MATH230: Gradient and Directional Derivatives

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Name: ______________________ Date: ______________ Score: / 30

Learning Objectives

- Calculate the mean, median, mode, and range of a data set
- Compute sample variance and standard deviation
- Construct quartiles, the IQR, and identify outliers
- Describe the shape, center, and spread of a distribution

Simplify each expression completely. Show all steps and circle your final answer.

Computing gradient components

1. For f(x,y) = 1x^2y^2, compute f_x(1,1) — the x-component of the gradient at (1,1).

$$f(x,y) = 1x^2y^2,\ (x_0,y_0) = (1,1)$$

Answer: ______________________________

2. For f(x,y) = 1x^2y^2, compute f_y(2,2) — the y-component of the gradient at (2,2).

$$f(x,y) = 1x^2y^2,\ (x_0,y_0) = (2,2)$$

Answer: ______________________________

3. For f(x,y) = 3x^3y^2, compute f_x(2,1) — the x-component of the gradient at (2,1).

$$f(x,y) = 3x^3y^2,\ (x_0,y_0) = (2,1)$$

Answer: ______________________________

4. For f(x,y) = 1x^2y^2, compute f_y(3,3) — the y-component of the gradient at (3,3).

$$f(x,y) = 1x^2y^2,\ (x_0,y_0) = (3,3)$$

Answer: ______________________________

5. For f(x,y) = 3x^2y^2, compute f_x(3,1) — the x-component of the gradient at (3,1).

$$f(x,y) = 3x^2y^2,\ (x_0,y_0) = (3,1)$$

Answer: ______________________________

6. For f(x,y) = 1x^2y^3, compute f_y(1,2) — the y-component of the gradient at (1,2).

$$f(x,y) = 1x^2y^3,\ (x_0,y_0) = (1,2)$$

Answer: ______________________________

7. For f(x,y) = 2x^2y^1, compute f_x(1,3) — the x-component of the gradient at (1,3).

$$f(x,y) = 2x^2y^1,\ (x_0, y_0) = (1,3)$$

Answer: ____

8. For f(x,y) = 3x^1y^3, compute f_y(2,2) — the y-component of the gradient at (2,2).

$$f(x,y) = 3x^1y^3,\ (x_0, y_0) = (2,2)$$

Answer: ____

9. For f(x,y) = 2x^3y^2, compute f_x(2,2) — the x-component of the gradient at (2,2).

$$f(x,y) = 2x^3y^2,\ (x_0, y_0) = (2,2)$$

Answer: ____

10. For f(x,y) = 1x^1y^2, compute f_y(3,2) — the y-component of the gradient at (3,2).

$$f(x,y) = 1x^1y^2,\ (x_0, y_0) = (3,2)$$

Answer: ____

11. For f(x,y) = 1x^3y^1, compute f_x(1,3) — the x-component of the gradient at (1,3).

$$f(x,y) = 1x^3y^1,\ (x_0, y_0) = (1,3)$$

Answer: ____

12. For f(x,y) = 2x^1y^2, compute f_y(1,1) — the y-component of the gradient at (1,1).

$$f(x,y) = 2x^1y^2,\ (x_0, y_0) = (1,1)$$

Answer: ____

13. For f(x,y) = 3x^3y^1, compute f_x(2,1) — the x-component of the gradient at (2,1).

$$f(x,y) = 3x^3y^1,\ (x_0, y_0) = (2,1)$$

Answer: ____

14. For f(x,y) = 4x^1y^2, compute f_y(2,2) — the y-component of the gradient at (2,2).

$$f(x,y) = 4x^1y^2,\ (x_0, y_0) = (2,2)$$

Answer: ____

15. For f(x,y) = 4x^3y^2, compute f_x(2,2) — the x-component of the gradient at (2,2).

$$f(x,y) = 4x^3y^2,\ (x_0, y_0) = (2,2)$$

Answer: ____

16. For f(x,y) = 4x^1y^3, compute f_y(3,3) — the y-component of the gradient at (3,3).

$$f(x,y) = 4x^1y^3,\ (x_0,y_0) = (3,3)$$

Answer: ______________________________________

17. For f(x,y) = 1x^3y^2, compute f_x(1,2) — the x-component of the gradient at (1,2).

$$f(x,y) = 1x^3y^2,\ (x_0,y_0) = (1,2)$$

