MATH 110: Confidence Intervals — Worksheet

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Learning Objectives

- Identify the critical value z* for a confidence level
- Compute the standard error and margin of error
- Construct a confidence interval for a population mean

Simplify each expression completely. Show all steps and circle your final answer.

Confidence interval for a mean (σ known)

1. A sample of n = 16 has a sample mean of 131. The population standard deviation is sigma = 10. Construct the 99% confidence interval for the population mean mu.

Answer: _____

2. A sample of n = 100 has a sample mean of 84. The population standard deviation is sigma = 20. Construct the 95% confidence interval for the population mean mu.

Answer: _____

3. A sample of n = 25 has a sample mean of 120. The population standard deviation is sigma = 15. Construct the 99% confidence interval for the population mean mu.

Answer: _____

4. A sample of n = 25 has a sample mean of 81. The population standard deviation is sigma = 12. Construct the 90% confidence interval for the population mean mu.

Answer: _____

5. A sample of n = 36 has a sample mean of 67. The population standard deviation is sigma = 20. Construct the 95% confidence interval for the population mean mu.

Answer: _____

6. A sample of n = 16 has a sample mean of 55. The population standard deviation is sigma = 15. Construct the 90% confidence interval for the population mean mu.

Answer: _____

7. A sample of n = 25 has a sample mean of 89. The population standard deviation is sigma = 15. Construct the 95% confidence interval for the population mean mu.

Answer: _____

8. A sample of n = 36 has a sample mean of 106. The population standard deviation is sigma = 20. Construct the 95% confidence interval for the population mean mu.

Answer: _____

9. A sample of n = 100 has a sample mean of 59. The population standard deviation is sigma = 15. Construct the 90% confidence interval for the population mean mu.

Answer: _____

10. A sample of n = 16 has a sample mean of 137. The population standard deviation is sigma = 10. Construct the 90% confidence interval for the population mean mu.

Answer: _____

Confidence Interval for a Population Proportion

11. In a random sample of 246 people, 39 had a particular trait. Construct a 99% confidence interval for the true proportion.

Answer: _____

12. In a random sample of 304 people, 49 had a particular trait. Construct a 90% confidence interval for the true proportion.

Answer: _____

13. In a random sample of 276 people, 47 had a particular trait. Construct a 95% confidence interval for the true proportion.

Answer: _____

14. In a random sample of 127 people, 98 had a particular trait. Construct a 99% confidence interval for the true proportion.

Answer: _____

15. In a random sample of 200 people, 53 had a particular trait. Construct a 90% confidence interval for the true proportion.

Answer: _____

16. In a random sample of 200 people, 178 had a particular trait. Construct a 90% confidence interval for the true proportion.

Answer: _____

17. In a random sample of 193 people, 156 had a particular trait. Construct a 99% confidence interval for the true proportion.

Answer: _____

18. In a random sample of 163 people, 151 had a particular trait. Construct a 99% confidence interval for the true proportion.

Answer: _____

19. In a random sample of 159 people, 49 had a particular trait. Construct a 95% confidence interval for the true proportion.

Answer: _____

20. In a random sample of 250 people, 60 had a particular trait. Construct a 99% confidence interval for the true proportion.

Answer: _____

Confidence Interval for a Mean (t-distribution)

21. A sample of n = 25 gives x-bar = 86 and s = 19. Build a 95% confidence interval for the population mean using the t-distribution.

Answer: _____

22. A sample of n = 20 gives x-bar = 82 and s = 5. Build a 95% confidence interval for the population mean using the t-distribution.

Answer: _____

23. A sample of n = 15 gives x-bar = 91 and s = 11. Build a 95% confidence interval for the population mean using the t-distribution.

Answer: _____

24. A sample of n = 20 gives x-bar = 99 and s = 9. Build a 95% confidence interval for the population mean using the t-distribution.

Answer: _____

25. A sample of n = 5 gives x-bar = 114 and s = 20. Build a 95% confidence interval for the population mean using the t-distribution.

Answer: _____

26. A sample of n = 30 gives x-bar = 74 and s = 5. Build a 95% confidence interval for the population mean using the t-distribution.

Answer: _____

27. A sample of n = 5 gives x-bar = 119 and s = 8. Build a 95% confidence interval for the population mean using the t-distribution.

Answer: _____

28. A sample of n = 15 gives x-bar = 86 and s = 12. Build a 95% confidence interval for the population mean using the t-distribution.

