MATH230: Partial Derivatives

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Name: _____ Date: _____ Score: / 30

Learning Objectives

- Calculate the mean, median, mode, and range of a data set
- Compute sample variance and standard deviation
- Construct quartiles, the IQR, and identify outliers
- Describe the shape, center, and spread of a distribution

Simplify each expression completely. Show all steps and circle your final answer.

Partial derivatives

1. Let f(x,y) = 1x^2y^3. Find the partial derivative f_x(2,1).

$$f(x,y) = 1x^2y^3,\ (x_0, y_0) = (2, 1)$$

Answer: _____

2. The temperature at point (x,y) on a metal plate is T(x,y) = 1x^3y^1 degrees. Find the rate of change in temperature with respect to x at (2,2).

$$f(x,y) = 1x^3y^1,\ (x_0, y_0) = (2, 2)$$

Answer: _____

3. Let f(x,y) = 4x^3y^2. Find the partial derivative f_y(1,2).

$$f(x,y) = 4x^3y^2,\ (x_0, y_0) = (1, 2)$$

Answer: _____

4. Production P(K,L) = 3K^2L^3. Find the marginal product of labor: P_L(2,1).

$$f(x,y) = 3x^2y^3,\ (x_0, y_0) = (2, 1)$$

Answer: _____

5. Let f(x,y) = 1x^3y^1. Find the partial derivative f_x(3,3).

$$f(x,y) = 1x^3y^1,\ (x_0, y_0) = (3, 3)$$

Answer: _____

6. The temperature at point (x,y) on a metal plate is T(x,y) = 3x^2y^2 degrees. Find the rate of change in temperature with respect to x at (3,2).

$$f(x,y) = 3x^2y^2,\ (x_0,y_0) = (3,2)$$

Answer: _____

7. Let f(x,y) = 5x^2y^2. Find the partial derivative f_y(3,3).

$$f(x,y) = 5x^2y^2,\ (x_0,y_0) = (3,3)$$

Answer: _____

8. Production P(K,L) = 1K^2L^3. Find the marginal product of labor: P_L(1,2).

$$f(x,y) = 1x^2y^3,\ (x_0,y_0) = (1,2)$$

Answer: _____

9. Let f(x,y) = 4x^3y^2. Find the partial derivative f_x(3,1).

$$f(x,y) = 4x^3y^2,\ (x_0,y_0) = (3,1)$$

Answer: _____

10. The temperature at point (x,y) on a metal plate is T(x,y) = 2x^2y^1 degrees. Find the rate of change in temperature with respect to x at (1,3).

$$f(x,y) = 2x^2y^1,\ (x_0,y_0) = (1,3)$$

Answer: _____

11. Let f(x,y) = 3x^1y^3. Find the partial derivative f_y(2,2).

$$f(x,y) = 3x^1y^3,\ (x_0,y_0) = (2,2)$$

Answer: _____

12. Production P(K,L) = 2K^2L^3. Find the marginal product of labor: P_L(3,3).

$$f(x,y) = 2x^2y^3,\ (x_0,y_0) = (3,3)$$

Answer: _____

13. Let f(x,y) = 2x^3y^3. Find the partial derivative f_x(2,2).

$$f(x,y) = 2x^3y^3,\ (x_0,y_0) = (2,2)$$

Answer: _____

14. The temperature at point (x,y) on a metal plate is T(x,y) = 1x^2y^1 degrees. Find the rate of change in temperature with respect to x at (3,2).

$$f(x,y) = 1x^2y^1,\ (x_0,y_0) = (3,2)$$

Answer: _____

15. Let f(x,y) = 5x^1y^3. Find the partial derivative f_y(3,3).

$$f(x,y) = 5x^1y^3,\ (x_0,y_0) = (3,3)$$

Answer: _____

16. Production P(K,L) = 1K^2L^2. Find the marginal product of labor: P_L(1,3).

$$f(x,y) = 1x^2y^2,\ (x_0,y_0) = (1,3)$$

Answer: _____

17. Let f(x,y) = 5x^2y^1. Find the partial derivative f_x(3,3).

$$f(x,y) = 5x^2y^1,\ (x_0,y_0) = (3,3)$$

Answer: _____

18. The temperature at point (x,y) on a metal plate is T(x,y) = 2x^2y^2 degrees. Find the rate of change in temperature with respect to x at (1,1).

