MATH240: Higher Order Odes

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Name: _____ Date: _____ Score: / 30

Learning Objectives

- Calculate the mean, median, mode, and range of a data set
- Compute sample variance and standard deviation
- Construct quartiles, the IQR, and identify outliers
- Describe the shape, center, and spread of a distribution

Simplify each expression completely. Show all steps and circle your final answer.

Characteristic equation

1. Find the roots of the characteristic equation for y'' + 4y' + -5y = 0.

$$x^2 + 4x - 5 = 0$$

Answer: _____

2. The motion of a spring-mass system satisfies y'' + 12y' + 32y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 12x + 32 = 0$$

Answer: _____

3. Find the characteristic roots of y'' + 4y' + 3y = 0. Write the general solution.

$$x^2 + 4x + 3 = 0$$

Answer: _____

4. Solve the characteristic equation: r^2 + 2r + 0 = 0.

$$x^2 + 2x = 0$$

Answer: _____

5. Find the roots of the characteristic equation for y'' + 1y' + -20y = 0.

$$x^2 + 1x - 20 = 0$$

Answer: _____

6. The motion of a spring-mass system satisfies y'' + 10y' + 21y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 10x + 21 = 0$$

Answer: _____

7. Find the characteristic roots of y'' + 8y' + 12y = 0. Write the general solution.

$$x^2 + 8x + 12 = 0$$

Answer: _____

8. Solve the characteristic equation: r^2 + 6r + 5 = 0.

$$x^2 + 6x + 5 = 0$$

Answer: _____

9. Find the roots of the characteristic equation for y'' + -1y' + -6y = 0.

$$x^2 - 1x - 6 = 0$$

Answer: _____

10. The motion of a spring-mass system satisfies y'' + 11y' + 30y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 11x + 30 = 0$$

Answer: _____

11. Find the characteristic roots of y'' + 10y' + 21y = 0. Write the general solution.

$$x^2 + 10x + 21 = 0$$

Answer: _____

12. Solve the characteristic equation: r^2 + 0r + -9 = 0.

$$x^2x - 9 = 0$$

Answer: _____

13. Find the roots of the characteristic equation for y'' + 0y' + -16y = 0.

$$x^2x - 16 = 0$$

Answer: _____

14. The motion of a spring-mass system satisfies y'' + 12y' + 35y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 12x + 35 = 0$$

Answer: _____

15. Find the characteristic roots of y'' + 11y' + 28y = 0. Write the general solution.

$$x^2 + 11x + 28 = 0$$

Answer: _____

16. Solve the characteristic equation: r^2 + 7r + 6 = 0.

$$x^2 + 7x + 6 = 0$$

Answer: _____

17. Find the roots of the characteristic equation for y'' + -1y' + -2y = 0.

$$x^2 - 1x - 2 = 0$$

Answer: _____

18. The motion of a spring-mass system satisfies y'' + 12y' + 35y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 12x + 35 = 0$$

Answer: _____

19. Find the characteristic roots of y'' + 8y' + 15y = 0. Write the general solution.

$$x^2 + 8x + 15 = 0$$

Answer: _____

20. Solve the characteristic equation: r^2 + 3r + -4 = 0.

$$x^2 + 3x - 4 = 0$$

Answer: _____

21. Find the roots of the characteristic equation for y'' + -1y' + -2y = 0.

$$x^2 - 1x - 2 = 0$$

Answer: _____

22. The motion of a spring-mass system satisfies y'' + 2y' + 1y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 2x + 1 = 0$$

Answer: _____

23. Find the characteristic roots of y'' + 7y' + 0y = 0. Write the general solution.

$$x^2 + 7x = 0$$

Answer: _____

24. Solve the characteristic equation: r^2 + 2r + 0 = 0.

$$x^2 + 2x = 0$$

Answer: _____

25. Find the roots of the characteristic equation for y'' + 2y' + -15y = 0.

$$x^2 + 2x - 15 = 0$$

Answer: _____

26. The motion of a spring-mass system satisfies y'' + 14y' + 49y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 14x + 49 = 0$$