Answer: ______________________________________

18. For f(x,y) = 1x^1y^3, compute f_y(2,2) — the y-component of the gradient at (2,2).

$$f(x,y) = 1x^1y^3,\ (x_0,y_0) = (2,2)$$

Answer: ______________________________________

19. For f(x,y) = 1x^2y^1, compute f_x(1,3) — the x-component of the gradient at (1,3).

$$f(x,y) = 1x^2y^1,\ (x_0,y_0) = (1,3)$$

Answer: ______________________________________

20. For f(x,y) = 1x^2y^3, compute f_y(2,1) — the y-component of the gradient at (2,1).

$$f(x,y) = 1x^2y^3,\ (x_0,y_0) = (2,1)$$

Answer: ______________________________________

Dot product in gradient context

21. The gradient ∇f at a point is (1, 3, 0) and the unit direction vector is (-2, 0, 0). Compute the directional derivative D_u f = ∇f · u.

$$(1,3,0) \cdot (-2,0,0)$$

Answer: ______________________________________

22. The gradient ∇f at a point is (0, -4, 0) and the unit direction vector is (1, 0, 0). Compute the directional derivative D_u f = ∇f · u.

$$(0,-4,0) \cdot (1,0,0)$$

Answer: ______________________________________

23. The gradient ∇f at a point is (2, -1, 0) and the unit direction vector is (2, -2, 0). Compute the directional derivative D_u f = ∇f · u.

$$(2,-1,0) \cdot (2,-2,0)$$

Answer: ______________________________________

24. The gradient ∇f at a point is (4, -2, 0) and the unit direction vector is (1, 2, 0). Compute the directional derivative D_u f = ∇f · u.

$$(4, -2, 0) \cdot (1, 2, 0)$$

Answer: ____

25. The gradient ∇f at a point is (3, -5, 0) and the unit direction vector is (2, 1, 0). Compute the directional derivative D_u f = ∇f · u.

$$(3, -5, 0) \cdot (2, 1, 0)$$

Answer: ____

26. The gradient ∇f at a point is (-2, -4, 0) and the unit direction vector is (-3, -2, 0). Compute the directional derivative D_u f = ∇f · u.

$$(-2, -4, 0) \cdot (-3, -2, 0)$$

Answer: ____

27. The gradient ∇f at a point is (4, -3, 0) and the unit direction vector is (3, -2, 0). Compute the directional derivative D_u f = ∇f · u.

$$(4, -3, 0) \cdot (3, -2, 0)$$

Answer: ____

28. The gradient ∇f at a point is (1, -1, 0) and the unit direction vector is (1, 3, 0). Compute the directional derivative D_u f = ∇f · u.

$$(1, -1, 0) \cdot (1, 3, 0)$$

Answer: ____

29. The gradient ∇f at a point is (2, 3, 0) and the unit direction vector is (1, 1, 0). Compute the directional derivative D_u f = ∇f · u.

$$(2, 3, 0) \cdot (1, 1, 0)$$

Answer: ____

30. The gradient ∇f at a point is (0, 3, 0) and the unit direction vector is (-1, -2, 0). Compute the directional derivative D_u f = ∇f · u.

$$(0, 3, 0) \cdot (-1, -2, 0)$$

Answer: ____

MATH230: Gradient and Directional Derivatives

ANSWER KEY & SOLUTIONS

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Topics: Computing gradient components, Dot product in gradient context. All answers verified by independent computation.

Solutions

Computing gradient components

1. For f(x,y) = 1x^2y^2, compute f_x(1,1) — the x-component of the gradient at (1,1).

$$f(x, y) = 1x^2y^2,\ (x_0, y_0) = (1, 1)$$

→ *Gradient = (f_x, f_y). Find x-component: f_x = 2x^1y^2.*
→ *f_x(1,1) = 2.*
→ *This is the x-component of ∇f at (1,1).*

