MATH240: Laplace Transforms

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Learning Objectives

- Calculate the mean, median, mode, and range of a data set
- Compute sample variance and standard deviation
- Construct quartiles, the IQR, and identify outliers
- Describe the shape, center, and spread of a distribution

Simplify each expression completely. Show all steps and circle your final answer.

Laplace transform of exponentials

1. Find the Laplace transform of f(t) = 6*e^(-4*t).

$$\mathcal{L}\{6e^{-4t}\}$$

Answer: _____

2. Find the Laplace transform of f(t) = 1*e^(1*t).

$$\mathcal{L}\{1e^{1t}\}$$

Answer: _____

3. Find the Laplace transform of f(t) = 4*e^(-1*t).

$$\mathcal{L}\{4e^{-1t}\}$$

Answer: _____

4. Find the Laplace transform of f(t) = 2*e^(2*t).

$$\mathcal{L}\{2e^{2t}\}$$

Answer: _____

5. Find the Laplace transform of f(t) = 5*e^(-2*t).

$$\mathcal{L}\{5e^{-2t}\}$$

Answer: _____

6. Find the Laplace transform of f(t) = 5*e^(-3*t).

$$\mathcal{L}\{5e^{-3t}\}$$

Answer: _____

7. Find the Laplace transform of f(t) = 1*e^(-1*t).

$$\mathcal{L}\{1e^{-1t}\}$$

Answer: _____

8. Find the Laplace transform of f(t) = 1*e^(-1*t).

$$\mathcal{L}\{1e^{-1t}\}$$

Answer: _____

Laplace transform of power functions

9. Find the Laplace transform of f(t) = 1*t^2.

$$\mathcal{L}\{1t^{2}\}$$

Answer: _____

10. A forcing function is f(t) = 4*t^3. Find the Laplace transform F(s) and identify the numerator coefficient.

$$\mathcal{L}\{4t^{3}\}$$

Answer: _____

11. Find the Laplace transform of f(t) = 3*t^0.

$$\mathcal{L}\{3t^{0}\}$$

Answer: _____

12. A forcing function is f(t) = 3*t^1. Find the Laplace transform F(s) and identify the numerator coefficient.

$$\mathcal{L}\{3t^{1}\}$$

Answer: _____

13. Find the Laplace transform of f(t) = 2*t^1.

$$\mathcal{L}\{2t^{1}\}$$

Answer: _____

14. A forcing function is f(t) = 3*t^1. Find the Laplace transform F(s) and identify the numerator coefficient.

$$\mathcal{L}\{3t^{1}\}$$

Answer: _____

15. Find the Laplace transform of f(t) = 1*t^1.

$$\mathcal{L}\{1t^1\}$$

Answer: _____

16. A forcing function is f(t) = 1*t^2. Find the Laplace transform F(s) and identify the numerator coefficient.

$$\mathcal{L}\{1t^2\}$$

Answer: _____

17. Find the Laplace transform of f(t) = 2*t^0.

$$\mathcal{L}\{2t^0\}$$

Answer: _____

18. A forcing function is f(t) = 3*t^2. Find the Laplace transform F(s) and identify the numerator coefficient.

$$\mathcal{L}\{3t^2\}$$

Answer: _____

19. Find the Laplace transform of f(t) = 4*t^3.

$$\mathcal{L}\{4t^3\}$$

Answer: _____

20. A forcing function is f(t) = 4*t^1. Find the Laplace transform F(s) and identify the numerator coefficient.

$$\mathcal{L}\{4t^1\}$$

Answer: _____

21. Find the Laplace transform of f(t) = 1*t^0.

$$\mathcal{L}\{1t^0\}$$

Answer: _____

22. A forcing function is f(t) = 4*t^3. Find the Laplace transform F(s) and identify the numerator coefficient.

