3 = 4^2 · 3^2 / 4 = 36.*

Answer: $= \frac{16}{2} \cdot \frac{9}{2} = 36$

8. Evaluate the iterated integral: integral_0^4 integral_0^2 xy dx dy. Use Fubini's theorem to evaluate as an iterated integral.

$$\iint_R xy\,dA,\ [0,2]\times[0,4]$$

→ *Integrate with respect to x first: integral_0^2 xy dx = 2^2 y/2.*
→ *Then with respect to y: integral_0^4 2^2 y/2 dy = 416/4 = 16.*

Answer: $= \frac{4}{2} \cdot \frac{16}{2} = 16$

9. Evaluate the double integral: integral from 2 to 4 integral from 1 to 5 of 8 dx dy.

$$\iint_R 8\,dA,\ [1,5]\times[2,4]$$

→ *Inner integral: integral from 1 to 5 of 8 dx = 8·(5-1) = 8·4.*
→ *Outer integral: integral from 2 to 4 of 8·4 dy = 8·4·(4-2) = 64.*

Answer: $= 8 \cdot 4 \cdot 2 = 64$

10. A density function rho = 5 kg/m^2 is uniform over the rectangle [0,3] x [0,4]. Find the total mass.

$$\iint_R 5\,dA,\ [0,3]\times[0,4]$$

→ *Mass = integral integral rho dA = 5 · (width) · (height).*
→ *= 5 · (3-0) · (4-0) = 5·3·4 = 60 kg.*

Answer: $= 5 \cdot 3 \cdot 4 = 60$

11. Evaluate: integral from 0 to 2 integral from 0 to 4 of xy dx dy.

$$\iint_R xy\,dA,\ [0,4]\times[0,2]$$

→ *Inner: integral_0^4 xy dx = x^2 y/2 from 0 to 4 = 4^2 y/2.*
→ *Outer: integral_0^2 (4^2 y/2) dy = 4^2/2 · y^2/2 from 0 to 2 = 4^2 · 2^2 / 4 = 16.*

Answer: $= \frac{16}{2} \cdot \frac{4}{2} = 16$

12. Evaluate the iterated integral: integral_0^3 integral_0^6 xy dx dy. Use Fubini's theorem to evaluate as an iterated integral.

$$\iint_R xy\,dA,\ [0,6]\times[0,3]$$

→ *Integrate with respect to x first: integral_0^6 xy dx = 6^2 y/2.*
→ *Then with respect to y: integral_0^3 6^2 y/2 dy = 369/4 = 81.*

Answer: $= \frac{36}{2}\cdot\frac{9}{2} = 81$

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13. Evaluate the double integral: integral from 1 to 6 integral from 1 to 5 of 3 dx dy.

$$\iint_R 3\,dA,\ [1,5]\times[1,6]$$

→ *Inner integral: integral from 1 to 5 of 3 dx = 3·(5-1) = 3·4.*
→ *Outer integral: integral from 1 to 6 of 3·4 dy = 3·4·(6-1) = 60.*

Answer: $= 3\cdot 4\cdot 5 = 60$

---

14. A density function rho = 3 kg/m^2 is uniform over the rectangle [0,3] x [1,2]. Find the total mass.

$$\iint_R 3\,dA,\ [0,3]\times[1,2]$$

→ *Mass = integral integral rho dA = 3 · (width) · (height).*
→ *= 3 · (3-0) · (2-1) = 3·3·1 = 9 kg.*

Answer: $= 3\cdot 3\cdot 1 = 9$

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15. Evaluate: integral from 0 to 5 integral from 0 to 2 of xy dx dy.

$$\iint_R xy\,dA,\ [0,2]\times[0,5]$$

→ *Inner: integral_0^2 xy dx = x^2 y/2 from 0 to 2 = 2^2 y/2.*
→ *Outer: integral_0^5 (2^2 y/2) dy = 2^2/2 · y^2/2 from 0 to 5 = 2^2 · 5^2 / 4 = 25.*

Answer: $= \frac{4}{2}\cdot\frac{25}{2} = 25$

---

16. Evaluate the iterated integral: integral_0^4 integral_0^2 xy dx dy. Use Fubini's theorem to evaluate as an iterated integral.

$$\iint_R xy\,dA,\ [0,2]\times[0,4]$$

→ *Integrate with respect to x first: integral_0^2 xy dx = 2^2 y/2.*
→ *Then with respect to y: integral_0^4 2^2 y/2 dy = 416/4 = 16.*

Answer: $= \frac{4}{2}\cdot\frac{16}{2} = 16$

---

17. Evaluate the double integral: integral from 0 to 4 integral from 0 to 4 of 4 dx dy.

$$\iint_R 4\,dA,\ [0,4]\times[0,4]$$

→ *Inner integral: integral from 0 to 4 of 4 dx = 4·(4-0) = 4·4.*
→ *Outer integral: integral from 0 to 4 of 4·4 dy = 4·4·(4-0) = 64.*

Answer: $= 4\cdot 4\cdot 4 = 64$

---

18. A density function rho = 5 kg/m^2 is uniform over the rectangle [0,5] x [0,3]. Find the total mass.

$$\iint_R 5\,dA,\ [0,5]\times[0,3]$$

→ *Mass = integral integral rho dA = 5 · (width) · (height).*
→ *= 5 · (5-0) · (3-0) = 5·5·3 = 75 kg.*

Answer: $= 5 \cdot 5 \cdot 3 = 75$

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19. Evaluate: integral from 0 to 4 integral from 0 to 4 of xy dx dy.

