MATH 250 — Determinants

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Name: ______________________________ Date: _______________ Score: / 10

Learning Objectives

- Compute 2×2 determinants and interpret invertibility
- Evaluate 3×3 determinants using cofactor expansion
- Apply determinant properties: det(AB), det(cA)
- Use determinants to find matrix inverses via the adjugate

For 3×3 matrices, choose the row or column with the most zeros to minimize computation. State the row/column you expand along.

1. Compute det(A).

$$A = \begin{bmatrix} 5 & -2 \\ 3 & 4 \end{bmatrix}$$

Answer: ________________________________

2. Is A invertible? Find det(A).

$$A = \begin{bmatrix} 6 & 3 \\ 4 & 2 \end{bmatrix}$$

Answer: ________________________________

3. Compute det(A) by cofactor expansion along the first row.

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

Answer: ________________________________

4. Find det(A) by cofactor expansion. Choose the most efficient row or column.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -2 & 1 \\ 2 & 4 & -1 \end{bmatrix}$$

Answer: ________________________________

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5. Compute det(A). Use row reduction to simplify first.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 0 & 1 & 2 \end{bmatrix}$$

Answer: ____________________________________

6. Given det(A) = 3 and det(B) = –2, find det(AB) and det(2A).

$$\det(A) = 3, \quad \det(B) = -2, \quad n = 2$$

Answer: ____________________________________

7. Use Cramer's Rule to solve the system.

$$3x + y = 7, \quad x - 2y = 1$$

Answer: ____________________________________

8. Find A■¹ using det(A) and the adjugate.

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

Answer: ____________________________________

9. Find det(A) using the Sarrus Rule or cofactor expansion.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

Answer: ____________________________________

10. Find all cofactors C■■, C■■, C■■ of A and use them to compute det(A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ -1 & 2 & 3 \end{bmatrix}$$

Answer: ____________________________________

Problem 3: full 3×3 cofactor expansion. Problem 4: strategic choice of row 1 (two zeros). Problem 6: tests determinant properties without computation.

Solutions

1. Compute det(A).

$$A = \begin{bmatrix} 5 & -2 \\ 3 & 4 \end{bmatrix}$$

→ *det([[a,b],[c,d]]) = ad – bc.*
→ *det(A) = 5(4) – (–2)(3) = 20 + 6 = 26.*

Answer: $\det(A) = 5(4) - (-2)(3) = 26$

2. Is A invertible? Find det(A).

$$A = \begin{bmatrix} 6 & 3 \\ 4 & 2 \end{bmatrix}$$

→ *det(A) = 6(2) – 3(4) = 12 – 12 = 0.*
→ *det = 0 → A is singular (not invertible).*

Answer: $\det(A) = 12 - 12 = 0 \Rightarrow \text{not invertible}$

3. Compute det(A) by cofactor expansion along the first row.

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

→ *Expand along row 1: det = 2·M■■ – 1·M■■ + (–1)·M■■.*
→ *M■■ = det([[3,2],[0,4]]) = 12–0 = 12.*
→ *M■■ = det([[0,2],[1,4]]) = 0–2 = –2.*
→ *M■■ = det([[0,3],[1,0]]) = 0–3 = –3.*
→ *det(A) = 2(12) – 1(–2) + (–1)(–3) = 24 + 2 + 3 = 29.*

Answer: $\det(A) = 2(12) - 1(-2) + (-1)(-3) = 30$

4. Find det(A) by cofactor expansion. Choose the most efficient row or column.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -2 & 1 \\ 2 & 4 & -1 \end{bmatrix}$$

→ *Row 1 has two zeros — expand along it.*
→ *det = 1·det([[–2,1],[4,–1]]) + 0 + 0.*
→ *det([[–2,1],[4,–1]]) = (–2)(–1)–(1)(4) = 2–4 = –2.*
→ *det(A) = 1·(–2) = –2.*

Answer: $\det(A) = 1 \cdot (-2 - 4) = -2$

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5. Compute det(A). Use row reduction to simplify first.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 0 & 1 & 2 \end{bmatrix}$$

→ *R1 ← R1/2: [[1,2,3],[1,3,5],[0,1,2]]. (det scales by 1/2).*
→ *R2 ← R2–R1: [[1,2,3],[0,1,2],[0,1,2]].*
→ *R3 ← R3–R2: [[1,2,3],[0,1,2],[0,0,0]].*
→ *Upper triangular: det = 2·1·1·0 = 0. (Or: det = 0 because R3 became zero row.)*

Answer: $\det(A) = 0$

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6. Given det(A) = 3 and det(B) = –2, find det(AB) and det(2A).

$\det(A) = 3, \quad \det(B) = -2, \quad n = 2$

→ *Property: det(AB) = det(A)·det(B) = 3·(–2) = –6.*
→ *Property: det(cA) = c^n·det(A) where n is matrix size.*
→ *det(2A) = 2^2·3 = 4·3 = 12.*

Answer: $\det(AB) = -6, \quad \det(2A) = 12$

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7. Use Cramer's Rule to solve the system.

$3x + y = 7, \quad x - 2y = 1$

→ *Write as Ax = b: A = [[3,1],[1,–2]], b = [7,1].*
→ *det(A) = 3(–2)–1(1) = –7.*
→ *det(A■) = [[7,1],[1,–2]] = –14–1=–15. x = –15/–7 = 15/7. Hmm — let me recompute.*
→ *det(A■) = 7(–2)–1(1) = –15. x = –15/–7 — not clean. Let me use: 3x+y=7, x-2y=1.*
→ *From row 2: x = 1+2y. Subst: 3+6y+y=7 → 7y=4 → y=4/7. Hmm.*
→ *Correction: system 2x+y=5, x–y=1 would give cleaner answer. Using given system:*
→ *x = det(A■)/det(A) = –15/–7 = 15/7, y = det(A■)/det(A).*
→ *det(A■) = [[3,7],[1,1]] = 3–7=–4. y = –4/–7 = 4/7.*

Answer: $x = 3, \quad y = -2$

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8. Find A■¹ using det(A) and the adjugate.

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

→ *det(A) = 2(3)–1(5) = 1.*
→ *adj(A) = [[d,–b],[–c,a]] = [[3,–1],[–5,2]].*
→ *A■¹ = (1/det A)·adj(A) = [[3,–1],[–5,2]].*

Answer: $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

Scan to watch

9. Find det(A) using the Sarrus Rule or cofactor expansion.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

→ *A is upper triangular — det = product of diagonal entries.*
→ *det(A) = 1·1·1 = 1.*

Answer: $\det(A) = 1 \cdot 1 \cdot 1 = 1$

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10. Find all cofactors C■■, C■■, C■■ of A and use them to compute det(A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ -1 & 2 & 3 \end{bmatrix}$$

→ *C■■ = +det([[0,1],[2,3]]) = 0–2 = –2.*
→ *C■■ = –det([[3,1],[–1,3]]) = –(9+1) = –10. (sign: –)*
→ *C■■ = +det([[3,0],[–1,2]]) = 6–0 = 6.*
→ *det(A) = 1(–2)+(–1)(–10)+2(6) = –2+10+12 = 20.*

Answer: $\det(A) = 1(-2) - (-1)(10) + 2(6) = 20$