MATH240: Numerical Methods

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Learning Objectives

- Calculate the mean, median, mode, and range of a data set
- Compute sample variance and standard deviation
- Construct quartiles, the IQR, and identify outliers
- Describe the shape, center, and spread of a distribution

Simplify each expression completely. Show all steps and circle your final answer.

Euler method

1. Use Euler's method with step size h = 1 to approximate y(x0 + h) for y' = 1x + -4, y(2) = 4.

$$y' = 1x - 4,\ x_0 = 2,\ y_0 = 4,\ h = 1$$

Answer: ____

2. Apply Euler's method (one step, h = 1) to y' = 2x + 5 starting at (x0, y0) = (2, 8). What is the approximation at x = x0 + 1?

$$y' = 2x + 5,\ x_0 = 2,\ y_0 = 8,\ h = 1$$

Answer: ____

3. A population model satisfies y' = 4t + 10. Starting at t = 0, y = 10, use one step of Euler's method with h = 2 to estimate y(0 + 2).

$$y' = 4x + 10,\ x_0 = 0,\ y_0 = 10,\ h = 2$$

Answer: ____

4. Use Euler's method with step size h = 1 to approximate y(x0 + h) for y' = 3x + 2, y(3) = 5.

$$y' = 3x + 2,\ x_0 = 3,\ y_0 = 5,\ h = 1$$

Answer: ____

5. Apply Euler's method (one step, h = 1) to y' = 2x + 7 starting at (x0, y0) = (0, 6). What is the approximation at x = x0 + 1?

$$y' = 2x + 7,\ x_0 = 0,\ y_0 = 6,\ h = 1$$

Answer: ____

6. A population model satisfies y' = 3t + 3. Starting at t = 3, y = 19, use one step of Euler's method with h = 2 to estimate y(3 + 2).

$$y' = 3x + 3,\ x_0 = 3,\ y_0 = 19,\ h = 2$$

Answer: ____

7. Use Euler's method with step size h = 2 to approximate y(x0 + h) for y' = 5x + 1, y(1) = 9.

$$y' = 5x + 1,\ x_0 = 1,\ y_0 = 9,\ h = 2$$

Answer: ____

8. Apply Euler's method (one step, h = 1) to y' = 2x + 6 starting at (x0, y0) = (1, 2). What is the approximation at x = x0 + 1?

$$y' = 2x + 6,\ x_0 = 1,\ y_0 = 2,\ h = 1$$

Answer: ____

9. A population model satisfies y' = 4t + 6. Starting at t = 2, y = 12, use one step of Euler's method with h = 2 to estimate y(2 + 2).

$$y' = 4x + 6,\ x_0 = 2,\ y_0 = 12,\ h = 2$$

Answer: ____

10. Use Euler's method with step size h = 1 to approximate y(x0 + h) for y' = 2x + 4, y(1) = 4.

$$y' = 2x + 4,\ x_0 = 1,\ y_0 = 4,\ h = 1$$

Answer: ____

11. Apply Euler's method (one step, h = 1) to y' = 4x + 1 starting at (x0, y0) = (2, 6). What is the approximation at x = x0 + 1?

$$y' = 4x + 1,\ x_0 = 2,\ y_0 = 6,\ h = 1$$

Answer: ____

12. A population model satisfies y' = 2t + 9. Starting at t = 3, y = 16, use one step of Euler's method with h = 2 to estimate y(3 + 2).

$$y' = 2x + 9,\ x_0 = 3,\ y_0 = 16,\ h = 2$$

Answer: ____

13. Use Euler's method with step size h = 2 to approximate y(x0 + h) for y' = 2x + 3, y(2) = 8.

$$y' = 2x + 3,\ x_0 = 2,\ y_0 = 8,\ h = 2$$

Answer: ____

14. Apply Euler's method (one step, h = 1) to y' = 2x + 4 starting at (x0, y0) = (0, 7). What is the approximation at x = x0 + 1?

$$y' = 2x + 4,\ x_0 = 0,\ y_0 = 7,\ h = 1$$

Answer: ____

15. A population model satisfies y' = 1t + 9. Starting at t = 2, y = 18, use one step of Euler's method with h = 1 to estimate y(2 + 1).

$$y' = 1x + 9,\ x_0 = 2,\ y_0 = 18,\ h = 1$$

Answer: ____

16. Use Euler's method with step size h = 2 to approximate y(x0 + h) for y' = 1x + 1, y(0) = 4.

$$y' = 1x + 1,\ x_0 = 0,\ y_0 = 4,\ h = 2$$

Answer: ____

17. Apply Euler's method (one step, h = 1) to y' = 6x + 4 starting at (x0, y0) = (0, 7). What is the approximation at x = x0 + 1?

