MATHTEACH: Number Theory and Operations

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Name: ________________________________ Date: ________________ Score: / 30

Learning Objectives

- Apply place value understanding with whole numbers and decimals
- Find the GCD and LCM of two numbers
- Identify prime and composite numbers; apply divisibility rules
- Use order of operations to evaluate expressions

Simplify each expression completely. Show all steps and circle your final answer.

GCD (Euclidean algorithm)

1. Find the greatest common divisor (GCD) of 93 and 13.

$$\gcd(93, 13)$$

Answer: ____

2. Two pieces of rope have lengths 22 cm and 27 cm. What is the longest piece length that can divide both ropes evenly with no leftover?

$$\gcd(22, 27)$$

Answer: ____

3. Find the greatest common divisor (GCD) of 90 and 19.

$$\gcd(90, 19)$$

Answer: ____

4. Two pieces of rope have lengths 35 cm and 37 cm. What is the longest piece length that can divide both ropes evenly with no leftover?

$$\gcd(35, 37)$$

Answer: ____

5. Find the greatest common divisor (GCD) of 72 and 13.

$$\gcd(72, 13)$$

Answer: ____

6. Two pieces of rope have lengths 71 cm and 27 cm. What is the longest piece length that can divide both ropes evenly with no leftover?

$$\gcd(71, 27)$$

Answer: ______________________________________

LCM of two numbers

7. Find the least common multiple (LCM) of 17 and 6.

$$\operatorname{lcm}(17, 6)$$

Answer: ______________________________________

8. Bus A arrives every 12 minutes and Bus B arrives every 9 minutes. Both arrive at the stop at time 0. After how many minutes will they next arrive together?

$$\operatorname{lcm}(12, 9)$$

Answer: ______________________________________

9. Find the least common multiple (LCM) of 6 and 9.

$$\operatorname{lcm}(6, 9)$$

Answer: ______________________________________

10. Bus A arrives every 4 minutes and Bus B arrives every 10 minutes. Both arrive at the stop at time 0. After how many minutes will they next arrive together?

$$\operatorname{lcm}(4, 10)$$

Answer: ______________________________________

11. Find the least common multiple (LCM) of 6 and 12.

$$\operatorname{lcm}(6, 12)$$

Answer: ______________________________________

12. Bus A arrives every 6 minutes and Bus B arrives every 4 minutes. Both arrive at the stop at time 0. After how many minutes will they next arrive together?

$$\operatorname{lcm}(6, 4)$$

Answer: ______________________________________

Order of operations

13. Evaluate using the order of operations: 15 + 2 x 8. Remember: multiply before you add.

$$15 + 2 \times 8$$

Answer: ____

14. A store sells 3 items at $9 each, plus a $12 fee. What is the total: 12 + 3 x 9?

$$12 + 3 \times 9$$

Answer: ____

15. Evaluate: (9 + 6) x 3. Parentheses come first!

$$(9 + 6) \times 3$$

Answer: ____

16. 8 groups each containing (3 + 7) students. How many students total? Evaluate (3 + 7) x 8.

$$(3 + 7) \times 8$$

Answer: ____

17. Evaluate using the order of operations: 1 + 4 x 2. Remember: multiply before you add.

$$1 + 4 \times 2$$

Answer: ____

18. A store sells 5 items at $6 each, plus a $19 fee. What is the total: 19 + 5 x 6?

$$19 + 5 \times 6$$

Answer: ____

19. Evaluate: (4 + 2) x 7. Parentheses come first!

$$(4 + 2) \times 7$$

Answer: ____

20. 4 groups each containing (6 + 6) students. How many students total? Evaluate (6 + 6) x 4.