Answer: _____

29. A sample of n = 30 gives x-bar = 107 and s = 17. Build a 95% confidence interval for the population mean using the t-distribution.

Answer: _____

30. A sample of n = 25 gives x-bar = 80 and s = 19. Build a 95% confidence interval for the population mean using the t-distribution.

Answer: _____

Answer Key & Solutions

Topics: Confidence Interval for a Mean (t-distribution), Confidence Interval for a Population Proportion, Confidence interval for a mean (σ known). All answers verified by independent computation.

Solutions

Confidence interval for a mean (σ known)

1. A sample of n = 16 has a sample mean of 131. The population standard deviation is sigma = 10. Construct the 99% confidence interval for the population mean mu.

    - Critical value z* for a 99% interval = 2.576.
    - Standard error = sigma / sqrt(n) = 10/4 = 2.5.
    - Margin of error E = z* x SE = 2.576 x 2.5 = 6.44.
    - CI = sample mean +/- E = 131 +/- 6.44 = (124.56, 137.44).

Answer: _____

2. A sample of n = 100 has a sample mean of 84. The population standard deviation is sigma = 20. Construct the 95% confidence interval for the population mean mu.

    - Critical value z* for a 95% interval = 1.96.
    - Standard error = sigma / sqrt(n) = 20/10 = 2.0.
    - Margin of error E = z* x SE = 1.96 x 2.0 = 3.92.
    - CI = sample mean +/- E = 84 +/- 3.92 = (80.08, 87.92).

Answer: _____

3. A sample of n = 25 has a sample mean of 120. The population standard deviation is sigma = 15. Construct the 99% confidence interval for the population mean mu.

    - Critical value z* for a 99% interval = 2.576.
    - Standard error = sigma / sqrt(n) = 15/5 = 3.0.
    - Margin of error E = z* x SE = 2.576 x 3.0 = 7.728.
    - CI = sample mean +/- E = 120 +/- 7.728 = (112.272, 127.728).

Answer: _____

4. A sample of n = 25 has a sample mean of 81. The population standard deviation is sigma = 12. Construct the 90% confidence interval for the population mean mu.

    - Critical value z* for a 90% interval = 1.645.
    - Standard error = sigma / sqrt(n) = 12/5 = 2.4.
    - Margin of error E = z* x SE = 1.645 x 2.4 = 3.948.
    - CI = sample mean +/- E = 81 +/- 3.948 = (77.052, 84.948).

Answer: _____

5. A sample of n = 36 has a sample mean of 67. The population standard deviation is sigma = 20. Construct the 95% confidence interval for the population mean mu.

    - Critical value z* for a 95% interval = 1.96.
    - Standard error = sigma / sqrt(n) = 20/6 = 3.3333.
    - Margin of error E = z* x SE = 1.96 x 3.3333 = 6.533.
    - CI = sample mean +/- E = 67 +/- 6.533 = (60.467, 73.533).

Answer: _____

6. A sample of n = 16 has a sample mean of 55. The population standard deviation is sigma = 15. Construct the 90% confidence interval for the population mean mu.

    - Critical value z* for a 90% interval = 1.645.
    - Standard error = sigma / sqrt(n) = 15/4 = 3.75.
    - Margin of error E = z* x SE = 1.645 x 3.75 = 6.169.
    - CI = sample mean +/- E = 55 +/- 6.169 = (48.831, 61.169).

Answer: _____

7. A sample of n = 25 has a sample mean of 89. The population standard deviation is sigma = 15. Construct the 95% confidence interval for the population mean mu.

    - Critical value z* for a 95% interval = 1.96.
    - Standard error = sigma / sqrt(n) = 15/5 = 3.0.
    - Margin of error E = z* x SE = 1.96 x 3.0 = 5.88.
    - CI = sample mean +/- E = 89 +/- 5.88 = (83.12, 94.88).

Answer: _____

8. A sample of n = 36 has a sample mean of 106. The population standard deviation is sigma = 20. Construct the 95% confidence interval for the population mean mu.

    - Critical value z* for a 95% interval = 1.96.
    - Standard error = sigma / sqrt(n) = 20/6 = 3.3333.
    - Margin of error E = z* x SE = 1.96 x 3.3333 = 6.533.
    - CI = sample mean +/- E = 106 +/- 6.533 = (99.467, 112.533).