Answer: $f_x = 2x^1y^2,\ f_x(1, 1) = 2$

2. For f(x,y) = 1x^2y^2, compute f_y(2,2) — the y-component of the gradient at (2,2).

$$f(x, y) = 1x^2y^2,\ (x_0, y_0) = (2, 2)$$

→ *Gradient = (f_x, f_y). Find y-component: f_y = 2x^2y^1.*
→ *f_y(2,2) = 16.*
→ *This is the y-component of ∇f at (2,2).*

Answer: $f_y = 2x^2y^1,\ f_y(2, 2) = 16$

3. For f(x,y) = 3x^3y^2, compute f_x(2,1) — the x-component of the gradient at (2,1).

$$f(x, y) = 3x^3y^2,\ (x_0, y_0) = (2, 1)$$

→ *Gradient = (f_x, f_y). Find x-component: f_x = 9x^2y^2.*
→ *f_x(2,1) = 36.*
→ *This is the x-component of ∇f at (2,1).*

Answer: $f_x = 9x^2y^2,\ f_x(2, 1) = 36$

4. For f(x,y) = 1x^2y^2, compute f_y(3,3) — the y-component of the gradient at (3,3).

$$f(x, y) = 1x^2y^2,\ (x_0, y_0) = (3, 3)$$

→ *Gradient = (f_x, f_y). Find y-component: f_y = 2x^2y^1.*
→ *f_y(3,3) = 54.*
→ *This is the y-component of ∇f at (3,3).*

Answer: $f_y = 2x^2y^1,\ f_y(3, 3) = 54$

5. For f(x,y) = 3x^2y^2, compute f_x(3,1) — the x-component of the gradient at (3,1).

$$f(x, y) = 3x^2y^2,\ (x_0, y_0) = (3, 1)$$

→ *Gradient = (f_x, f_y). Find x-component: f_x = 6x^1y^2.*
→ *f_x(3,1) = 18.*
→ *This is the x-component of ∇f at (3,1).*

Answer: $f_x = 6x^1y^2,\ f_x(3, 1) = 18$

6. For f(x,y) = 1x^2y^3, compute f_y(1,2) — the y-component of the gradient at (1,2).

$$f(x,y) = 1x^2y^3,\ (x_0,y_0) = (1,2)$$

→ Gradient = (f_x, f_y). Find y-component: f_y = 3x^2y^2.
→ f_y(1,2) = 12.
→ This is the y-component of ∇f at (1,2).

Answer: $f_y = 3x^2y^2,\ f_y(1,2) = 12$

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7. For f(x,y) = 2x^2y^1, compute f_x(1,3) — the x-component of the gradient at (1,3).

$$f(x,y) = 2x^2y^1,\ (x_0,y_0) = (1,3)$$

→ Gradient = (f_x, f_y). Find x-component: f_x = 4x^1y^1.
→ f_x(1,3) = 12.
→ This is the x-component of ∇f at (1,3).

Answer: $f_x = 4x^1y^1,\ f_x(1,3) = 12$

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8. For f(x,y) = 3x^1y^3, compute f_y(2,2) — the y-component of the gradient at (2,2).

$$f(x,y) = 3x^1y^3,\ (x_0,y_0) = (2,2)$$

→ Gradient = (f_x, f_y). Find y-component: f_y = 9x^1y^2.
→ f_y(2,2) = 72.
→ This is the y-component of ∇f at (2,2).

Answer: $f_y = 9x^1y^2,\ f_y(2,2) = 72$

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9. For f(x,y) = 2x^3y^2, compute f_x(2,2) — the x-component of the gradient at (2,2).

$$f(x,y) = 2x^3y^2,\ (x_0,y_0) = (2,2)$$

→ Gradient = (f_x, f_y). Find x-component: f_x = 6x^2y^2.
→ f_x(2,2) = 96.
→ This is the x-component of ∇f at (2,2).

Answer: $f_x = 6x^2y^2,\ f_x(2,2) = 96$

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10. For f(x,y) = 1x^1y^2, compute f_y(3,2) — the y-component of the gradient at (3,2).

$$f(x,y) = 1x^1y^2,\ (x_0,y_0) = (3,2)$$

→ Gradient = (f_x, f_y). Find y-component: f_y = 2x^1y^1.
→ f_y(3,2) = 12.
→ This is the y-component of ∇f at (3,2).