$$(6 + 6) \times 4$$

Answer: ____

21. Evaluate using the order of operations: 12 + 5 x 3. Remember: multiply before you add.

$$12 + 5 \times 3$$

Answer: ____

22. A store sells 3 items at $5 each, plus a $2 fee. What is the total: 2 + 3 x 5?

$$2 + 3 \times 5$$

Answer: ____

23. Evaluate: (2 + 5) x 7. Parentheses come first!

$$(2 + 5) \times 7$$

Answer: ____

24. 2 groups each containing (7 + 10) students. How many students total? Evaluate (7 + 10) x 2.

$$(7 + 10) \times 2$$

Answer: ____

Prime factorization & prime numbers

25. Write the prime factorization of 45 (use exponents).

$$n = 45$$

Answer: ____

26. Write the prime factorization of 63 (use exponents).

$$n = 63$$

Answer: ____

27. Write the prime factorization of 84 (use exponents).

$$n = 84$$

Answer: ____

28. Write the prime factorization of 45 (use exponents).

$$n = 45$$

Answer: ____

29. Write the prime factorization of 90 (use exponents).

$$n = 90$$

Answer: ____

30. Write the prime factorization of 99 (use exponents).

$$n = 99$$

Answer: ____

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ANSWER KEY & SOLUTIONS

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Topics: LCM of two numbers, GCD (Euclidean algorithm), Prime factorization & prime numbers, Order of operations. All answers verified by independent computation.

Solutions

GCD (Euclidean algorithm)

1. Find the greatest common divisor (GCD) of 93 and 13.

$$\gcd(93, 13)$$

→ *Use the Euclidean algorithm: repeatedly replace (a, b) with (b, a mod b) until b = 0.*
→ *The last nonzero remainder is the GCD.*
→ *GCD(93, 13) = 1.*

Answer: $\gcd(93, 13) = 1$

2. Two pieces of rope have lengths 22 cm and 27 cm. What is the longest piece length that can divide both ropes evenly with no leftover?

$$\gcd(22, 27)$$

→ *We need the greatest common divisor, which is the largest number dividing both evenly.*
→ *Apply the Euclidean algorithm to 22 and 27.*
→ *GCD(22, 27) = 1 cm.*

Answer: $\gcd(22, 27) = 1$

3. Find the greatest common divisor (GCD) of 90 and 19.

$$\gcd(90, 19)$$

→ *Use the Euclidean algorithm: repeatedly replace (a, b) with (b, a mod b) until b = 0.*
→ *The last nonzero remainder is the GCD.*
→ *GCD(90, 19) = 1.*

Answer: $\gcd(90, 19) = 1$

4. Two pieces of rope have lengths 35 cm and 37 cm. What is the longest piece length that can divide both ropes evenly with no leftover?

$$\gcd(35, 37)$$

→ *We need the greatest common divisor, which is the largest number dividing both evenly.*
→ *Apply the Euclidean algorithm to 35 and 37.*
→ *GCD(35, 37) = 1 cm.*

Answer: $\gcd(35, 37) = 1$

5. Find the greatest common divisor (GCD) of 72 and 13.

$$\gcd(72, 13)$$

→ *Use the Euclidean algorithm: repeatedly replace (a, b) with (b, a mod b) until b = 0.*
→ *The last nonzero remainder is the GCD.*
→ *GCD(72, 13) = 1.*

Answer: $\gcd(72, 13) = 1$

6. Two pieces of rope have lengths 71 cm and 27 cm. What is the longest piece length that can divide both ropes evenly with no leftover?

$$\gcd(71, 27)$$

→ *We need the greatest common divisor, which is the largest number dividing both evenly.*
→ *Apply the Euclidean algorithm to 71 and 27.*
→ *GCD(71, 27) = 1 cm.*

Answer: $\gcd(71, 27) = 1$

LCM of two numbers

7. Find the least common multiple (LCM) of 17 and 6.

$$\text{lcm}(17, 6)$$

→ *Use the formula: LCM(a,b) = a*b / GCD(a,b).*
→ *GCD(17,6) = 1.*
→ *LCM(17,6) = (17 * 6) / 1 = 102 / 1 = 102.*