$$y' = 6x + 4,\ x_0 = 0,\ y_0 = 7,\ h = 1$$

Answer: ____

18. A population model satisfies y' = 2t + 3. Starting at t = 3, y = 5, use one step of Euler's method with h = 1 to estimate y(3 + 1).

$$y' = 2x + 3,\ x_0 = 3,\ y_0 = 5,\ h = 1$$

Answer: ____

19. Use Euler's method with step size h = 1 to approximate y(x0 + h) for y' = 3x + 0, y(1) = 5.

$$y' = 3x + 0,\ x_0 = 1,\ y_0 = 5,\ h = 1$$

Answer: ____

20. Apply Euler's method (one step, h = 1) to y' = 5x + 3 starting at (x0, y0) = (2, 3). What is the approximation at x = x0 + 1?

$$y' = 5x + 3,\ x_0 = 2,\ y_0 = 3,\ h = 1$$

Answer: ____

21. A population model satisfies y' = 2t + 3. Starting at t = 1, y = 19, use one step of Euler's method with h = 2 to estimate y(1 + 2).

$$y' = 2x + 3,\ x_0 = 1,\ y_0 = 19,\ h = 2$$

Answer: ____

22. Use Euler's method with step size h = 2 to approximate y(x0 + h) for y' = 4x + 3, y(2) = 5.

$$y' = 4x + 3,\ x_0 = 2,\ y_0 = 5,\ h = 2$$

Answer: ______________________________________

23. Apply Euler's method (one step, h = 1) to y' = 5x + 2 starting at (x0, y0) = (2, 6). What is the approximation at x = x0 + 1?

$$y' = 5x + 2,\ x_0 = 2,\ y_0 = 6,\ h = 1$$

Answer: ______________________________________

24. A population model satisfies y' = 4t + 6. Starting at t = 2, y = 12, use one step of Euler's method with h = 2 to estimate y(2 + 2).

$$y' = 4x + 6,\ x_0 = 2,\ y_0 = 12,\ h = 2$$

Answer: ______________________________________

25. Use Euler's method with step size h = 2 to approximate y(x0 + h) for y' = 1x + 0, y(3) = 4.

$$y' = 1x + 0,\ x_0 = 3,\ y_0 = 4,\ h = 2$$

Answer: ______________________________________

26. Apply Euler's method (one step, h = 1) to y' = 2x + 2 starting at (x0, y0) = (1, 5). What is the approximation at x = x0 + 1?

$$y' = 2x + 2,\ x_0 = 1,\ y_0 = 5,\ h = 1$$

Answer: ______________________________________

27. A population model satisfies y' = 4t + 10. Starting at t = 0, y = 12, use one step of Euler's method with h = 2 to estimate y(0 + 2).

$$y' = 4x + 10,\ x_0 = 0,\ y_0 = 12,\ h = 2$$

Answer: ______________________________________

28. Use Euler's method with step size h = 2 to approximate y(x0 + h) for y' = 1x + -3, y(1) = 2.

$$y' = 1x - 3,\ x_0 = 1,\ y_0 = 2,\ h = 2$$

Answer: ______________________________________

29. Apply Euler's method (one step, h = 1) to y' = 2x + 5 starting at (x0, y0) = (2, 5). What is the approximation at x = x0 + 1?

$$y' = 2x + 5,\ x_0 = 2,\ y_0 = 5,\ h = 1$$

Answer: ______________________________________

30. A population model satisfies y' = 3t + 10. Starting at t = 0, y = 13, use one step of Euler's method with h = 1 to estimate y(0 + 1).

$$y' = 3x + 10,\ x_0 = 0,\ y_0 = 13,\ h = 1$$

Answer: ____

Topics: Euler method. All answers verified by independent computation.