Answer: $f_y = 2x^1y^1,\ f_y(3,2) = 12$

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11. For f(x,y) = 1x^3y^1, compute f_x(1,3) — the x-component of the gradient at (1,3).

$$f(x,y) = 1x^3y^1,\ (x_0, y_0) = (1, 3)$$

→ Gradient = (f_x, f_y). Find x-component: f_x = 3x^2y^1.
→ f_x(1,3) = 9.
→ This is the x-component of ∇f at (1,3).

Answer: $f_x = 3x^2y^1,\ f_x(1,3) = 9$

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12. For f(x,y) = 2x^1y^2, compute f_y(1,1) — the y-component of the gradient at (1,1).

$$f(x,y) = 2x^1y^2,\ (x_0, y_0) = (1, 1)$$

→ Gradient = (f_x, f_y). Find y-component: f_y = 4x^1y^1.
→ f_y(1,1) = 4.
→ This is the y-component of ∇f at (1,1).

Answer: $f_y = 4x^1y^1,\ f_y(1,1) = 4$

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13. For f(x,y) = 3x^3y^1, compute f_x(2,1) — the x-component of the gradient at (2,1).

$$f(x,y) = 3x^3y^1,\ (x_0, y_0) = (2, 1)$$

→ Gradient = (f_x, f_y). Find x-component: f_x = 9x^2y^1.
→ f_x(2,1) = 36.
→ This is the x-component of ∇f at (2,1).

Answer: $f_x = 9x^2y^1,\ f_x(2,1) = 36$

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14. For f(x,y) = 4x^1y^2, compute f_y(2,2) — the y-component of the gradient at (2,2).

$$f(x,y) = 4x^1y^2,\ (x_0, y_0) = (2, 2)$$

→ Gradient = (f_x, f_y). Find y-component: f_y = 8x^1y^1.
→ f_y(2,2) = 32.
→ This is the y-component of ∇f at (2,2).

Answer: $f_y = 8x^1y^1,\ f_y(2,2) = 32$

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15. For f(x,y) = 4x^3y^2, compute f_x(2,2) — the x-component of the gradient at (2,2).

$$f(x,y) = 4x^3y^2,\ (x_0, y_0) = (2, 2)$$

→ Gradient = (f_x, f_y). Find x-component: f_x = 12x^2y^2.
→ f_x(2,2) = 192.
→ This is the x-component of ∇f at (2,2).

Answer: $f_x = 12x^2y^2,\ f_x(2,2) = 192$

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16. For f(x,y) = 4x^1y^3, compute f_y(3,3) — the y-component of the gradient at (3,3).

$$f(x,y) = 4x^1y^3,\ (x_0,y_0) = (3,3)$$

→ *Gradient = (f_x, f_y). Find y-component: f_y = 12x^1y^2.*
→ *f_y(3,3) = 324.*
→ *This is the y-component of ∇f at (3,3).*

Answer: $f_y = 12x^1y^2,\ f_y(3,3) = 324$

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17. For f(x,y) = 1x^3y^2, compute f_x(1,2) — the x-component of the gradient at (1,2).

$$f(x,y) = 1x^3y^2,\ (x_0,y_0) = (1,2)$$

→ *Gradient = (f_x, f_y). Find x-component: f_x = 3x^2y^2.*
→ *f_x(1,2) = 12.*
→ *This is the x-component of ∇f at (1,2).*

Answer: $f_x = 3x^2y^2,\ f_x(1,2) = 12$

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18. For f(x,y) = 1x^1y^3, compute f_y(2,2) — the y-component of the gradient at (2,2).

$$f(x,y) = 1x^1y^3,\ (x_0,y_0) = (2,2)$$

→ *Gradient = (f_x, f_y). Find y-component: f_y = 3x^1y^2.*
→ *f_y(2,2) = 24.*
→ *This is the y-component of ∇f at (2,2).*

Answer: $f_y = 3x^1y^2,\ f_y(2,2) = 24$

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19. For f(x,y) = 1x^2y^1, compute f_x(1,3) — the x-component of the gradient at (1,3).

$$f(x,y) = 1x^2y^1,\ (x_0,y_0) = (1,3)$$

→ *Gradient = (f_x, f_y). Find x-component: f_x = 2x^1y^1.*
→ *f_x(1,3) = 6.*
→ *This is the x-component of ∇f at (1,3).*