$$f(x,y) = 2x^2y^2,\ (x_0,y_0) = (1,1)$$

Answer: _____

19. Let f(x,y) = 3x^2y^2. Find the partial derivative f_y(2,1).

$$f(x,y) = 3x^2y^2,\ (x_0,y_0) = (2,1)$$

Answer: _____

20. Production P(K,L) = 4K^1L^2. Find the marginal product of labor: P_L(2,2).

$$f(x,y) = 4x^1y^2,\ (x_0,y_0) = (2,2)$$

Answer: _____

21. Let f(x,y) = 5x^2y^1. Find the partial derivative f_x(1,2).

$$f(x,y) = 5x^2y^1,\ (x_0,y_0) = (1,2)$$

Answer: _____

22. The temperature at point (x,y) on a metal plate is T(x,y) = 4x^3y^2 degrees. Find the rate of change in temperature with respect to x at (2,2).

$$f(x,y) = 4x^3y^2,\ (x_0,y_0) = (2,2)$$

Answer: _____

23. Let f(x,y) = 4x^1y^3. Find the partial derivative f_y(3,3).

$$f(x,y) = 4x^1y^3,\ (x_0,y_0) = (3,3)$$

Answer: _____

24. Production P(K,L) = 4K^2L^3. Find the marginal product of labor: P_L(3,1).

$$f(x,y) = 4x^2y^3,\ (x_0,y_0) = (3,1)$$

Answer: _____

25. Let f(x,y) = 1x^3y^2. Find the partial derivative f_x(1,2).

$$f(x,y) = 1x^3y^2,\ (x_0,y_0) = (1,2)$$

Answer: _____

26. The temperature at point (x,y) on a metal plate is T(x,y) = 1x^2y^2 degrees. Find the rate of change in temperature with respect to x at (2,2).

$$f(x,y) = 1x^2y^2,\ (x_0,y_0) = (2,2)$$

Answer: _____

27. Let f(x,y) = 4x^3y^2. Find the partial derivative f_y(3,3).

$$f(x,y) = 4x^3y^2,\ (x_0,y_0) = (3,3)$$

Answer: _____

28. Production P(K,L) = 1K^1L^2. Find the marginal product of labor: P_L(1,3).

$$f(x,y) = 1x^1y^2,\ (x_0,y_0) = (1,3)$$

Answer: _____

29. Let f(x,y) = 1x^3y^3. Find the partial derivative f_x(2,2).

$$f(x,y) = 1x^3y^3,\ (x_0,y_0) = (2,2)$$

Answer: _____

30. The temperature at point (x,y) on a metal plate is T(x,y) = 3x^2y^2 degrees. Find the rate of change in temperature with respect to x at (1,1).

$$f(x,y) = 3x^2y^2,\ (x_0, y_0) = (1,1)$$

Answer: _____

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ANSWER KEY & SOLUTIONS

Topics: Partial derivatives. All answers verified by independent computation.

Solutions

Partial derivatives

1. Let f(x,y) = 1x^2y^3. Find the partial derivative f_x(2,1).

$$f(x,y) = 1x^2y^3,\ (x_0,y_0) = (2,1)$$

→ *Differentiate with respect to x, treating y as constant.*
→ *f_x = 2x^1y^3.*
→ *f_x(2,1) = 2·2^1·1^3 = 4.*

Answer: $f_x = 2x^1y^3,\ f_x(2,1) = 4$

2. The temperature at point (x,y) on a metal plate is T(x,y) = 1x^3y^1 degrees. Find the rate of change in temperature with respect to x at (2,2).

$$f(x,y) = 1x^3y^1,\ (x_0,y_0) = (2,2)$$

→ *T_x = 3x^2y^1 (rate of change in x-direction).*
→ *T_x(2,2) = 24 degrees per unit.*

Answer: $f_x = 3x^2y^1,\ f_x(2,2) = 24$

3. Let f(x,y) = 4x^3y^2. Find the partial derivative f_y(1,2).

$$f(x,y) = 4x^3y^2,\ (x_0,y_0) = (1,2)$$

→ *Differentiate with respect to y, treating x as constant.*
→ *f_y = 8x^3y^1.*
→ *f_y(1,2) = 8·1^3·2^1 = 16.*

