MATH230: Line Integrals and Vector Calculus

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Learning Objectives

- Calculate the mean, median, mode, and range of a data set
- Compute sample variance and standard deviation
- Construct quartiles, the IQR, and identify outliers
- Describe the shape, center, and spread of a distribution

Simplify each expression completely. Show all steps and circle your final answer.

Flux and divergence

1. The divergence of a vector field is constant div F = 2 over a region [0,4] x [0,3]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 2\, dA,\ [0,4] \times [0,3]$$

Answer: ____

2. The divergence of a vector field is constant div F = 5 over a region [1,5] x [1,2]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 5\, dA,\ [1,5] \times [1,2]$$

Answer: ____

3. The divergence of a vector field is constant div F = 6 over a region [0,4] x [0,5]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 6\, dA,\ [0,4] \times [0,5]$$

Answer: ____

4. The divergence of a vector field is constant div F = 4 over a region [0,3] x [0,4]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 4\, dA,\ [0,3] \times [0,4]$$

Answer: ____

5. The divergence of a vector field is constant div F = 3 over a region [1,4] x [1,5]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 3\, dA,\ [1,4] \times [1,5]$$

Answer: ____

6. The divergence of a vector field is constant div F = 1 over a region [1,2] x [0,4]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 1\, dA,\ [1,2] \times [0,4]$$

Answer: ____

7. The divergence of a vector field is constant div F = 5 over a region [1,3] x [1,3]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 5\, dA,\ [1,3] \times [1,3]$$

Answer: ____

8. The divergence of a vector field is constant div F = 8 over a region [1,4] x [1,5]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 8\, dA,\ [1,4] \times [1,5]$$

Answer: ____

9. The divergence of a vector field is constant div F = 2 over a region [1,5] x [0,5]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 2\, dA,\ [1,5] \times [0,5]$$

Answer: ____

10. The divergence of a vector field is constant div F = 1 over a region [0,3] x [0,4]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 1\, dA,\ [0,3] \times [0,4]$$

Answer: ____

Green's theorem (flux form)

11. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [1,3] x [1,4]. If the curl is constant = 1, find the circulation.

$$\iint_R 1\, dA,\ [1,3] \times [1,4]$$

Answer: ____

12. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [1,2] x [0,3]. If the curl is constant = 1, find the circulation.

$$\iint_R 1\, dA,\ [1,2] \times [0,3]$$

Answer: ____

13. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [1,5] x [0,4]. If the curl is constant = 1, find the circulation.

$$\iint_R 1\, dA,\ [1,5] \times [0,4]$$

Answer: ____

14. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [0,5] x [1,5]. If the curl is constant = 3, find the circulation.

$$\iint_R 3\, dA,\ [0,5] \times [1,5]$$

Answer: ____

15. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [0,3] x [1,2]. If the curl is constant = 1, find the circulation.

$$\iint_R 1\, dA,\ [0,3] \times [1,2]$$

Answer: ____

16. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [0,3] x [0,2]. If the curl is constant = 5, find the circulation.

$$\iint_R 5\, dA,\ [0,3] \times [0,2]$$

Answer: ____

17. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [0,3] x [1,4]. If the curl is constant = 4, find the circulation.

$$\iint_R 4\, dA,\ [0,3] \times [1,4]$$

Answer: ____

18. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [0,5] x [0,3]. If the curl is constant = 4, find the circulation.

$$\iint_R 4\, dA,\ [0,5] \times [0,3]$$

Answer: ____

19. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [0,5] x [1,5]. If the curl is constant = 1, find the circulation.

$$\iint_R 1\, dA,\ [0,5] \times [1,5]$$

Answer: ____

20. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [1,5] x [1,3]. If the curl is constant = 1, find the circulation.

$$\iint_R 1\, dA,\ [1,5] \times [1,3]$$

Answer: ____

Work along a path

21. A vector field F = (1, 3, 0) acts along a straight path with displacement vector d = (2, 4, 0). Find the work done W = F · d.

$$(1,3,0) \cdot (2,4,0)$$

Answer: ____

22. A vector field F = (0, -4, 3) acts along a straight path with displacement vector d = (5, 4, 2). Find the work done W = F · d.