Solutions

Euler method

1. Use Euler's method with step size h = 1 to approximate y(x0 + h) for y' = 1x + -4, y(2) = 4.

$$y' = 1x - 4,\ x_0 = 2,\ y_0 = 4,\ h = 1$$

→ *f(x0) = 1·2 + -4 = -2.*
→ *y1 = y0 + h·f(x0) = 4 + 1·-2 = 2.*

Answer: $y_1 \approx 4 + 1 \cdot -2 = 2$

---

2. Apply Euler's method (one step, h = 1) to y' = 2x + 5 starting at (x0, y0) = (2, 8). What is the approximation at x = x0 + 1?

$$y' = 2x + 5,\ x_0 = 2,\ y_0 = 8,\ h = 1$$

→ *Evaluate slope: f(2) = 2·2 + 5 = 9.*
→ *Next approximation: y1 = 8 + 1·9 = 17.*

Answer: $y_1 \approx 8 + 1 \cdot 9 = 17$

---

3. A population model satisfies y' = 4t + 10. Starting at t = 0, y = 10, use one step of Euler's method with h = 2 to estimate y(0 + 2).

$$y' = 4x + 10,\ x_0 = 0,\ y_0 = 10,\ h = 2$$

→ *Rate at t = 0: f(0) = 4·0 + 10 = 10.*
→ *Euler step: y1 = 10 + 2·10 = 30.*

Answer: $y_1 \approx 10 + 2 \cdot 10 = 30$

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4. Use Euler's method with step size h = 1 to approximate y(x0 + h) for y' = 3x + 2, y(3) = 5.

$$y' = 3x + 2,\ x_0 = 3,\ y_0 = 5,\ h = 1$$

→ *f(x0) = 3·3 + 2 = 11.*
→ *y1 = y0 + h·f(x0) = 5 + 1·11 = 16.*

Answer: $y_1 \approx 5 + 1 \cdot 11 = 16$

---

5. Apply Euler's method (one step, h = 1) to y' = 2x + 7 starting at (x0, y0) = (0, 6). What is the approximation at x = x0 + 1?

$$y' = 2x + 7,\ x_0 = 0,\ y_0 = 6,\ h = 1$$

→ *Evaluate slope: f(0) = 2·0 + 7 = 7.*
→ *Next approximation: y1 = 6 + 1·7 = 13.*

Answer: $y_1 \approx 6 + 1 \cdot 7 = 13$

---

6. A population model satisfies y' = 3t + 3. Starting at t = 3, y = 19, use one step of Euler's method with h = 2 to estimate y(3 + 2).

$$y' = 3x + 3,\ x_0 = 3,\ y_0 = 19,\ h = 2$$

→ *Rate at t = 3: f(3) = 3·3 + 3 = 12.*
→ *Euler step: y1 = 19 + 2·12 = 43.*

Answer: $y_1 \approx 19 + 2 \cdot 12 = 43$

---

7. Use Euler's method with step size h = 2 to approximate y(x0 + h) for y' = 5x + 1, y(1) = 9.

$$y' = 5x + 1,\ x_0 = 1,\ y_0 = 9,\ h = 2$$

→ f(x0) = 5·1 + 1 = 6.
→ y1 = y0 + h·f(x0) = 9 + 2·6 = 21.

Answer: $y_1 \approx 9 + 2 \cdot 6 = 21$

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8. Apply Euler's method (one step, h = 1) to y' = 2x + 6 starting at (x0, y0) = (1, 2). What is the approximation at x = x0 + 1?

$$y' = 2x + 6,\ x_0 = 1,\ y_0 = 2,\ h = 1$$

→ Evaluate slope: f(1) = 2·1 + 6 = 8.
→ Next approximation: y1 = 2 + 1·8 = 10.

Answer: $y_1 \approx 2 + 1 \cdot 8 = 10$

---

9. A population model satisfies y' = 4t + 6. Starting at t = 2, y = 12, use one step of Euler's method with h = 2 to estimate y(2 + 2).

$$y' = 4x + 6,\ x_0 = 2,\ y_0 = 12,\ h = 2$$

→ Rate at t = 2: f(2) = 4·2 + 6 = 14.
→ Euler step: y1 = 12 + 2·14 = 40.

Answer: $y_1 \approx 12 + 2 \cdot 14 = 40$

---

10. Use Euler's method with step size h = 1 to approximate y(x0 + h) for y' = 2x + 4, y(1) = 4.

$$y' = 2x + 4,\ x_0 = 1,\ y_0 = 4,\ h = 1$$

→ f(x0) = 2·1 + 4 = 6.
→ y1 = y0 + h·f(x0) = 4 + 1·6 = 10.

Answer: $y_1 \approx 4 + 1 \cdot 6 = 10$

---

11. Apply Euler's method (one step, h = 1) to y' = 4x + 1 starting at (x0, y0) = (2, 6). What is the approximation at x = x0 + 1?

$$y' = 4x + 1,\ x_0 = 2,\ y_0 = 6,\ h = 1$$

→ Evaluate slope: f(2) = 4·2 + 1 = 9.
→ Next approximation: y1 = 6 + 1·9 = 15.