Answer: _____

9. A sample of n = 100 has a sample mean of 59. The population standard deviation is sigma = 15. Construct the 90% confidence interval for the population mean mu.

    - Critical value z* for a 90% interval = 1.645.
    - Standard error = sigma / sqrt(n) = 15/10 = 1.5.
    - Margin of error E = z* x SE = 1.645 x 1.5 = 2.468.
    - CI = sample mean +/- E = 59 +/- 2.468 = (56.532, 61.468).

Answer: _____

10. A sample of n = 16 has a sample mean of 137. The population standard deviation is sigma = 10. Construct the 90% confidence interval for the population mean mu.

    - Critical value z* for a 90% interval = 1.645.
    - Standard error = sigma / sqrt(n) = 10/4 = 2.5.
    - Margin of error E = z* x SE = 1.645 x 2.5 = 4.112.
    - CI = sample mean +/- E = 137 +/- 4.112 = (132.887, 141.113).

Answer: _____

Confidence Interval for a Population Proportion

11. In a random sample of 246 people, 39 had a particular trait. Construct a 99% confidence interval for the true proportion.

    - Sample proportion: p-hat = x/n = 39/246 = 0.1585.
    - Critical value: z* = 2.576 for 99% CI.
    - Standard error: SE = sqrt(p-hat*(1-p-hat)/n) = 0.0233.
    - Margin of error: E = z* * SE = 2.576 * 0.0233 = 0.06.
    - 99% CI: (0.1585 - 0.06, 0.1585 + 0.06) = (0.0985, 0.2185).

Answer: _____

12. In a random sample of 304 people, 49 had a particular trait. Construct a 90% confidence interval for the true proportion.

    - Sample proportion: p-hat = x/n = 49/304 = 0.1612.
    - Critical value: z* = 1.645 for 90% CI.
    - Standard error: SE = sqrt(p-hat*(1-p-hat)/n) = 0.0211.

- Margin of error: E = z* * SE = 1.645 * 0.0211 = 0.0347.
- 90% CI: (0.1612 - 0.0347, 0.1612 + 0.0347) = (0.1265, 0.1959).

Answer: _____

13. In a random sample of 276 people, 47 had a particular trait. Construct a 95% confidence interval for the true proportion.

- Sample proportion: p-hat = x/n = 47/276 = 0.1703.
- Critical value: z* = 1.96 for 95% CI.
- Standard error: SE = sqrt(p-hat*(1-p-hat)/n) = 0.0226.
- Margin of error: E = z* * SE = 1.96 * 0.0226 = 0.0443.
- 95% CI: (0.1703 - 0.0443, 0.1703 + 0.0443) = (0.126, 0.2146).

Answer: _____

14. In a random sample of 127 people, 98 had a particular trait. Construct a 99% confidence interval for the true proportion.

- Sample proportion: p-hat = x/n = 98/127 = 0.7717.
- Critical value: z* = 2.576 for 99% CI.
- Standard error: SE = sqrt(p-hat*(1-p-hat)/n) = 0.0372.
- Margin of error: E = z* * SE = 2.576 * 0.0372 = 0.0959.
- 99% CI: (0.7717 - 0.0959, 0.7717 + 0.0959) = (0.6758, 0.8676).

Answer: _____

15. In a random sample of 200 people, 53 had a particular trait. Construct a 90% confidence interval for the true proportion.

- Sample proportion: p-hat = x/n = 53/200 = 0.265.
- Critical value: z* = 1.645 for 90% CI.
- Standard error: SE = sqrt(p-hat*(1-p-hat)/n) = 0.0312.
- Margin of error: E = z* * SE = 1.645 * 0.0312 = 0.0513.
- 90% CI: (0.265 - 0.0513, 0.265 + 0.0513) = (0.2137, 0.3163).

Answer: _____

16. In a random sample of 200 people, 178 had a particular trait. Construct a 90% confidence interval for the true proportion.

- Sample proportion: p-hat = x/n = 178/200 = 0.89.
- Critical value: z* = 1.645 for 90% CI.
- Standard error: SE = sqrt(p-hat*(1-p-hat)/n) = 0.0221.
- Margin of error: E = z* * SE = 1.645 * 0.0221 = 0.0364.
- 90% CI: (0.89 - 0.0364, 0.89 + 0.0364) = (0.8536, 0.9264).