Answer: _____

27. Find the characteristic roots of y'' + 2y' + 0y = 0. Write the general solution.

$$x^2 + 2x = 0$$

Answer: _____

28. Solve the characteristic equation: r^2 + 2r + -24 = 0.

$$x^2 + 2x - 24 = 0$$

Answer: _____

29. Find the roots of the characteristic equation for y'' + 2y' + -15y = 0.

$$x^2 + 2x - 15 = 0$$

Answer: _____

30. The motion of a spring-mass system satisfies y'' + 11y' + 24y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 11x + 24 = 0$$

Answer: _____

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ANSWER KEY & SOLUTIONS

Topics: Characteristic equation. All answers verified by independent computation.

Solutions

Characteristic equation

1. Find the roots of the characteristic equation for y'' + 4y' + -5y = 0.

$$x^2 + 4x - 5 = 0$$

→ *Characteristic equation: r^2 + 4r + -5 = 0.*
→ *Factor or use quadratic formula.*
→ *Roots: r = -5, r = 1.*
→ *General solution: y = C1·e^(-5t) + C2·e^(1t).*

Answer: $x = -5$ or $x = 1$

2. The motion of a spring-mass system satisfies y'' + 12y' + 32y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 12x + 32 = 0$$

→ *Characteristic equation: r^2 + 12r + 32 = 0.*
→ *Roots: r1 = -8, r2 = -4.*
→ *Both roots real and negative: overdamped system.*

Answer: $x = -8$ or $x = -4$

3. Find the characteristic roots of y'' + 4y' + 3y = 0. Write the general solution.

$$x^2 + 4x + 3 = 0$$

→ *r^2 + 4r + 3 = 0.*
→ *r1 = -3, r2 = -1.*
→ *General solution: y = C1*e^(-3t) + C2*e^(-1t).*

Answer: $x = -3$ or $x = -1$

4. Solve the characteristic equation: r^2 + 2r + 0 = 0.

$$x^2 + 2x = 0$$

→ *Factor: (r - -2)(r - 0) = 0.*
→ *Roots: r = -2 and r = 0.*

Answer: $x = -2$ or $x = 0$

5. Find the roots of the characteristic equation for y'' + 1y' + -20y = 0.

$$x^2 + 1x - 20 = 0$$

→ *Characteristic equation: r^2 + 1r + -20 = 0.*
→ *Factor or use quadratic formula.*
→ *Roots: r = -5, r = 4.*
→ *General solution: y = C1·e^(-5t) + C2·e^(4t).*

Answer: $x = -5$ or $x = 4$

6. The motion of a spring-mass system satisfies y'' + 10y' + 21y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 10x + 21 = 0$$

→ *Characteristic equation: r^2 + 10r + 21 = 0.*
→ *Roots: r1 = -7, r2 = -3.*
→ *Both roots real and negative: overdamped system.*

Answer: $x = -7$ or $x = -3$

---

7. Find the characteristic roots of y'' + 8y' + 12y = 0. Write the general solution.

$$x^2 + 8x + 12 = 0$$

→ *r^2 + 8r + 12 = 0.*
→ *r1 = -6, r2 = -2.*
→ *General solution: y = C1*e^(-6t) + C2*e^(-2t).*

Answer: $x = -6$ or $x = -2$

---

8. Solve the characteristic equation: r^2 + 6r + 5 = 0.

$$x^2 + 6x + 5 = 0$$

→ *Factor: (r - -5)(r - -1) = 0.*
→ *Roots: r = -5 and r = -1.*

Answer: $x = -5$ or $x = -1$

---

9. Find the roots of the characteristic equation for y'' + -1y' + -6y = 0.

$$x^2 - 1x - 6 = 0$$

→ *Characteristic equation: r^2 + -1r + -6 = 0.*
→ *Factor or use quadratic formula.*
→ *Roots: r = -2, r = 3.*
→ *General solution: y = C1·e^(-2t) + C2·e^(3t).*