Answer: $y_1 \approx 6 + 1 \cdot 9 = 15$

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12. A population model satisfies y' = 2t + 9. Starting at t = 3, y = 16, use one step of Euler's method with h = 2 to estimate y(3 + 2).

$$y' = 2x + 9,\ x_0 = 3,\ y_0 = 16,\ h = 2$$

→ Rate at t = 3: f(3) = 2·3 + 9 = 15.
→ Euler step: y1 = 16 + 2·15 = 46.

Answer: $y_1 \approx 16 + 2 \cdot 15 = 46$

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13. Use Euler's method with step size h = 2 to approximate y(x0 + h) for y' = 2x + 3, y(2) = 8.

$$y' = 2x + 3,\ x_0 = 2,\ y_0 = 8,\ h = 2$$

→ f(x0) = 2·2 + 3 = 7.
→ y1 = y0 + h·f(x0) = 8 + 2·7 = 22.

Answer: $y_1 \approx 8 + 2 \cdot 7 = 22$

---

14. Apply Euler's method (one step, h = 1) to y' = 2x + 4 starting at (x0, y0) = (0, 7). What is the approximation at x = x0 + 1?

$$y' = 2x + 4,\ x_0 = 0,\ y_0 = 7,\ h = 1$$

→ Evaluate slope: f(0) = 2·0 + 4 = 4.
→ Next approximation: y1 = 7 + 1·4 = 11.

Answer: $y_1 \approx 7 + 1 \cdot 4 = 11$

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15. A population model satisfies y' = 1t + 9. Starting at t = 2, y = 18, use one step of Euler's method with h = 1 to estimate y(2 + 1).

$$y' = 1x + 9,\ x_0 = 2,\ y_0 = 18,\ h = 1$$

→ Rate at t = 2: f(2) = 1·2 + 9 = 11.
→ Euler step: y1 = 18 + 1·11 = 29.

Answer: $y_1 \approx 18 + 1 \cdot 11 = 29$

---

16. Use Euler's method with step size h = 2 to approximate y(x0 + h) for y' = 1x + 1, y(0) = 4.

$$y' = 1x + 1,\ x_0 = 0,\ y_0 = 4,\ h = 2$$

→ f(x0) = 1·0 + 1 = 1.
→ y1 = y0 + h·f(x0) = 4 + 2·1 = 6.

Answer: $y_1 \approx 4 + 2 \cdot 1 = 6$

---

17. Apply Euler's method (one step, h = 1) to y' = 6x + 4 starting at (x0, y0) = (0, 7). What is the approximation at x = x0 + 1?

$$y' = 6x + 4,\ x_0 = 0,\ y_0 = 7,\ h = 1$$

→ Evaluate slope: f(0) = 6·0 + 4 = 4.
→ Next approximation: y1 = 7 + 1·4 = 11.

Answer: $y_1 \approx 7 + 1 \cdot 4 = 11$

---

18. A population model satisfies y' = 2t + 3. Starting at t = 3, y = 5, use one step of Euler's method with h = 1 to estimate y(3 + 1).

$$y' = 2x + 3,\ x_0 = 3,\ y_0 = 5,\ h = 1$$

→ Rate at t = 3: f(3) = 2·3 + 3 = 9.
→ Euler step: y1 = 5 + 1·9 = 14.

Answer: $y_1 \approx 5 + 1 \cdot 9 = 14$

---

19. Use Euler's method with step size h = 1 to approximate y(x0 + h) for y' = 3x + 0, y(1) = 5.

$$y' = 3x + 0,\ x_0 = 1,\ y_0 = 5,\ h = 1$$

→ f(x0) = 3·1 + 0 = 3.
→ y1 = y0 + h·f(x0) = 5 + 1·3 = 8.

Answer: $y_1 \approx 5 + 1 \cdot 3 = 8$

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20. Apply Euler's method (one step, h = 1) to y' = 5x + 3 starting at (x0, y0) = (2, 3). What is the approximation at x = x0 + 1?

$$y' = 5x + 3,\ x_0 = 2,\ y_0 = 3,\ h = 1$$

→ Evaluate slope: f(2) = 5·2 + 3 = 13.
→ Next approximation: y1 = 3 + 1·13 = 16.

Answer: $y_1 \approx 3 + 1 \cdot 13 = 16$

---

21. A population model satisfies y' = 2t + 3. Starting at t = 1, y = 19, use one step of Euler's method with h = 2 to estimate y(1 + 2).

$$y' = 2x + 3,\ x_0 = 1,\ y_0 = 19,\ h = 2$$

→ Rate at t = 1: f(1) = 2·1 + 3 = 5.
→ Euler step: y1 = 19 + 2·5 = 29.