Answer: $$\text{lcm}(17, 6) = \frac{102}{1} = 102$$

---

8. Bus A arrives every 12 minutes and Bus B arrives every 9 minutes. Both arrive at the stop at time 0. After how many minutes will they next arrive together?

$$\text{lcm}(12, 9)$$

→ *The next time both buses arrive together is the LCM of their intervals.*
→ *GCD(12,9) = 3.*
→ *LCM(12,9) = 108 / 3 = 36 minutes.*

Answer: $$\text{lcm}(12, 9) = \frac{108}{3} = 36$$

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9. Find the least common multiple (LCM) of 6 and 9.

$$\text{lcm}(6, 9)$$

→ *Use the formula: LCM(a,b) = a*b / GCD(a,b).*
→ *GCD(6,9) = 3.*
→ *LCM(6,9) = (6 * 9) / 3 = 54 / 3 = 18.*

Answer: $$\text{lcm}(6, 9) = \frac{54}{3} = 18$$

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10. Bus A arrives every 4 minutes and Bus B arrives every 10 minutes. Both arrive at the stop at time 0. After how many minutes will they next arrive together?

$$\text{lcm}(4, 10)$$

→ *The next time both buses arrive together is the LCM of their intervals.*
→ *GCD(4,10) = 2.*
→ *LCM(4,10) = 40 / 2 = 20 minutes.*

Answer: $$\text{lcm}(4, 10) = \frac{40}{2} = 20$$

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11. Find the least common multiple (LCM) of 6 and 12.

$$\text{lcm}(6, 12)$$

→ *Use the formula: LCM(a,b) = a*b / GCD(a,b).*
→ *GCD(6,12) = 6.*
→ *LCM(6,12) = (6 * 12) / 6 = 72 / 6 = 12.*

Answer: $$\text{lcm}(6, 12) = \frac{72}{6} = 12$$

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12. Bus A arrives every 6 minutes and Bus B arrives every 4 minutes. Both arrive at the stop at time 0. After how many minutes will they next arrive together?

$$\operatorname{lcm}(6,4)$$

→ The next time both buses arrive together is the LCM of their intervals.
→ GCD(6,4) = 2.
→ LCM(6,4) = 24 / 2 = 12 minutes.

Answer: $$\operatorname{lcm}(6,4) = \frac{24}{2} = 12$$

Order of operations

13. Evaluate using the order of operations: 15 + 2 x 8. Remember: multiply before you add.

$$15 + 2 \times 8$$

→ *Step 1: Multiply first: 2 x 8 = 16.*
→ *Step 2: Then add: 15 + 16 = 31.*

Answer: $15 + 16 = 31$

14. A store sells 3 items at $9 each, plus a $12 fee. What is the total: 12 + 3 x 9?

$$12 + 3 \times 9$$

→ *Multiply first: 3 x 9 = 27. Then add: 12 + 27 = 39.*

Answer: $12 + 27 = 39$

15. Evaluate: (9 + 6) x 3. Parentheses come first!

$$(9 + 6) \times 3$$

→ *Step 1: Add inside parentheses: 9 + 6 = 15.*
→ *Step 2: Multiply: 15 x 3 = 45.*

Answer: $15 \times 3 = 45$

16. 8 groups each containing (3 + 7) students. How many students total? Evaluate (3 + 7) x 8.

