MATH240: Applications Growth Decay

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Learning Objectives

- Calculate the mean, median, mode, and range of a data set
- Compute sample variance and standard deviation
- Construct quartiles, the IQR, and identify outliers
- Describe the shape, center, and spread of a distribution

Simplify each expression completely. Show all steps and circle your final answer.

Exponential growth and decay

1. Newton's Law of Cooling: T'(t) = -4(T(t) - 20), where 20 is the ambient temperature. If the object is currently T = 20 + 27 degrees above ambient (so T = 20 + 27), find T'(t).

$$f(x) = -4x + 20, \quad x = 27$$

Answer: _____

2. A population satisfies y' = 1y. If the initial population is y(0) = 46, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 1x + 46, \quad x = 0$$

Answer: _____

3. A radioactive substance decays. The amount Q(t) = -2t^2 + 183 grams at time t seconds. Find Q(2).

$$-2x^2 + 183, \; x = 2$$

Answer: _____

4. A tank contains Q grams of salt. The rate of change is Q'(t) = -2Q + 8. Find Q'(t) when Q = 20 grams.

$$f(x) = -2x + 8, \quad x = 20$$

Answer: _____

5. Newton's Law of Cooling: T'(t) = -4(T(t) - 23), where 23 is the ambient temperature. If the object is currently T = 23 + 12 degrees above ambient (so T = 23 + 12), find T'(t).

$$f(x) = -4x + 23, \quad x = 12$$

Answer: _____

6. A population satisfies y' = 3y. If the initial population is y(0) = 18, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 3x + 18, \quad x = 0$$

Answer: _____

7. A radioactive substance decays. The amount Q(t) = -1t^2 + 130 grams at time t seconds. Find Q(3).

$$-1x^2 + 130, \; x = 3$$

Answer: _____

8. A tank contains Q grams of salt. The rate of change is Q'(t) = -3Q + 7. Find Q'(t) when Q = 18 grams.

$$f(x) = -3x + 7, \quad x = 18$$

Answer: _____

9. Newton's Law of Cooling: T'(t) = -1(T(t) - 22), where 22 is the ambient temperature. If the object is currently T = 22 + 33 degrees above ambient (so T = 22 + 33), find T'(t).

$$f(x) = -1x + 22, \quad x = 33$$

Answer: _____

10. A population satisfies y' = 2y. If the initial population is y(0) = 74, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 2x + 74, \quad x = 0$$

Answer: _____

11. A radioactive substance decays. The amount Q(t) = -3t^2 + 63 grams at time t seconds. Find Q(5).

$$-3x^2 + 63, \; x = 5$$

Answer: _____

12. A tank contains Q grams of salt. The rate of change is Q'(t) = -1Q + 15. Find Q'(t) when Q = 11 grams.

$$f(x) = -1x + 15, \quad x = 11$$

Answer: _____

13. Newton's Law of Cooling: T'(t) = -3(T(t) - 23), where 23 is the ambient temperature. If the object is currently T = 23 + 45 degrees above ambient (so T = 23 + 45), find T'(t).

$$f(x) = -3x + 23, \quad x = 45$$

Answer: _____

14. A population satisfies y' = 1y. If the initial population is y(0) = 35, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 1x + 35, \quad x = 0$$

Answer: _____

15. A radioactive substance decays. The amount Q(t) = -1t^2 + 52 grams at time t seconds. Find Q(5).

$$-1x^2 + 52, \; x = 5$$

Answer: _____

16. A tank contains Q grams of salt. The rate of change is Q'(t) = -3Q + 7. Find Q'(t) when Q = 5 grams.

$$f(x) = -3x + 7, \quad x = 5$$

Answer: _____

17. Newton's Law of Cooling: T'(t) = -3(T(t) - 21), where 21 is the ambient temperature. If the object is currently T = 21 + 22 degrees above ambient (so T = 21 + 22), find T'(t).

$$f(x) = -3x + 21, \quad x = 22$$

Answer: _____

18. A population satisfies y' = 2y. If the initial population is y(0) = 20, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 2x + 20, \quad x = 0$$

Answer: _____

19. A radioactive substance decays. The amount Q(t) = -3t^2 + 122 grams at time t seconds. Find Q(3).

$$-3x^2 + 122, \; x = 3$$

Answer: _____

20. A tank contains Q grams of salt. The rate of change is Q'(t) = -2Q + 4. Find Q'(t) when Q = 11 grams.

$$f(x) = -2x + 4, \quad x = 11$$

Answer: _____

21. Newton's Law of Cooling: T'(t) = -3(T(t) - 20), where 20 is the ambient temperature. If the object is currently T = 20 + 25 degrees above ambient (so T = 20 + 25), find T'(t).