Answer: $y_1 \approx 19 + 2 \cdot 5 = 29$

---

22. Use Euler's method with step size h = 2 to approximate y(x0 + h) for y' = 4x + 3, y(2) = 5.

$$y' = 4x + 3,\ x_0 = 2,\ y_0 = 5,\ h = 2$$

→ f(x0) = 4·2 + 3 = 11.
→ y1 = y0 + h·f(x0) = 5 + 2·11 = 27.

Answer: $y_1 \approx 5 + 2 \cdot 11 = 27$

---

23. Apply Euler's method (one step, h = 1) to y' = 5x + 2 starting at (x0, y0) = (2, 6). What is the approximation at x = x0 + 1?

$$y' = 5x + 2,\ x_0 = 2,\ y_0 = 6,\ h = 1$$

→ Evaluate slope: f(2) = 5·2 + 2 = 12.
→ Next approximation: y1 = 6 + 1·12 = 18.

Answer: $y_1 \approx 6 + 1 \cdot 12 = 18$

---

24. A population model satisfies y' = 4t + 6. Starting at t = 2, y = 12, use one step of Euler's method with h = 2 to estimate y(2 + 2).

$$y' = 4x + 6,\ x_0 = 2,\ y_0 = 12,\ h = 2$$

→ Rate at t = 2: f(2) = 4·2 + 6 = 14.
→ Euler step: y1 = 12 + 2·14 = 40.

Answer: $y_1 \approx 12 + 2 \cdot 14 = 40$

---

25. Use Euler's method with step size h = 2 to approximate y(x0 + h) for y' = 1x + 0, y(3) = 4.

$$y' = 1x + 0,\ x_0 = 3,\ y_0 = 4,\ h = 2$$

→ f(x0) = 1·3 + 0 = 3.
→ y1 = y0 + h·f(x0) = 4 + 2·3 = 10.

Answer: $y_1 \approx 4 + 2 \cdot 3 = 10$

26. Apply Euler's method (one step, h = 1) to y' = 2x + 2 starting at (x0, y0) = (1, 5). What is the approximation at x = x0 + 1?

$$y' = 2x + 2,\ x_0 = 1,\ y_0 = 5,\ h = 1$$

→ Evaluate slope: f(1) = 2·1 + 2 = 4.
→ Next approximation: y1 = 5 + 1·4 = 9.

Answer: $y_1 \approx 5 + 1 \cdot 4 = 9$

27. A population model satisfies y' = 4t + 10. Starting at t = 0, y = 12, use one step of Euler's method with h = 2 to estimate y(0 + 2).

$$y' = 4x + 10,\ x_0 = 0,\ y_0 = 12,\ h = 2$$

→ Rate at t = 0: f(0) = 4·0 + 10 = 10.
→ Euler step: y1 = 12 + 2·10 = 32.

Answer: $y_1 \approx 12 + 2 \cdot 10 = 32$

28. Use Euler's method with step size h = 2 to approximate y(x0 + h) for y' = 1x + -3, y(1) = 2.

$$y' = 1x - 3,\ x_0 = 1,\ y_0 = 2,\ h = 2$$

→ f(x0) = 1·1 + -3 = -2.
→ y1 = y0 + h·f(x0) = 2 + 2·-2 = -2.

Answer: $y_1 \approx 2 + 2 \cdot -2 = -2$

29. Apply Euler's method (one step, h = 1) to y' = 2x + 5 starting at (x0, y0) = (2, 5). What is the approximation at x = x0 + 1?

$$y' = 2x + 5,\ x_0 = 2,\ y_0 = 5,\ h = 1$$

→ Evaluate slope: f(2) = 2·2 + 5 = 9.
→ Next approximation: y1 = 5 + 1·9 = 14.

Answer: $y_1 \approx 5 + 1 \cdot 9 = 14$

30. A population model satisfies y' = 3t + 10. Starting at t = 0, y = 13, use one step of Euler's method with h = 1 to estimate y(0 + 1).

$$y' = 3x + 10,\ x_0 = 0,\ y_0 = 13,\ h = 1$$

→ Rate at t = 0: f(0) = 3·0 + 10 = 10.
→ Euler step: y1 = 13 + 1·10 = 23.

Answer: $y_1 \approx 13 + 1 \cdot 10 = 23$