$$\mathcal{L}\{4t^3\}$$

Answer: _____

23. Find the Laplace transform of f(t) = 3*t^0.

$$\mathcal{L}\{3t^0\}$$

Answer: _____

Laplace transform of sine

24. Find the Laplace transform of f(t) = 3*sin(4*t).

$$\mathcal{L}\{3\sin(4t)\}$$

Answer: _____

25. Find the Laplace transform of f(t) = 1*sin(3*t).

$$\mathcal{L}\{1\sin(3t)\}$$

Answer: _____

26. Find the Laplace transform of f(t) = 5*sin(2*t).

$$\mathcal{L}\{5\sin(2t)\}$$

Answer: _____

27. Find the Laplace transform of f(t) = 1*sin(3*t).

$$\mathcal{L}\{1\sin(3t)\}$$

Answer: _____

28. Find the Laplace transform of f(t) = 4*sin(2*t).

$$\mathcal{L}\{4\sin(2t)\}$$

Answer: _____

29. Find the Laplace transform of f(t) = 4*sin(3*t).

$$\mathcal{L}\{4\sin(3t)\}$$

Answer: _____

30. Find the Laplace transform of f(t) = 1*sin(1*t).

$$\mathcal{L}\{1\sin(1t)\}$$

Answer: _____

MATH240: Laplace Transforms

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ANSWER KEY & SOLUTIONS

Topics: Laplace transform of sine, Laplace transform of power functions, Laplace transform of exponentials. All answers verified by independent computation.

Solutions

Laplace transform of exponentials

1. Find the Laplace transform of f(t) = 6*e^(-4*t).

$$\mathcal{L}\{6e^{-4t}\}$$

→ *Formula: L[e^(bt)] = 1/(s-b).*
→ *L[6*e^(-4t)] = 6/(s - -4).*
→ *The denominator shift is s = -4.*

Answer: $= \frac{6}{s--4}$

2. Find the Laplace transform of f(t) = 1*e^(1*t).

$$\mathcal{L}\{1e^{1t}\}$$

→ *Formula: L[e^(bt)] = 1/(s-b).*
→ *L[1*e^(1t)] = 1/(s - 1).*
→ *The denominator shift is s = 1.*

Answer: $= \frac{1}{s-1}$

3. Find the Laplace transform of f(t) = 4*e^(-1*t).

$$\mathcal{L}\{4e^{-1t}\}$$

→ *Formula: L[e^(bt)] = 1/(s-b).*
→ *L[4*e^(-1t)] = 4/(s - -1).*
→ *The denominator shift is s = -1.*

Answer: $= \frac{4}{s--1}$

4. Find the Laplace transform of f(t) = 2*e^(2*t).

$$\mathcal{L}\{2e^{2t}\}$$

→ *Formula: L[e^(bt)] = 1/(s-b).*
→ *L[2*e^(2t)] = 2/(s - 2).*
→ *The denominator shift is s = 2.*

Answer: $= \frac{2}{s-2}$

5. Find the Laplace transform of f(t) = 5*e^(-2*t).

$$\mathcal{L}\{5e^{-2t}\}$$

→ *Formula: L[e^(bt)] = 1/(s-b).*
→ *L[5*e^(-2t)] = 5/(s - -2).*
→ *The denominator shift is s = -2.*

Answer: $= \frac{5}{s--2}$

6. Find the Laplace transform of f(t) = 5*e^(-3*t).

$$\mathcal{L}\{5e^{-3t}\}$$

→ *Formula: L[e^(bt)] = 1/(s-b).*
→ *L[5*e^(-3t)] = 5/(s - -3).*
→ *The denominator shift is s = -3.*

Answer: $= \frac{5}{s--3}$

---

7. Find the Laplace transform of f(t) = 1*e^(-1*t).

$$\mathcal{L}\{1e^{-1t}\}$$

→ *Formula: L[e^(bt)] = 1/(s-b).*
→ *L[1*e^(-1t)] = 1/(s - -1).*
→ *The denominator shift is s = -1.*

Answer: $= \frac{1}{s--1}$

---

8. Find the Laplace transform of f(t) = 1*e^(-1*t).

$$\mathcal{L}\{1e^{-1t}\}$$

→ *Formula: L[e^(bt)] = 1/(s-b).*
→ *L[1*e^(-1t)] = 1/(s - -1).*
→ *The denominator shift is s = -1.*