$$f(x) = -3x + 20, \quad x = 25$$

Answer: _____

22. A population satisfies y' = 4y. If the initial population is y(0) = 66, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 4x + 66, \quad x = 0$$

Answer: _____

23. A radioactive substance decays. The amount Q(t) = -2t^2 + 81 grams at time t seconds. Find Q(5).

$$-2x^2 + 81, \; x = 5$$

Answer: _____

24. A tank contains Q grams of salt. The rate of change is Q'(t) = -2Q + 6. Find Q'(t) when Q = 14 grams.

$$f(x) = -2x + 6, \quad x = 14$$

Answer: _____

25. Newton's Law of Cooling: T'(t) = -4(T(t) - 22), where 22 is the ambient temperature. If the object is currently T = 22 + 37 degrees above ambient (so T = 22 + 37), find T'(t).

$$f(x) = -4x + 22, \quad x = 37$$

Answer: _____

26. A population satisfies y' = 1y. If the initial population is y(0) = 24, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 1x + 24, \quad x = 0$$

Answer: _____

27. A radioactive substance decays. The amount Q(t) = -2t^2 + 178 grams at time t seconds. Find Q(2).

$$-2x^2 + 178, \; x = 2$$

Answer: _____

28. A tank contains Q grams of salt. The rate of change is Q'(t) = -1Q + 2. Find Q'(t) when Q = 8 grams.

$$f(x) = -1x + 2, \quad x = 8$$

Answer: _____

29. Newton's Law of Cooling: T'(t) = -4(T(t) - 22), where 22 is the ambient temperature. If the object is currently T = 22 + 39 degrees above ambient (so T = 22 + 39), find T'(t).

$$f(x) = -4x + 22, \quad x = 39$$

Answer: _____

30. A population satisfies y' = 3y. If the initial population is y(0) = 75, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 3x + 75, \quad x = 0$$

Answer: _____

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ANSWER KEY & SOLUTIONS

Topics: Exponential growth and decay. All answers verified by independent computation.

Solutions

Exponential growth and decay

1. Newton's Law of Cooling: T'(t) = -4(T(t) - 20), where 20 is the ambient temperature. If the object is currently T = 20 + 27 degrees above ambient (so T = 20 + 27), find T'(t).

$$f(x) = -4x + 20, \quad x = 27$$

→ *T'(t) = -4·(T - 20) = -4·27 = -88 degrees/sec.*
→ *Negative rate means the object is cooling toward ambient.*

Answer: $f(27) = -88$

2. A population satisfies y' = 1y. If the initial population is y(0) = 46, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 1x + 46, \quad x = 0$$

→ *y'(t) = 1y(t).*
→ *At t=0: y'(0) = 1·y(0) = 1·46 = 46.*

Answer: $f(0) = 46$

3. A radioactive substance decays. The amount Q(t) = -2t^2 + 183 grams at time t seconds. Find Q(2).

$$-2x^2 + 183, \; x = 2$$

→ *Q(2) = -2(2)^2 + 183 = -8 + 183 = 175 grams.*

Answer: $-2(2)^2 + 183 = -8 + 183 = 175$

4. A tank contains Q grams of salt. The rate of change is Q'(t) = -2Q + 8. Find Q'(t) when Q = 20 grams.

$$f(x) = -2x + 8, \quad x = 20$$

→ *Substitute Q = 20: Q'(20) = -2·20 + 8 = -40 + 8 = -32 g/min.*

Answer: $f(20) = -32$

5. Newton's Law of Cooling: T'(t) = -4(T(t) - 23), where 23 is the ambient temperature. If the object is currently T = 23 + 12 degrees above ambient (so T = 23 + 12), find T'(t).

$$f(x) = -4x + 23, \quad x = 12$$

→ *T'(t) = -4·(T - 23) = -4·12 = -25 degrees/sec.*
→ *Negative rate means the object is cooling toward ambient.*

Answer: $f(12) = -25$

6. A population satisfies y' = 3y. If the initial population is y(0) = 18, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 3x + 18, \quad x = 0$$