Answer: $f_x = 2x^1y^1,\ f_x(1,3) = 6$

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20. For f(x,y) = 1x^2y^3, compute f_y(2,1) — the y-component of the gradient at (2,1).

$$f(x,y) = 1x^2y^3,\ (x_0,y_0) = (2,1)$$

→ *Gradient = (f_x, f_y). Find y-component: f_y = 3x^2y^2.*
→ *f_y(2,1) = 12.*
→ *This is the y-component of ∇f at (2,1).*

Answer: $f_y = 3x^2y^2,\ f_y(2,1) = 12$

Dot product in gradient context

21. The gradient ∇f at a point is (1, 3, 0) and the unit direction vector is (-2, 0, 0). Compute the directional derivative D_u f = ∇f · u.

$$(1, 3, 0) \cdot (-2, 0, 0)$$

→ D_u f = ∇f · u = (1)(-2) + (3)(0) + 0.
→ = -2.

Answer: $= -2$

22. The gradient ∇f at a point is (0, -4, 0) and the unit direction vector is (1, 0, 0). Compute the directional derivative D_u f = ∇f · u.

$$(0, -4, 0) \cdot (1, 0, 0)$$

→ D_u f = ∇f · u = (0)(1) + (-4)(0) + 0.
→ = 0.

Answer: $= 0$

23. The gradient ∇f at a point is (2, -1, 0) and the unit direction vector is (2, -2, 0). Compute the directional derivative D_u f = ∇f · u.

$$(2, -1, 0) \cdot (2, -2, 0)$$

→ D_u f = ∇f · u = (2)(2) + (-1)(-2) + 0.
→ = 6.

Answer: $= 6$

24. The gradient ∇f at a point is (4, -2, 0) and the unit direction vector is (1, 2, 0). Compute the directional derivative D_u f = ∇f · u.

$$(4, -2, 0) \cdot (1, 2, 0)$$

→ D_u f = ∇f · u = (4)(1) + (-2)(2) + 0.
→ = 0.

Answer: $= 0$

25. The gradient ∇f at a point is (3, -5, 0) and the unit direction vector is (2, 1, 0). Compute the directional derivative D_u f = ∇f · u.

$$(3, -5, 0) \cdot (2, 1, 0)$$

→ D_u f = ∇f · u = (3)(2) + (-5)(1) + 0.
→ = 1.

Answer: $= 1$

26. The gradient ∇f at a point is (-2, -4, 0) and the unit direction vector is (-3, -2, 0). Compute the directional derivative D_u f = ∇f · u.

$$(-2, -4, 0) \cdot (-3, -2, 0)$$

→ D_u f = ∇f · u = (-2)(-3) + (-4)(-2) + 0.
→ = 14.

Answer: $= 14$

27. The gradient ∇f at a point is (4, -3, 0) and the unit direction vector is (3, -2, 0). Compute the directional derivative D_u f = ∇f · u.

$$(4, -3, 0) \cdot (3, -2, 0)$$

→ D_u f = ∇f · u = (4)(3) + (-3)(-2) + 0.
→ = 18.

Answer: $= 18$

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28. The gradient ∇f at a point is (1, -1, 0) and the unit direction vector is (1, 3, 0). Compute the directional derivative D_u f = ∇f · u.

$$(1, -1, 0) \cdot (1, 3, 0)$$

→ D_u f = ∇f · u = (1)(1) + (-1)(3) + 0.
→ = -2.

Answer: $= -2$

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29. The gradient ∇f at a point is (2, 3, 0) and the unit direction vector is (1, 1, 0). Compute the directional derivative D_u f = ∇f · u.

$$(2, 3, 0) \cdot (1, 1, 0)$$

→ D_u f = ∇f · u = (2)(1) + (3)(1) + 0.
→ = 5.

Answer: $= 5$

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30. The gradient ∇f at a point is (0, 3, 0) and the unit direction vector is (-1, -2, 0). Compute the directional derivative D_u f = ∇f · u.

$$(0, 3, 0) \cdot (-1, -2, 0)$$

→ D_u f = ∇f · u = (0)(-1) + (3)(-2) + 0.
→ = -6.

Answer: $= -6$