Answer: $f_y = 8x^3y^1,\ f_y(1,2) = 16$

4. Production P(K,L) = 3K^2L^3. Find the marginal product of labor: P_L(2,1).

$$f(x,y) = 3x^2y^3,\ (x_0,y_0) = (2,1)$$

→ *Differentiate with respect to L (treating K as constant).*
→ *P_L = 9K^2L^2.*
→ *P_L(2,1) = 36 units per unit labor.*

Answer: $f_y = 9x^2y^2,\ f_y(2,1) = 36$

5. Let f(x,y) = 1x^3y^1. Find the partial derivative f_x(3,3).

$$f(x,y) = 1x^3y^1,\ (x_0,y_0) = (3,3)$$

→ *Differentiate with respect to x, treating y as constant.*
→ *f_x = 3x^2y^1.*
→ *f_x(3,3) = 3·3^2·3^1 = 81.*

Answer: $f_x = 3x^2y^1,\ f_x(3,3) = 81$

6. The temperature at point (x,y) on a metal plate is T(x,y) = 3x^2y^2 degrees. Find the rate of change in temperature with respect to x at (3,2).

$$f(x,y) = 3x^2y^2,\ (x_0,y_0) = (3,2)$$

→ *T_x = 6x^1y^2 (rate of change in x-direction).*
→ *T_x(3,2) = 72 degrees per unit.*

Answer: $f_x = 6x^1y^2,\ f_x(3,2) = 72$

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7. Let f(x,y) = 5x^2y^2. Find the partial derivative f_y(3,3).

$$f(x,y) = 5x^2y^2,\ (x_0,y_0) = (3,3)$$

→ *Differentiate with respect to y, treating x as constant.*
→ *f_y = 10x^2y^1.*
→ *f_y(3,3) = 10·3^2·3^1 = 270.*

Answer: $f_y = 10x^2y^1,\ f_y(3,3) = 270$

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8. Production P(K,L) = 1K^2L^3. Find the marginal product of labor: P_L(1,2).

$$f(x,y) = 1x^2y^3,\ (x_0,y_0) = (1,2)$$

→ *Differentiate with respect to L (treating K as constant).*
→ *P_L = 3K^2L^2.*
→ *P_L(1,2) = 12 units per unit labor.*

Answer: $f_y = 3x^2y^2,\ f_y(1,2) = 12$

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9. Let f(x,y) = 4x^3y^2. Find the partial derivative f_x(3,1).

$$f(x,y) = 4x^3y^2,\ (x_0,y_0) = (3,1)$$

→ *Differentiate with respect to x, treating y as constant.*
→ *f_x = 12x^2y^2.*
→ *f_x(3,1) = 12·3^2·1^2 = 108.*

Answer: $f_x = 12x^2y^2,\ f_x(3,1) = 108$

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10. The temperature at point (x,y) on a metal plate is T(x,y) = 2x^2y^1 degrees. Find the rate of change in temperature with respect to x at (1,3).

$$f(x,y) = 2x^2y^1,\ (x_0,y_0) = (1,3)$$

→ *T_x = 4x^1y^1 (rate of change in x-direction).*
→ *T_x(1,3) = 12 degrees per unit.*

Answer: $f_x = 4x^1y^1,\ f_x(1,3) = 12$

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11. Let f(x,y) = 3x^1y^3. Find the partial derivative f_y(2,2).

$$f(x,y) = 3x^1y^3,\ (x_0,y_0) = (2,2)$$

→ *Differentiate with respect to y, treating x as constant.*
→ *f_y = 9x^1y^2.*
→ *f_y(2,2) = 9·2^1·2^2 = 72.*

Answer: $f_y = 9x^1y^2,\ f_y(2,2) = 72$

---

12. Production P(K,L) = 2K^2L^3. Find the marginal product of labor: P_L(3,3).

$$f(x,y) = 2x^2y^3,\ (x_0,y_0) = (3,3)$$

→ *Differentiate with respect to L (treating K as constant).*
→ *P_L = 6K^2L^2.*
→ *P_L(3,3) = 486 units per unit labor.*