→ *y'(t) = 3y(t).*
→ *At t=0: y'(0) = 3·y(0) = 3·18 = 18.*

Answer: $f(0) = 18$

7. A radioactive substance decays. The amount Q(t) = -1t^2 + 130 grams at time t seconds. Find Q(3).

$$-1x^2 + 130,\ x = 3$$

→ *Q(3) = -1(3)^2 + 130 = -9 + 130 = 121 grams.*

Answer: $-1(3)^2 + 130 = -9 + 130 = 121$

8. A tank contains Q grams of salt. The rate of change is Q'(t) = -3Q + 7. Find Q'(t) when Q = 18 grams.

$$f(x) = -3x + 7, \quad x = 18$$

→ *Substitute Q = 18: Q'(18) = -3·18 + 7 = -54 + 7 = -47 g/min.*

Answer: $f(18) = -47$

9. Newton's Law of Cooling: T'(t) = -1(T(t) - 22), where 22 is the ambient temperature. If the object is currently T = 22 + 33 degrees above ambient (so T = 22 + 33), find T'(t).

$$f(x) = -1x + 22, \quad x = 33$$

→ *T'(t) = -1·(T - 22) = -1·33 = -11 degrees/sec.*
→ *Negative rate means the object is cooling toward ambient.*

Answer: $f(33) = -11$

10. A population satisfies y' = 2y. If the initial population is y(0) = 74, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 2x + 74, \quad x = 0$$

→ *y'(t) = 2y(t).*
→ *At t=0: y'(0) = 2·y(0) = 2·74 = 74.*

Answer: $f(0) = 74$

11. A radioactive substance decays. The amount Q(t) = -3t^2 + 63 grams at time t seconds. Find Q(5).

$$-3x^2 + 63,\ x = 5$$

→ *Q(5) = -3(5)^2 + 63 = -75 + 63 = -12 grams.*

Answer: $-3(5)^2 + 63 = -75 + 63 = -12$

12. A tank contains Q grams of salt. The rate of change is Q'(t) = -1Q + 15. Find Q'(t) when Q = 11 grams.

$$f(x) = -1x + 15, \quad x = 11$$

→ *Substitute Q = 11: Q'(11) = -1·11 + 15 = -11 + 15 = 4 g/min.*

Answer: $f(11) = 4$

13. Newton's Law of Cooling: T'(t) = -3(T(t) - 23), where 23 is the ambient temperature. If the object is currently T = 23 + 45 degrees above ambient (so T = 23 + 45), find T'(t).

$$f(x) = -3x + 23, \quad x = 45$$

→ *T'(t) = -3·(T - 23) = -3·45 = -112 degrees/sec.*
→ *Negative rate means the object is cooling toward ambient.*

Answer: $f(45) = -112$

14. A population satisfies y' = 1y. If the initial population is y(0) = 35, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 1x + 35, \quad x = 0$$

→ *y'(t) = 1y(t).*
→ *At t=0: y'(0) = 1·y(0) = 1·35 = 35.*

Answer: $f(0) = 35$

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15. A radioactive substance decays. The amount Q(t) = -1t^2 + 52 grams at time t seconds. Find Q(5).

$$-1x^2 + 52, \; x = 5$$

→ *Q(5) = -1(5)^2 + 52 = -25 + 52 = 27 grams.*

Answer: $-1(5)^2 + 52 = -25 + 52 = 27$

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16. A tank contains Q grams of salt. The rate of change is Q'(t) = -3Q + 7. Find Q'(t) when Q = 5 grams.

$$f(x) = -3x + 7, \quad x = 5$$

→ *Substitute Q = 5: Q'(5) = -3·5 + 7 = -15 + 7 = -8 g/min.*

Answer: $f(5) = -8$

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17. Newton's Law of Cooling: T'(t) = -3(T(t) - 21), where 21 is the ambient temperature. If the object is currently T = 21 + 22 degrees above ambient (so T = 21 + 22), find T'(t).

$$f(x) = -3x + 21, \quad x = 22$$

→ *T'(t) = -3·(T - 21) = -3·22 = -45 degrees/sec.*
→ *Negative rate means the object is cooling toward ambient.*

Answer: $f(22) = -45$

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18. A population satisfies y' = 2y. If the initial population is y(0) = 20, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 2x + 20, \quad x = 0$$