Answer: $= \frac{1}{s--1}$

Laplace transform of power functions

9. Find the Laplace transform of f(t) = 1*t^2.

$$\mathcal{L}\{1t^2\}$$

→ *Formula: L[t^n] = n! / s^(n+1).*
→ *L[1*t^2] = 1*2 / s^3 = 2 / s^3.*
→ *Numerator coefficient: 2.*

Answer: $= \frac{2}{s^3}$

10. A forcing function is f(t) = 4*t^3. Find the Laplace transform F(s) and identify the numerator coefficient.

$$\mathcal{L}\{4t^3\}$$

→ *L[4*t^3] = 4*3! / s^4.*
→ *Numerator: 4*6 = 24.*
→ *F(s) = 24/s^4.*

Answer: $= \frac{24}{s^4}$

11. Find the Laplace transform of f(t) = 3*t^0.

$$\mathcal{L}\{3t^0\}$$

→ *Formula: L[t^n] = n! / s^(n+1).*
→ *L[3*t^0] = 3*1 / s^1 = 3 / s^1.*
→ *Numerator coefficient: 3.*

Answer: $= \frac{3}{s^1}$

12. A forcing function is f(t) = 3*t^1. Find the Laplace transform F(s) and identify the numerator coefficient.

$$\mathcal{L}\{3t^1\}$$

→ *L[3*t^1] = 3*1! / s^2.*
→ *Numerator: 3*1 = 3.*
→ *F(s) = 3/s^2.*

Answer: $= \frac{3}{s^2}$

13. Find the Laplace transform of f(t) = 2*t^1.

$$\mathcal{L}\{2t^1\}$$

→ *Formula: L[t^n] = n! / s^(n+1).*
→ *L[2*t^1] = 2*1 / s^2 = 2 / s^2.*
→ *Numerator coefficient: 2.*

Answer: $= \frac{2}{s^2}$

14. A forcing function is f(t) = 3*t^1. Find the Laplace transform F(s) and identify the numerator coefficient.

$$\mathcal{L}\{3t^1\}$$

→ *L[3*t^1] = 3*1! / s^2.*
→ *Numerator: 3*1 = 3.*
→ *F(s) = 3/s^2.*

Answer: $= \frac{3}{s^2}$

---

15. Find the Laplace transform of f(t) = 1*t^1.

$$\mathcal{L}\{1t^1\}$$

→ *Formula: L[t^n] = n! / s^(n+1).*
→ *L[1*t^1] = 1*1 / s^2 = 1 / s^2.*
→ *Numerator coefficient: 1.*

Answer: $= \frac{1}{s^2}$

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16. A forcing function is f(t) = 1*t^2. Find the Laplace transform F(s) and identify the numerator coefficient.

$$\mathcal{L}\{1t^2\}$$

→ *L[1*t^2] = 1*2! / s^3.*
→ *Numerator: 1*2 = 2.*
→ *F(s) = 2/s^3.*

Answer: $= \frac{2}{s^3}$

---

17. Find the Laplace transform of f(t) = 2*t^0.

$$\mathcal{L}\{2t^0\}$$

→ *Formula: L[t^n] = n! / s^(n+1).*
→ *L[2*t^0] = 2*1 / s^1 = 2 / s^1.*
→ *Numerator coefficient: 2.*

Answer: $= \frac{2}{s^1}$

---

18. A forcing function is f(t) = 3*t^2. Find the Laplace transform F(s) and identify the numerator coefficient.

$$\mathcal{L}\{3t^2\}$$

→ *L[3*t^2] = 3*2! / s^3.*
→ *Numerator: 3*2 = 6.*
→ *F(s) = 6/s^3.*

Answer: $= \frac{6}{s^3}$

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19. Find the Laplace transform of f(t) = 4*t^3.

$$\mathcal{L}\{4t^3\}$$

→ *Formula: L[t^n] = n! / s^(n+1).*
→ *L[4*t^3] = 4*6 / s^4 = 24 / s^4.*
→ *Numerator coefficient: 24.*