$$(0,-4,3) \cdot (5,4,2)$$

Answer: ____

23. A vector field F = (2, -1, 2) acts along a straight path with displacement vector d = (6, 2, 2). Find the work done W = F · d.

$$(2,-1,2) \cdot (6,2,2)$$

Answer: ____

24. A vector field F = (4, -2, 3) acts along a straight path with displacement vector d = (5, 6, 1). Find the work done W = F · d.

$$(4,-2,3) \cdot (5,6,1)$$

Answer: ____

25. A vector field F = (3, -5, 3) acts along a straight path with displacement vector d = (6, 5, 1). Find the work done W = F · d.

$$(3, -5, 3) \cdot (6, 5, 1)$$

Answer: ____

26. A vector field F = (-2, -4, 3) acts along a straight path with displacement vector d = (1, 2, 1). Find the work done W = F · d.

$$(-2, -4, 3) \cdot (1, 2, 1)$$

Answer: ____

27. A vector field F = (4, -3, 0) acts along a straight path with displacement vector d = (2, 4, 1). Find the work done W = F · d.

$$(4, -3, 0) \cdot (2, 4, 1)$$

Answer: ____

28. A vector field F = (1, -1, 2) acts along a straight path with displacement vector d = (5, 2, 1). Find the work done W = F · d.

$$(1, -1, 2) \cdot (5, 2, 1)$$

Answer: ____

29. A vector field F = (2, 3, 0) acts along a straight path with displacement vector d = (5, 5, 0). Find the work done W = F · d.

$$(2, 3, 0) \cdot (5, 5, 0)$$

Answer: ____

30. A vector field F = (0, 3, 0) acts along a straight path with displacement vector d = (3, 2, 0). Find the work done W = F · d.

$$(0, 3, 0) \cdot (3, 2, 0)$$

Answer: ____

Topics: Green's theorem (flux form), Work along a path, Flux and divergence. All answers verified by independent computation.

Solutions

Flux and divergence

1. The divergence of a vector field is constant div F = 2 over a region [0,4] x [0,3]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 2\, dA,\ [0,4] \times [0,3]$$

→ Flux = integral integral (div F) dA = 2 · (4-0) · (3-0).
→ = 2·4·3 = 24.

Answer: $= 2 \cdot 4 \cdot 3 = 24$

2. The divergence of a vector field is constant div F = 5 over a region [1,5] x [1,2]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 5\, dA,\ [1,5] \times [1,2]$$

→ Flux = integral integral (div F) dA = 5 · (5-1) · (2-1).
→ = 5·4·1 = 20.

Answer: $= 5 \cdot 4 \cdot 1 = 20$

3. The divergence of a vector field is constant div F = 6 over a region [0,4] x [0,5]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 6\, dA,\ [0,4] \times [0,5]$$

→ Flux = integral integral (div F) dA = 6 · (4-0) · (5-0).
→ = 6·4·5 = 120.

Answer: $= 6 \cdot 4 \cdot 5 = 120$

4. The divergence of a vector field is constant div F = 4 over a region [0,3] x [0,4]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 4\, dA,\ [0,3] \times [0,4]$$

→ Flux = integral integral (div F) dA = 4 · (3-0) · (4-0).
→ = 4·3·4 = 48.

Answer: $= 4 \cdot 3 \cdot 4 = 48$

5. The divergence of a vector field is constant div F = 3 over a region [1,4] x [1,5]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 3\, dA,\ [1,4] \times [1,5]$$

→ Flux = integral integral (div F) dA = 3 · (4-1) · (5-1).
→ = 3·3·4 = 36.

Answer: $= 3 \cdot 3 \cdot 4 = 36$

6. The divergence of a vector field is constant div F = 1 over a region [1,2] x [0,4]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 1\, dA,\ [1,2] \times [0,4]$$

→ Flux = integral integral (div F) dA = 1 · (2-1) · (4-0).
→ = 1·1·4 = 4.

Answer: $= 1 \cdot 1 \cdot 4 = 4$

---

7. The divergence of a vector field is constant div F = 5 over a region [1,3] x [1,3]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 5\, dA,\ [1,3] \times [1,3]$$

→ Flux = integral integral (div F) dA = 5 · (3-1) · (3-1).
→ = 5·2·2 = 20.