Answer: $f_y = 6x^2y^2,\ f_y(3,3) = 486$

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13. Let f(x,y) = 2x^3y^3. Find the partial derivative f_x(2,2).

$$f(x,y) = 2x^3y^3,\ (x_0,y_0) = (2,2)$$

→ *Differentiate with respect to x, treating y as constant.*
→ *f_x = 6x^2y^3.*
→ *f_x(2,2) = 6·2^2·2^3 = 192.*

Answer: $f_x = 6x^2y^3,\ f_x(2,2) = 192$

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14. The temperature at point (x,y) on a metal plate is T(x,y) = 1x^2y^1 degrees. Find the rate of change in temperature with respect to x at (3,2).

$$f(x,y) = 1x^2y^1,\ (x_0,y_0) = (3,2)$$

→ *T_x = 2x^1y^1 (rate of change in x-direction).*
→ *T_x(3,2) = 12 degrees per unit.*

Answer: $f_x = 2x^1y^1,\ f_x(3,2) = 12$

---

15. Let f(x,y) = 5x^1y^3. Find the partial derivative f_y(3,3).

$$f(x,y) = 5x^1y^3,\ (x_0,y_0) = (3,3)$$

→ *Differentiate with respect to y, treating x as constant.*
→ *f_y = 15x^1y^2.*
→ *f_y(3,3) = 15·3^1·3^2 = 405.*

Answer: $f_y = 15x^1y^2,\ f_y(3,3) = 405$

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16. Production P(K,L) = 1K^2L^2. Find the marginal product of labor: P_L(1,3).

$$f(x,y) = 1x^2y^2,\ (x_0,y_0) = (1,3)$$

→ *Differentiate with respect to L (treating K as constant).*
→ *P_L = 2K^2L^1.*
→ *P_L(1,3) = 6 units per unit labor.*

Answer: $f_y = 2x^2y^1,\ f_y(1,3) = 6$

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17. Let f(x,y) = 5x^2y^1. Find the partial derivative f_x(3,3).

$$f(x,y) = 5x^2y^1,\ (x_0,y_0) = (3,3)$$

→ *Differentiate with respect to x, treating y as constant.*
→ *f_x = 10x^1y^1.*
→ *f_x(3,3) = 10·3^1·3^1 = 90.*

Answer: $f_x = 10x^1y^1,\ f_x(3,3) = 90$

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18. The temperature at point (x,y) on a metal plate is T(x,y) = 2x^2y^2 degrees. Find the rate of change in temperature with respect to x at (1,1).

$$f(x,y) = 2x^2y^2,\ (x_0,y_0) = (1,1)$$

→ *T_x = 4x^1y^2 (rate of change in x-direction).*
→ *T_x(1,1) = 4 degrees per unit.*

Answer: $f_x = 4x^1y^2,\ f_x(1,1) = 4$

---

19. Let f(x,y) = 3x^2y^2. Find the partial derivative f_y(2,1).

$$f(x,y) = 3x^2y^2,\ (x_0,y_0) = (2,1)$$

→ *Differentiate with respect to y, treating x as constant.*
→ *f_y = 6x^2y^1.*
→ *f_y(2,1) = 6·2^2·1^1 = 24.*

Answer: $f_y = 6x^2y^1,\ f_y(2,1) = 24$

---

20. Production P(K,L) = 4K^1L^2. Find the marginal product of labor: P_L(2,2).

$$f(x,y) = 4x^1y^2,\ (x_0,y_0) = (2,2)$$

→ *Differentiate with respect to L (treating K as constant).*
→ *P_L = 8K^1L^1.*
→ *P_L(2,2) = 32 units per unit labor.*

Answer: $f_y = 8x^1y^1,\ f_y(2,2) = 32$

---

21. Let f(x,y) = 5x^2y^1. Find the partial derivative f_x(1,2).

$$f(x,y) = 5x^2y^1,\ (x_0,y_0) = (1,2)$$

→ *Differentiate with respect to x, treating y as constant.*
→ *f_x = 10x^1y^1.*
→ *f_x(1,2) = 10·1^1·2^1 = 20.*