Answer: $= \frac{24}{s^4}$

---

20. A forcing function is f(t) = 4*t^1. Find the Laplace transform F(s) and identify the numerator coefficient.

$$\mathcal{L}\{4t^1\}$$

→ *L[4*t^1] = 4*1! / s^2.*
→ *Numerator: 4*1 = 4.*
→ *F(s) = 4/s^2.*

Answer: $= \frac{4}{s^2}$

---

21. Find the Laplace transform of f(t) = 1*t^0.

$$\mathcal{L}\{1t^0\}$$

→ *Formula: L[t^n] = n! / s^(n+1).*
→ *L[1*t^0] = 1*1 / s^1 = 1 / s^1.*
→ *Numerator coefficient: 1.*

Answer: $= \frac{1}{s^1}$

---

22. A forcing function is f(t) = 4*t^3. Find the Laplace transform F(s) and identify the numerator coefficient.

$$\mathcal{L}\{4t^3\}$$

→ *L[4*t^3] = 4*3! / s^4.*
→ *Numerator: 4*6 = 24.*
→ *F(s) = 24/s^4.*

Answer: $= \frac{24}{s^4}$

---

23. Find the Laplace transform of f(t) = 3*t^0.

$$\mathcal{L}\{3t^0\}$$

→ *Formula: L[t^n] = n! / s^(n+1).*
→ *L[3*t^0] = 3*1 / s^1 = 3 / s^1.*
→ *Numerator coefficient: 3.*

Answer: $= \frac{3}{s^1}$

Laplace transform of sine

24. Find the Laplace transform of f(t) = 3*sin(4*t).

$$\mathcal{L}\{3\sin(4t)\}$$

→ *Formula: L[sin(bt)] = b/(s^2 + b^2).*
→ *L[3*sin(4t)] = 3*4/(s^2 + 16) = 12/(s^2 + 16).*

Answer: $= \frac{12}{s^2 + 16}$

25. Find the Laplace transform of f(t) = 1*sin(3*t).

$$\mathcal{L}\{1\sin(3t)\}$$

→ *Formula: L[sin(bt)] = b/(s^2 + b^2).*
→ *L[1*sin(3t)] = 1*3/(s^2 + 9) = 3/(s^2 + 9).*

Answer: $= \frac{3}{s^2 + 9}$

26. Find the Laplace transform of f(t) = 5*sin(2*t).

$$\mathcal{L}\{5\sin(2t)\}$$

→ *Formula: L[sin(bt)] = b/(s^2 + b^2).*
→ *L[5*sin(2t)] = 5*2/(s^2 + 4) = 10/(s^2 + 4).*

Answer: $= \frac{10}{s^2 + 4}$

27. Find the Laplace transform of f(t) = 1*sin(3*t).

$$\mathcal{L}\{1\sin(3t)\}$$

→ *Formula: L[sin(bt)] = b/(s^2 + b^2).*
→ *L[1*sin(3t)] = 1*3/(s^2 + 9) = 3/(s^2 + 9).*

Answer: $= \frac{3}{s^2 + 9}$

28. Find the Laplace transform of f(t) = 4*sin(2*t).

$$\mathcal{L}\{4\sin(2t)\}$$

→ *Formula: L[sin(bt)] = b/(s^2 + b^2).*
→ *L[4*sin(2t)] = 4*2/(s^2 + 4) = 8/(s^2 + 4).*

Answer: $= \frac{8}{s^2 + 4}$

29. Find the Laplace transform of f(t) = 4*sin(3*t).

$$\mathcal{L}\{4\sin(3t)\}$$

→ *Formula: L[sin(bt)] = b/(s^2 + b^2).*
→ *L[4*sin(3t)] = 4*3/(s^2 + 9) = 12/(s^2 + 9).*

Answer: $= \frac{12}{s^2 + 9}$

30. Find the Laplace transform of f(t) = 1*sin(1*t).

$$\mathcal{L}\{1\sin(1t)\}$$

→ *Formula: L[sin(bt)] = b/(s^2 + b^2).*
→ *L[1*sin(1t)] = 1*1/(s^2 + 1) = 1/(s^2 + 1).*

Answer: $= \frac{1}{s^2+1}$