Answer: $x = -2$ or $x = 3$

---

10. The motion of a spring-mass system satisfies y'' + 11y' + 30y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 11x + 30 = 0$$

→ *Characteristic equation: r^2 + 11r + 30 = 0.*
→ *Roots: r1 = -6, r2 = -5.*
→ *Both roots real and negative: overdamped system.*

Answer: $x = -6$ or $x = -5$

---

11. Find the characteristic roots of y'' + 10y' + 21y = 0. Write the general solution.

$$x^2 + 10x + 21 = 0$$

→ *r^2 + 10r + 21 = 0.*
→ *r1 = -7, r2 = -3.*
→ *General solution: y = C1*e^(-7t) + C2*e^(-3t).*

Answer: $x = -7$ or $x = -3$

---

12. Solve the characteristic equation: r^2 + 0r + -9 = 0.

$$x^2x - 9 = 0$$

→ *Factor: (r - -3)(r - 3) = 0.*
→ *Roots: r = -3 and r = 3.*

Answer: $x = -3$ or $x = 3$

---

13. Find the roots of the characteristic equation for y'' + 0y' + -16y = 0.

$$x^2x - 16 = 0$$

→ *Characteristic equation: r^2 + 0r + -16 = 0.*
→ *Factor or use quadratic formula.*
→ *Roots: r = -4, r = 4.*
→ *General solution: y = C1·e^(-4t) + C2·e^(4t).*

Answer: $x = -4$ or $x = 4$

---

14. The motion of a spring-mass system satisfies y'' + 12y' + 35y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 12x + 35 = 0$$

→ *Characteristic equation: r^2 + 12r + 35 = 0.*
→ *Roots: r1 = -7, r2 = -5.*
→ *Both roots real and negative: overdamped system.*

Answer: $x = -7$ or $x = -5$

---

15. Find the characteristic roots of y'' + 11y' + 28y = 0. Write the general solution.

$$x^2 + 11x + 28 = 0$$

→ *r^2 + 11r + 28 = 0.*
→ *r1 = -7, r2 = -4.*
→ *General solution: y = C1*e^(-7t) + C2*e^(-4t).*

Answer: $x = -7$ or $x = -4$

---

16. Solve the characteristic equation: r^2 + 7r + 6 = 0.

$$x^2 + 7x + 6 = 0$$

→ *Factor: (r - -6)(r - -1) = 0.*
→ *Roots: r = -6 and r = -1.*

Answer: $x = -6$ or $x = -1$

---

17. Find the roots of the characteristic equation for y'' + -1y' + -2y = 0.

$$x^2 - 1x - 2 = 0$$

→ *Characteristic equation: r^2 + -1r + -2 = 0.*
→ *Factor or use quadratic formula.*
→ *Roots: r = -1, r = 2.*
→ *General solution: y = C1·e^(-1t) + C2·e^(2t).*

Answer: $x = -1$ or $x = 2$

---

18. The motion of a spring-mass system satisfies y'' + 12y' + 35y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 12x + 35 = 0$$

→ *Characteristic equation: r^2 + 12r + 35 = 0.*
→ *Roots: r1 = -7, r2 = -5.*
→ *Both roots real and negative: overdamped system.*

Answer: $x = -7$ or $x = -5$

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19. Find the characteristic roots of y'' + 8y' + 15y = 0. Write the general solution.

$$x^2 + 8x + 15 = 0$$

→ *r^2 + 8r + 15 = 0.*
→ *r1 = -5, r2 = -3.*
→ *General solution: y = C1*e^(-5t) + C2*e^(-3t).*

Answer: $x = -5$ or $x = -3$

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20. Solve the characteristic equation: r^2 + 3r + -4 = 0.