$$\iint_R xy\,dA,\ [0,4]\times[0,4]$$

→ *Inner: integral_0^4 xy dx = x^2 y/2 from 0 to 4 = 4^2 y/2.*
→ *Outer: integral_0^4 (4^2 y/2) dy = 4^2/2 · y^2/2 from 0 to 4 = 4^2 · 4^2 / 4 = 64.*

Answer: $= \frac{16}{2} \cdot \frac{16}{2} = 64$

---

20. Evaluate the iterated integral: integral_0^3 integral_0^5 xy dx dy. Use Fubini's theorem to evaluate as an iterated integral.

$$\iint_R xy\,dA,\ [0,5]\times[0,3]$$

→ *Integrate with respect to x first: integral_0^5 xy dx = 5^2 y/2.*
→ *Then with respect to y: integral_0^3 5^2 y/2 dy = 259/4 = 225/4.*

Answer: $= \frac{25}{2} \cdot \frac{9}{2} = \frac{225}{4}$

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21. Evaluate the double integral: integral from 1 to 5 integral from 0 to 4 of 3 dx dy.

$$\iint_R 3\,dA,\ [0,4]\times[1,5]$$

→ *Inner integral: integral from 0 to 4 of 3 dx = 3·(4-0) = 3·4.*
→ *Outer integral: integral from 1 to 5 of 3·4 dy = 3·4·(5-1) = 48.*

Answer: $= 3 \cdot 4 \cdot 4 = 48$

---

22. A density function rho = 9 kg/m^2 is uniform over the rectangle [1,4] x [1,5]. Find the total mass.

$$\iint_R 9\,dA,\ [1,4]\times[1,5]$$

→ *Mass = integral integral rho dA = 9 · (width) · (height).*
→ *= 9 · (4-1) · (5-1) = 9·3·4 = 108 kg.*

Answer: $= 9 \cdot 3 \cdot 4 = 108$

---

23. Evaluate: integral from 0 to 2 integral from 0 to 5 of xy dx dy.

$$\iint_R xy\,dA,\ [0,5]\times[0,2]$$

→ *Inner: integral_0^5 xy dx = x^2 y/2 from 0 to 5 = 5^2 y/2.*
→ *Outer: integral_0^2 (5^2 y/2) dy = 5^2/2 · y^2/2 from 0 to 2 = 5^2 · 2^2 / 4 = 25.*

Answer: $= \frac{25}{2} \cdot \frac{4}{2} = 25$

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24. Evaluate the iterated integral: integral_0^4 integral_0^5 xy dx dy. Use Fubini's theorem to evaluate as an iterated integral.

$$\iint_R xy\,dA,\ [0,5]\times[0,4]$$

→ *Integrate with respect to x first: integral_0^5 xy dx = 5^2 y/2.*
→ *Then with respect to y: integral_0^4 5^2 y/2 dy = 2516/4 = 100.*

Answer: $= \frac{25}{2}\cdot\frac{16}{2} = 100$

---

25. Evaluate the double integral: integral from 0 to 6 integral from 1 to 6 of 2 dx dy.

$$\iint_R 2\,dA,\ [1,6]\times[0,6]$$

→ *Inner integral: integral from 1 to 6 of 2 dx = 2·(6-1) = 2·5.*
→ *Outer integral: integral from 0 to 6 of 2·5 dy = 2·5·(6-0) = 60.*

Answer: $= 2\cdot 5\cdot 6 = 60$

---

26. A density function rho = 3 kg/m^2 is uniform over the rectangle [0,5] x [1,5]. Find the total mass.

$$\iint_R 3\,dA,\ [0,5]\times[1,5]$$

→ *Mass = integral integral rho dA = 3 · (width) · (height).*
→ *= 3 · (5-0) · (5-1) = 3·5·4 = 60 kg.*

Answer: $= 3\cdot 5\cdot 4 = 60$

---

27. Evaluate: integral from 0 to 2 integral from 0 to 5 of xy dx dy.

$$\iint_R xy\,dA,\ [0,5]\times[0,2]$$

→ *Inner: integral_0^5 xy dx = x^2 y/2 from 0 to 5 = 5^2 y/2.*
→ *Outer: integral_0^2 (5^2 y/2) dy = 5^2/2 · y^2/2 from 0 to 2 = 5^2 · 2^2 / 4 = 25.*

Answer: $= \frac{25}{2}\cdot\frac{4}{2} = 25$

---

28. Evaluate the iterated integral: integral_0^2 integral_0^2 xy dx dy. Use Fubini's theorem to evaluate as an iterated integral.

$$\iint_R xy\,dA,\ [0,2]\times[0,2]$$

→ *Integrate with respect to x first: integral_0^2 xy dx = 2^2 y/2.*
→ *Then with respect to y: integral_0^2 2^2 y/2 dy = 44/4 = 4.*

Answer: $= \frac{4}{2}\cdot\frac{4}{2} = 4$

---

29. Evaluate the double integral: integral from 1 to 4 integral from 1 to 6 of 2 dx dy.

$$\iint_R 2\,dA,\ [1,6]\times[1,4]$$

→ *Inner integral: integral from 1 to 6 of 2 dx = 2·(6-1) = 2·5.*
→ *Outer integral: integral from 1 to 4 of 2·5 dy = 2·5·(4-1) = 30.*

Answer: $= 2\cdot 5\cdot 3 = 30$

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30. A density function rho = 7 kg/m^2 is uniform over the rectangle [0,4] x [0,3]. Find the total mass.

$$\iint_R 7\,dA,\ [0,4]\times[0,3]$$

→ *Mass = integral integral rho dA = 7 · (width) · (height).*
→ *= 7 · (4-0) · (3-0) = 7·4·3 = 84 kg.*

Answer: $= 7 \cdot 4 \cdot 3 = 84$