Answer: $= 5 \cdot 2 \cdot 2 = 20$

---

8. The divergence of a vector field is constant div F = 8 over a region [1,4] x [1,5]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 8\, dA,\ [1,4] \times [1,5]$$

→ Flux = integral integral (div F) dA = 8 · (4-1) · (5-1).
→ = 8·3·4 = 96.

Answer: $= 8 \cdot 3 \cdot 4 = 96$

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9. The divergence of a vector field is constant div F = 2 over a region [1,5] x [0,5]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 2\, dA,\ [1,5] \times [0,5]$$

→ Flux = integral integral (div F) dA = 2 · (5-1) · (5-0).
→ = 2·4·5 = 40.

Answer: $= 2 \cdot 4 \cdot 5 = 40$

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10. The divergence of a vector field is constant div F = 1 over a region [0,3] x [0,4]. By the Divergence Theorem (2D), find the net flux out of the region.

$$\iint_R 1\, dA,\ [0,3] \times [0,4]$$

→ Flux = integral integral (div F) dA = 1 · (3-0) · (4-0).
→ = 1·3·4 = 12.

Answer: $= 1 \cdot 3 \cdot 4 = 12$

Green's theorem (flux form)

11. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [1,3] x [1,4]. If the curl is constant = 1, find the circulation.

$$\iint_R 1\,dA,\ [1,3]\times[1,4]$$

→ By Green's theorem: circulation = integral integral (curl) dA.
→ = 1 · (3-1) · (4-1) = 1·2·3 = 6.

Answer: $= 1 \cdot 2 \cdot 3 = 6$

---

12. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [1,2] x [0,3]. If the curl is constant = 1, find the circulation.

$$\iint_R 1\,dA,\ [1,2]\times[0,3]$$

→ By Green's theorem: circulation = integral integral (curl) dA.
→ = 1 · (2-1) · (3-0) = 1·1·3 = 3.

Answer: $= 1 \cdot 1 \cdot 3 = 3$

---

13. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [1,5] x [0,4]. If the curl is constant = 1, find the circulation.

$$\iint_R 1\,dA,\ [1,5]\times[0,4]$$

→ By Green's theorem: circulation = integral integral (curl) dA.
→ = 1 · (5-1) · (4-0) = 1·4·4 = 16.

Answer: $= 1 \cdot 4 \cdot 4 = 16$

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14. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [0,5] x [1,5]. If the curl is constant = 3, find the circulation.

$$\iint_R 3\,dA,\ [0,5]\times[1,5]$$

→ By Green's theorem: circulation = integral integral (curl) dA.
→ = 3 · (5-0) · (5-1) = 3·5·4 = 60.

Answer: $= 3 \cdot 5 \cdot 4 = 60$

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15. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [0,3] x [1,2]. If the curl is constant = 1, find the circulation.

$$\iint_R 1\,dA,\ [0,3]\times[1,2]$$

→ By Green's theorem: circulation = integral integral (curl) dA.
→ = 1 · (3-0) · (2-1) = 1·3·1 = 3.

Answer: $= 1 \cdot 3 \cdot 1 = 3$

---

16. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [0,3] x [0,2]. If the curl is constant = 5, find the circulation.

$$\iint_R 5\, dA,\ [0,3]\times[0,2]$$

→ *By Green's theorem: circulation = integral integral (curl) dA.*
→ *= 5 · (3-0) · (2-0) = 5·3·2 = 30.*

Answer: $= 5 \cdot 3 \cdot 2 = 30$

---

17. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [0,3] x [1,4]. If the curl is constant = 4, find the circulation.

$$\iint_R 4\, dA,\ [0,3]\times[1,4]$$

→ *By Green's theorem: circulation = integral integral (curl) dA.*
→ *= 4 · (3-0) · (4-1) = 4·3·3 = 36.*

Answer: $= 4 \cdot 3 \cdot 3 = 36$

---

18. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [0,5] x [0,3]. If the curl is constant = 4, find the circulation.

$$\iint_R 4\, dA,\ [0,5]\times[0,3]$$

→ *By Green's theorem: circulation = integral integral (curl) dA.*
→ *= 4 · (5-0) · (3-0) = 4·5·3 = 60.*