→ *y'(t) = 2y(t).*
→ *At t=0: y'(0) = 2·y(0) = 2·20 = 20.*

Answer: $f(0) = 20$

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19. A radioactive substance decays. The amount Q(t) = -3t^2 + 122 grams at time t seconds. Find Q(3).

$$-3x^2 + 122, \; x = 3$$

→ *Q(3) = -3(3)^2 + 122 = -27 + 122 = 95 grams.*

Answer: $-3(3)^2 + 122 = -27 + 122 = 95$

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20. A tank contains Q grams of salt. The rate of change is Q'(t) = -2Q + 4. Find Q'(t) when Q = 11 grams.

$$f(x) = -2x + 4, \quad x = 11$$

→ *Substitute Q = 11: Q'(11) = -2·11 + 4 = -22 + 4 = -18 g/min.*

Answer: $f(11) = -18$

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21. Newton's Law of Cooling: T'(t) = -3(T(t) - 20), where 20 is the ambient temperature. If the object is currently T = 20 + 25 degrees above ambient (so T = 20 + 25), find T'(t).

$$f(x) = -3x + 20, \quad x = 25$$

→ *T'(t) = -3·(T - 20) = -3·25 = -55 degrees/sec.*
→ *Negative rate means the object is cooling toward ambient.*

Answer: $f(25) = -55$

22. A population satisfies y' = 4y. If the initial population is y(0) = 66, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 4x + 66, \quad x = 0$$

→ *y'(t) = 4y(t).*
→ *At t=0: y'(0) = 4·y(0) = 4·66 = 66.*

Answer: $f(0) = 66$

23. A radioactive substance decays. The amount Q(t) = -2t^2 + 81 grams at time t seconds. Find Q(5).

$$-2x^2 + 81, \; x = 5$$

→ *Q(5) = -2(5)^2 + 81 = -50 + 81 = 31 grams.*

Answer: $-2(5)^2 + 81 = -50 + 81 = 31$

24. A tank contains Q grams of salt. The rate of change is Q'(t) = -2Q + 6. Find Q'(t) when Q = 14 grams.

$$f(x) = -2x + 6, \quad x = 14$$

→ *Substitute Q = 14: Q'(14) = -2·14 + 6 = -28 + 6 = -22 g/min.*

Answer: $f(14) = -22$

25. Newton's Law of Cooling: T'(t) = -4(T(t) - 22), where 22 is the ambient temperature. If the object is currently T = 22 + 37 degrees above ambient (so T = 22 + 37), find T'(t).

$$f(x) = -4x + 22, \quad x = 37$$

→ *T'(t) = -4·(T - 22) = -4·37 = -126 degrees/sec.*
→ *Negative rate means the object is cooling toward ambient.*

Answer: $f(37) = -126$

26. A population satisfies y' = 1y. If the initial population is y(0) = 24, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 1x + 24, \quad x = 0$$

→ *y'(t) = 1y(t).*
→ *At t=0: y'(0) = 1·y(0) = 1·24 = 24.*

Answer: $f(0) = 24$

27. A radioactive substance decays. The amount Q(t) = -2t^2 + 178 grams at time t seconds. Find Q(2).

$$-2x^2 + 178, \; x = 2$$

→ *Q(2) = -2(2)^2 + 178 = -8 + 178 = 170 grams.*

Answer: $-2(2)^2 + 178 = -8 + 178 = 170$

28. A tank contains Q grams of salt. The rate of change is Q'(t) = -1Q + 2. Find Q'(t) when Q = 8 grams.

$$f(x) = -1x + 2, \quad x = 8$$

→ *Substitute Q = 8: Q'(8) = -1·8 + 2 = -8 + 2 = -6 g/min.*

Answer: $f(8) = -6$

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29. Newton's Law of Cooling: T'(t) = -4(T(t) - 22), where 22 is the ambient temperature. If the object is currently T = 22 + 39 degrees above ambient (so T = 22 + 39), find T'(t).

$$f(x) = -4x + 22, \quad x = 39$$

→ *T'(t) = -4·(T - 22) = -4·39 = -134 degrees/sec.*

→ *Negative rate means the object is cooling toward ambient.*

Answer: $f(39) = -134$

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30. A population satisfies y' = 3y. If the initial population is y(0) = 75, find the initial rate of growth dy/dt at t = 0.

$$f(x) = 3x + 75, \quad x = 0$$

→ *y'(t) = 3y(t).*

→ *At t=0: y'(0) = 3·y(0) = 3·75 = 75.*

Answer: $f(0) = 75$
