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Learning Objectives

- Perform matrix addition, subtraction, and scalar multiplication
- Multiply matrices of compatible sizes (2×2 , $2 \times 3 \times 3 \times 2$, etc.)
- Find the transpose and verify properties like $(AB)^T = B^T A^T$
- Compute the inverse of a 2×2 matrix and verify $AA^{-1} = I$

For matrix multiplication, always verify dimensions first. A is $m \times n$ and B is $n \times p \rightarrow AB$ is $m \times p$.

1. Compute $A + B$. (2×2 matrices)

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 4 \\ 0 & -3 \end{bmatrix}$$

Answer: _____

2. Compute $2A - B$. (3×3 matrices)

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix}$$

Answer: _____

3. Compute $A \cdot B$. ($2 \times 2 \times 2 \times 2$)

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$$

Answer: _____

4. Compute $A \cdot B$. State the dimensions of the result. ($2 \times 3 \times 3 \times 2$)

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 4 & -1 \end{bmatrix}$$

Answer: _____



5. Compute the matrix product. What type of result do you get? ($1 \times 3 \times 3 \times 1$)

$$A = \begin{bmatrix} 2 & -1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$$

Answer: _____

6. Compute $A \cdot B$ and state the size of the result. ($3 \times 1 \times 1 \times 3$)

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -1 & 2 \end{bmatrix}$$

Answer: _____

7. Find A^{\blacksquare} . State the dimensions of A and A^{\blacksquare} . ($2 \times 4 \rightarrow 4 \times 2$)

$$A = \begin{bmatrix} 1 & -2 & 0 & 3 \\ 4 & 1 & -1 & 2 \end{bmatrix}$$

Answer: _____

8. Compute $A \cdot B$. State the size of the result. ($3 \times 2 \times 2 \times 4$)

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 1 & -1 & 3 & 2 \end{bmatrix}$$

Answer: _____

9. Find $A^{\blacksquare^{-1}}$ using the 2×2 inverse formula.

$$A = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}$$

Answer: _____

10. Verify $AB = I^{\blacksquare}$ (confirm $B = A^{\blacksquare^{-1}}$).

$$A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

Answer: _____





Problems 3–4 cover matrix multiplication with non-square matrices. Problems 7–8 cover the 2×2 inverse formula and verification.

Solutions

1. Compute $A + B$. (2×2 matrices)

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 4 \\ 0 & -3 \end{bmatrix}$$

→ Add corresponding entries.

→ Row 1: $3+(-1)=2$, $-1+4=3$. Row 2: $2+0=2$, $5+(-3)=2$.

Answer:
$$\begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$$

2. Compute $2A - B$. (3×3 matrices)

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix}$$

→ $2A = [[2,0,4],[-2,6,2],[0,4,-2]]$.

→ Row 1: $2-2=0$, $0-1=-1$, $4-0=4$.

→ Row 2: $-2-0=-2$, $6-(-1)=7$, $2-4=-2$.

→ Row 3: $0-3=-3$, $4-0=4$, $-2-2=-4$.

Answer:
$$\begin{bmatrix} 0 & -1 & 4 \\ -2 & 7 & -2 \\ -3 & 4 & -4 \end{bmatrix}$$

3. Compute $A \cdot B$. ($2 \times 2 \times 2 \times 2$)

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$$

→ Row 1·Col 1: $2(1)+1(2)=4$. Row 1·Col 2: $2(-1)+1(0)=-2$.

→ Row 2·Col 1: $3(1)+4(2)=11$. Row 2·Col 2: $3(-1)+4(0)=-3$.

Answer:
$$\begin{bmatrix} 4 & -2 \\ 11 & -3 \end{bmatrix}$$



4. Compute A·B. State the dimensions of the result. ($2 \times 3 \times 3 \times 2$)

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 4 & -1 \end{bmatrix}$$

→ A is 2×3 , B is 3×2 → product AB is 2×2 .

→ Row 1·Col 1: $1(2)+2(0)+0(4)=2$. Row 1·Col 2: $1(1)+2(3)+0(-1)=7$.