Answer: $= 4 \cdot 5 \cdot 3 = 60$

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19. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [0,5] x [1,5]. If the curl is constant = 1, find the circulation.

$$\iint_R 1\, dA,\ [0,5]\times[1,5]$$

→ *By Green's theorem: circulation = integral integral (curl) dA.*
→ *= 1 · (5-0) · (5-1) = 1·5·4 = 20.*

Answer: $= 1 \cdot 5 \cdot 4 = 20$

---

20. By Green's theorem, the circulation integral around a closed curve equals the double integral of curl over the enclosed region R = [1,5] x [1,3]. If the curl is constant = 1, find the circulation.

$$\iint_R 1\, dA,\ [1,5]\times[1,3]$$

→ *By Green's theorem: circulation = integral integral (curl) dA.*
→ *= 1 · (5-1) · (3-1) = 1·4·2 = 8.*

Answer: $= 1 \cdot 4 \cdot 2 = 8$

Work along a path

21. A vector field F = (1, 3, 0) acts along a straight path with displacement vector d = (2, 4, 0). Find the work done W = F · d.

$$(1, 3, 0) \cdot (2, 4, 0)$$

→ *W = F · d = (1)(2) + (3)(4) + (0)(0) = 14.*

Answer: $= 14$

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22. A vector field F = (0, -4, 3) acts along a straight path with displacement vector d = (5, 4, 2). Find the work done W = F · d.

$$(0, -4, 3) \cdot (5, 4, 2)$$

→ *W = F · d = (0)(5) + (-4)(4) + (3)(2) = -10.*

Answer: $= -10$

---

23. A vector field F = (2, -1, 2) acts along a straight path with displacement vector d = (6, 2, 2). Find the work done W = F · d.

$$(2, -1, 2) \cdot (6, 2, 2)$$

→ *W = F · d = (2)(6) + (-1)(2) + (2)(2) = 14.*

Answer: $= 14$

---

24. A vector field F = (4, -2, 3) acts along a straight path with displacement vector d = (5, 6, 1). Find the work done W = F · d.

$$(4, -2, 3) \cdot (5, 6, 1)$$

→ *W = F · d = (4)(5) + (-2)(6) + (3)(1) = 11.*

Answer: $= 11$

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25. A vector field F = (3, -5, 3) acts along a straight path with displacement vector d = (6, 5, 1). Find the work done W = F · d.

$$(3, -5, 3) \cdot (6, 5, 1)$$

→ *W = F · d = (3)(6) + (-5)(5) + (3)(1) = -4.*

Answer: $= -4$

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26. A vector field F = (-2, -4, 3) acts along a straight path with displacement vector d = (1, 2, 1). Find the work done W = F · d.

$$(-2, -4, 3) \cdot (1, 2, 1)$$

→ *W = F · d = (-2)(1) + (-4)(2) + (3)(1) = -7.*

Answer: $= -7$

---

27. A vector field F = (4, -3, 0) acts along a straight path with displacement vector d = (2, 4, 1). Find the work done W = F · d.

$$(4, -3, 0) \cdot (2, 4, 1)$$

→ *W = F · d = (4)(2) + (-3)(4) + (0)(1) = -4.*

Answer: $= -4$

---

28. A vector field F = (1, -1, 2) acts along a straight path with displacement vector d = (5, 2, 1). Find the work done W = F · d.

$$(1, -1, 2) \cdot (5, 2, 1)$$

→ *W = F · d = (1)(5) + (-1)(2) + (2)(1) = 5.*

Answer: $= 5$

---

29. A vector field F = (2, 3, 0) acts along a straight path with displacement vector d = (5, 5, 0). Find the work done W = F · d.

$$(2, 3, 0) \cdot (5, 5, 0)$$

→ *W = F · d = (2)(5) + (3)(5) + (0)(0) = 25.*

Answer: $= 25$

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30. A vector field F = (0, 3, 0) acts along a straight path with displacement vector d = (3, 2, 0). Find the work done W = F · d.

$$(0, 3, 0) \cdot (3, 2, 0)$$

→ *W = F · d = (0)(3) + (3)(2) + (0)(0) = 6.*

Answer: $= 6$