Answer: $f_x = 10x^1y^1,\ f_x(1,2) = 20$

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22. The temperature at point (x,y) on a metal plate is T(x,y) = 4x^3y^2 degrees. Find the rate of change in temperature with respect to x at (2,2).

$$f(x,y) = 4x^3y^2,\ (x_0,y_0) = (2,2)$$

→ *T_x = 12x^2y^2 (rate of change in x-direction).*
→ *T_x(2,2) = 192 degrees per unit.*

Answer: $f_x = 12x^2y^2,\ f_x(2,2) = 192$

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23. Let f(x,y) = 4x^1y^3. Find the partial derivative f_y(3,3).

$$f(x,y) = 4x^1y^3,\ (x_0,y_0) = (3,3)$$

→ *Differentiate with respect to y, treating x as constant.*
→ *f_y = 12x^1y^2.*
→ *f_y(3,3) = 12·3^1·3^2 = 324.*

Answer: $f_y = 12x^1y^2,\ f_y(3,3) = 324$

---

24. Production P(K,L) = 4K^2L^3. Find the marginal product of labor: P_L(3,1).

$$f(x,y) = 4x^2y^3,\ (x_0,y_0) = (3,1)$$

→ *Differentiate with respect to L (treating K as constant).*
→ *P_L = 12K^2L^2.*
→ *P_L(3,1) = 108 units per unit labor.*

Answer: $f_y = 12x^2y^2,\ f_y(3,1) = 108$

---

25. Let f(x,y) = 1x^3y^2. Find the partial derivative f_x(1,2).

$$f(x,y) = 1x^3y^2,\ (x_0,y_0) = (1,2)$$

→ *Differentiate with respect to x, treating y as constant.*
→ *f_x = 3x^2y^2.*
→ *f_x(1,2) = 3·1^2·2^2 = 12.*

Answer: $f_x = 3x^2y^2,\ f_x(1,2) = 12$

---

26. The temperature at point (x,y) on a metal plate is T(x,y) = 1x^2y^2 degrees. Find the rate of change in temperature with respect to x at (2,2).

$$f(x,y) = 1x^2y^2,\ (x_0,y_0) = (2,2)$$

→ *T_x = 2x^1y^2 (rate of change in x-direction).*
→ *T_x(2,2) = 16 degrees per unit.*

Answer: $f_x = 2x^1y^2,\ f_x(2,2) = 16$

---

27. Let f(x,y) = 4x^3y^2. Find the partial derivative f_y(3,3).

$$f(x,y) = 4x^3y^2,\ (x_0,y_0) = (3,3)$$

→ *Differentiate with respect to y, treating x as constant.*
→ *f_y = 8x^3y^1.*
→ *f_y(3,3) = 8·3^3·3^1 = 648.*

Answer: $f_y = 8x^3y^1,\ f_y(3,3) = 648$

---

28. Production P(K,L) = 1K^1L^2. Find the marginal product of labor: P_L(1,3).

$$f(x,y) = 1x^1y^2,\ (x_0,y_0) = (1,3)$$

→ *Differentiate with respect to L (treating K as constant).*
→ *P_L = 2K^1L^1.*
→ *P_L(1,3) = 6 units per unit labor.*

Answer: $f_y = 2x^1y^1,\ f_y(1,3) = 6$

---

29. Let f(x,y) = 1x^3y^3. Find the partial derivative f_x(2,2).

$$f(x,y) = 1x^3y^3,\ (x_0,y_0) = (2,2)$$

→ *Differentiate with respect to x, treating y as constant.*
→ *f_x = 3x^2y^3.*
→ *f_x(2,2) = 3·2^2·2^3 = 96.*

Answer: $f_x = 3x^2y^3,\ f_x(2,2) = 96$

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30. The temperature at point (x,y) on a metal plate is T(x,y) = 3x^2y^2 degrees. Find the rate of change in temperature with respect to x at (1,1).

$$f(x,y) = 3x^2y^2,\ (x_0,y_0) = (1,1)$$

→ *T_x = 6x^1y^2 (rate of change in x-direction).*
→ *T_x(1,1) = 6 degrees per unit.*

Answer: $f_x = 6x^1y^2,\ f_x(1,1) = 6$