→ Row 2·Col 1: $3(2)+1(0)+(-1)(4)=2$. Row 2·Col 2: $3(1)+1(3)+(-1)(-1)=7$.

Answer: $\begin{bmatrix} 2 & 7 \\ 2 & 7 \end{bmatrix}$

5. Compute the matrix product. What type of result do you get? ($1 \times 3 \times 3 \times 1$)

$$A = \begin{bmatrix} 2 & -1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$$

→ A is 1×3 , B is 3×1 → product AB is 1×1 (a scalar).

→ $2(1)+(-1)(4)+3(-2) = 2-4-6 = -8$.

Answer: $AB = 2(1) + (-1)(4) + 3(-2) = -8$ (1 times 1 scalar)

6. Compute A·B and state the size of the result. ($3 \times 1 \times 1 \times 3$)

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -1 & 2 \end{bmatrix}$$

→ A is 3×1 , B is 1×3 → product AB is 3×3 (outer product).

→ Row i , Col j of $AB = A[i] \cdot B[j]$.

→ Row 1: $1 \cdot [4, -1, 2] = [4, -1, 2]$. Row 2: $2 \cdot [4, -1, 2] = [8, -2, 4]$. Row 3: $3 \cdot [4, -1, 2] = [12, -3, 6]$.

Answer: $\begin{bmatrix} 4 & -1 & 2 \\ 8 & -2 & 4 \\ 12 & -3 & 6 \end{bmatrix}$



7. Find A^T . State the dimensions of A and A^T . ($2 \times 4 \rightarrow 4 \times 2$)

$$A = \begin{bmatrix} 1 & -2 & 0 & 3 \\ 4 & 1 & -1 & 2 \end{bmatrix}$$

$\rightarrow A$ is $2 \times 4 \rightarrow A^T$ is 4×2 (rows become columns).

\rightarrow Col 1 of A^T = Row 1 of A : $[1, -2, 0, 3]^T$.

\rightarrow Col 2 of A^T = Row 2 of A : $[4, 1, -1, 2]^T$.

Answer:

$$\begin{bmatrix} 1 & 4 \\ -2 & 1 \\ 0 & -1 \\ 3 & 2 \end{bmatrix}$$

8. Compute $A \cdot B$. State the size of the result. ($3 \times 2 \times 2 \times 4$)

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 1 & -1 & 3 & 2 \end{bmatrix}$$

$\rightarrow A$ is 3×2 , B is $2 \times 4 \rightarrow$ product is 3×4 .

\rightarrow Row 1: $1 \cdot [2, 1, 0, -1] + 0 \cdot [1, -1, 3, 2] = [2, 1, 0, -1]$.

\rightarrow Row 2: $2 \cdot [2, 1, 0, -1] + 1 \cdot [1, -1, 3, 2] = [5, 1, 3, 0]$.

\rightarrow Row 3: $0 \cdot [2, 1, 0, -1] + 3 \cdot [1, -1, 3, 2] = [3, -3, 9, 6]$.

Answer:

$$\begin{bmatrix} 2 & 1 & 0 & -1 \\ 5 & 1 & 3 & 0 \\ 3 & -3 & 9 & 6 \end{bmatrix}$$

9. Find A^{-1} using the 2×2 inverse formula.

$$A = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}$$

$\rightarrow A^{-1} = (1/\det A) \cdot [[d, -b], [-c, a]]$.

$\rightarrow \det A = 4(6) - 7(2) = 10$.

$\rightarrow A^{-1} = (1/10) \cdot [[6, -7], [-2, 4]] = [[3/5, -7/10], [-1/5, 2/5]]$.

Answer:

$$\begin{bmatrix} 3/5 & -7/10 \\ -1/5 & 2/5 \end{bmatrix}$$



10. Verify $AB = I$ (confirm $B = A^{-1}$).

$$A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

→ Row 1·Col 1: $3(2)+1(-5)=1$. Row 1·Col 2: $3(-1)+1(3)=0$.

→ Row 2·Col 1: $5(2)+2(-5)=0$. Row 2·Col 2: $5(-1)+2(3)=1$.

→ $AB = I$. ✓ B is the inverse of A .

Answer: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