$$(3 + 7) \times 8$$

→ *(3 + 7) x 8 = 10 x 8 = 80.*

Answer: $10 \times 8 = 80$

17. Evaluate using the order of operations: 1 + 4 x 2. Remember: multiply before you add.

$$1 + 4 \times 2$$

→ *Step 1: Multiply first: 4 x 2 = 8.*
→ *Step 2: Then add: 1 + 8 = 9.*

Answer: $1 + 8 = 9$

18. A store sells 5 items at $6 each, plus a $19 fee. What is the total: 19 + 5 x 6?

$$19 + 5 \times 6$$

→ *Multiply first: 5 x 6 = 30. Then add: 19 + 30 = 49.*

Answer: $19 + 30 = 49$

19. Evaluate: (4 + 2) x 7. Parentheses come first!

$$(4 + 2) \times 7$$

→ *Step 1: Add inside parentheses: 4 + 2 = 6.*
→ *Step 2: Multiply: 6 x 7 = 42.*

Answer: $6 \times 7 = 42$

20. 4 groups each containing (6 + 6) students. How many students total? Evaluate (6 + 6) x 4.

$$(6 + 6) \times 4$$

→ (6 + 6) x 4 = 12 x 4 = 48.

Answer: $12 \times 4 = 48$

---

21. Evaluate using the order of operations: 12 + 5 x 3. Remember: multiply before you add.

$$12 + 5 \times 3$$

→ Step 1: Multiply first: 5 x 3 = 15.
→ Step 2: Then add: 12 + 15 = 27.

Answer: $12 + 15 = 27$

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22. A store sells 3 items at $5 each, plus a $2 fee. What is the total: 2 + 3 x 5?

$$2 + 3 \times 5$$

→ Multiply first: 3 x 5 = 15. Then add: 2 + 15 = 17.

Answer: $2 + 15 = 17$

---

23. Evaluate: (2 + 5) x 7. Parentheses come first!

$$(2 + 5) \times 7$$

→ Step 1: Add inside parentheses: 2 + 5 = 7.
→ Step 2: Multiply: 7 x 7 = 49.

Answer: $7 \times 7 = 49$

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24. 2 groups each containing (7 + 10) students. How many students total? Evaluate (7 + 10) x 2.

$$(7 + 10) \times 2$$

→ (7 + 10) x 2 = 17 x 2 = 34.

Answer: $17 \times 2 = 34$

Prime factorization & prime numbers

25. Write the prime factorization of 45 (use exponents).

$n = 45$

→ *Divide 45 by the smallest prime that goes in evenly; repeat on each quotient.*
→ *Collect repeated primes as powers.*
→ *45 = 3 x 3 x 5 = 3^2 x 5.*

Answer: $45 = 3^2 \times 5$

---

26. Write the prime factorization of 63 (use exponents).

$n = 63$

→ *Divide 63 by the smallest prime that goes in evenly; repeat on each quotient.*
→ *Collect repeated primes as powers.*
→ *63 = 3 x 3 x 7 = 3^2 x 7.*

Answer: $63 = 3^2 \times 7$

---

27. Write the prime factorization of 84 (use exponents).

$n = 84$

→ *Divide 84 by the smallest prime that goes in evenly; repeat on each quotient.*
→ *Collect repeated primes as powers.*
→ *84 = 2 x 2 x 3 x 7 = 2^2 x 3 x 7.*

Answer: $84 = 2^2 \times 3 \times 7$

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28. Write the prime factorization of 45 (use exponents).

$n = 45$

→ *Divide 45 by the smallest prime that goes in evenly; repeat on each quotient.*
→ *Collect repeated primes as powers.*
→ *45 = 3 x 3 x 5 = 3^2 x 5.*

Answer: $45 = 3^2 \times 5$

---

29. Write the prime factorization of 90 (use exponents).

$n = 90$

→ *Divide 90 by the smallest prime that goes in evenly; repeat on each quotient.*
→ *Collect repeated primes as powers.*
→ *90 = 2 x 3 x 3 x 5 = 2 x 3^2 x 5.*

Answer: $90 = 2 \times 3^2 \times 5$

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30. Write the prime factorization of 99 (use exponents).

$n = 99$

→ *Divide 99 by the smallest prime that goes in evenly; repeat on each quotient.*
→ *Collect repeated primes as powers.*
→ *99 = 3 x 3 x 11 = 3^2 x 11.*

Answer: $99 = 3^2 \times 11$