$$x^2 + 3x - 4 = 0$$

→ *Factor: (r - -4)(r - 1) = 0.*
→ *Roots: r = -4 and r = 1.*

Answer: $x = -4$ or $x = 1$

---

21. Find the roots of the characteristic equation for y'' + -1y' + -2y = 0.

$$x^2 - 1x - 2 = 0$$

→ *Characteristic equation: r^2 + -1r + -2 = 0.*
→ *Factor or use quadratic formula.*
→ *Roots: r = -1, r = 2.*
→ *General solution: y = C1·e^(-1t) + C2·e^(2t).*

Answer: $x = -1$ or $x = 2$

---

22. The motion of a spring-mass system satisfies y'' + 2y' + 1y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 2x + 1 = 0$$

→ *Characteristic equation: r^2 + 2r + 1 = 0.*
→ *Roots: r1 = -1, r2 = -1.*
→ *Both roots real and negative: overdamped system.*

Answer: $x = -1$ or $x = -1$

---

23. Find the characteristic roots of y'' + 7y' + 0y = 0. Write the general solution.

$$x^2 + 7x = 0$$

→ *r^2 + 7r + 0 = 0.*
→ *r1 = -7, r2 = 0.*
→ *General solution: y = C1*e^(-7t) + C2*e^(0t).*

Answer: $x = -7$ or $x = 0$

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24. Solve the characteristic equation: r^2 + 2r + 0 = 0.

$$x^2 + 2x = 0$$

→ *Factor: (r - -2)(r - 0) = 0.*
→ *Roots: r = -2 and r = 0.*

Answer: $x = -2$ or $x = 0$

---

25. Find the roots of the characteristic equation for y'' + 2y' + -15y = 0.

$$x^2 + 2x - 15 = 0$$

→ *Characteristic equation: r^2 + 2r + -15 = 0.*
→ *Factor or use quadratic formula.*
→ *Roots: r = -5, r = 3.*
→ *General solution: y = C1·e^(-5t) + C2·e^(3t).*

Answer: $x = -5$ or $x = 3$

---

26. The motion of a spring-mass system satisfies y'' + 14y' + 49y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 14x + 49 = 0$$

→ *Characteristic equation: r^2 + 14r + 49 = 0.*
→ *Roots: r1 = -7, r2 = -7.*
→ *Both roots real and negative: overdamped system.*

Answer: $x = -7$ or $x = -7$

---

27. Find the characteristic roots of y'' + 2y' + 0y = 0. Write the general solution.

$$x^2 + 2x = 0$$

→ *r^2 + 2r + 0 = 0.*
→ *r1 = -2, r2 = 0.*
→ *General solution: y = C1*e^(-2t) + C2*e^(0t).*

Answer: $x = -2$ or $x = 0$

---

28. Solve the characteristic equation: r^2 + 2r + -24 = 0.

$$x^2 + 2x - 24 = 0$$

→ *Factor: (r - -6)(r - 4) = 0.*
→ *Roots: r = -6 and r = 4.*

Answer: $x = -6$ or $x = 4$

---

29. Find the roots of the characteristic equation for y'' + 2y' + -15y = 0.

$$x^2 + 2x - 15 = 0$$

→ *Characteristic equation: r^2 + 2r + -15 = 0.*
→ *Factor or use quadratic formula.*
→ *Roots: r = -5, r = 3.*
→ *General solution: y = C1·e^(-5t) + C2·e^(3t).*

Answer: $x = -5$ or $x = 3$

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30. The motion of a spring-mass system satisfies y'' + 11y' + 24y = 0. Find the characteristic roots and identify the type of damping.

$$x^2 + 11x + 24 = 0$$

→ *Characteristic equation: r^2 + 11r + 24 = 0.*
→ *Roots: r1 = -8, r2 = -3.*
→ *Both roots real and negative: overdamped system.*

Answer: $x = -8$ or $x = -3$