Answer: _____

17. In a random sample of 193 people, 156 had a particular trait. Construct a 99% confidence interval for the true proportion.

- Sample proportion: p-hat = x/n = 156/193 = 0.8083.
- Critical value: z* = 2.576 for 99% CI.
- Standard error: SE = sqrt(p-hat*(1-p-hat)/n) = 0.0283.
- Margin of error: E = z* * SE = 2.576 * 0.0283 = 0.073.
- 99% CI: (0.8083 - 0.073, 0.8083 + 0.073) = (0.7353, 0.8813).

Answer: _____

18. In a random sample of 163 people, 151 had a particular trait. Construct a 99% confidence interval for the true proportion.

- Sample proportion: p-hat = x/n = 151/163 = 0.9264.
- Critical value: z* = 2.576 for 99% CI.
- Standard error: SE = sqrt(p-hat*(1-p-hat)/n) = 0.0205.
- Margin of error: E = z* * SE = 2.576 * 0.0205 = 0.0527.
- 99% CI: (0.9264 - 0.0527, 0.9264 + 0.0527) = (0.8737, 0.9791).

Answer: _____

19. In a random sample of 159 people, 49 had a particular trait. Construct a 95% confidence interval for the true proportion.

- Sample proportion: p-hat = x/n = 49/159 = 0.3082.
- Critical value: z* = 1.96 for 95% CI.
- Standard error: SE = sqrt(p-hat*(1-p-hat)/n) = 0.0366.
- Margin of error: E = z* * SE = 1.96 * 0.0366 = 0.0718.
- 95% CI: (0.3082 - 0.0718, 0.3082 + 0.0718) = (0.2364, 0.38).

Answer: _____

20. In a random sample of 250 people, 60 had a particular trait. Construct a 99% confidence interval for the true proportion.

- Sample proportion: p-hat = x/n = 60/250 = 0.24.
- Critical value: z* = 2.576 for 99% CI.
- Standard error: SE = sqrt(p-hat*(1-p-hat)/n) = 0.027.
- Margin of error: E = z* * SE = 2.576 * 0.027 = 0.0696.
- 99% CI: (0.24 - 0.0696, 0.24 + 0.0696) = (0.1704, 0.3096).

Answer: _____

Confidence Interval for a Mean (t-distribution)

21. A sample of n = 25 gives x-bar = 86 and s = 19. Build a 95% confidence interval for the population mean using the t-distribution.

- Degrees of freedom: df = n - 1 = 25 - 1 = 24.
- Critical value: t* = 2.064 (95% CI, df=24).
- Standard error: SE = s / sqrt(n) = 19 / sqrt(25) = 3.8.
- Margin of error: E = t* * SE = 2.064 * 3.8 = 7.843.
- 95% CI: (86 - 7.843, 86 + 7.843) = (78.157, 93.843).

Answer: _____

22. A sample of n = 20 gives x-bar = 82 and s = 5. Build a 95% confidence interval for the population mean using the t-distribution.

- Degrees of freedom: df = n - 1 = 20 - 1 = 19.
- Critical value: t* = 2.093 (95% CI, df=19).
- Standard error: SE = s / sqrt(n) = 5 / sqrt(20) = 1.118.
- Margin of error: E = t* * SE = 2.093 * 1.118 = 2.34.
- 95% CI: (82 - 2.34, 82 + 2.34) = (79.66, 84.34).

Answer: _____

23. A sample of n = 15 gives x-bar = 91 and s = 11. Build a 95% confidence interval for the population mean using the t-distribution.

- Degrees of freedom: df = n - 1 = 15 - 1 = 14.
- Critical value: t* = 2.145 (95% CI, df=14).
- Standard error: SE = s / sqrt(n) = 11 / sqrt(15) = 2.8402.
- Margin of error: E = t* * SE = 2.145 * 2.8402 = 6.092.
- 95% CI: (91 - 6.092, 91 + 6.092) = (84.908, 97.092).

Answer: _____

24. A sample of n = 20 gives x-bar = 99 and s = 9. Build a 95% confidence interval for the population mean using the t-distribution.

- Degrees of freedom: df = n - 1 = 20 - 1 = 19.
- Critical value: t* = 2.093 (95% CI, df=19).
- Standard error: SE = s / sqrt(n) = 9 / sqrt(20) = 2.0125.
- Margin of error: E = t* * SE = 2.093 * 2.0125 = 4.212.
- 95% CI: (99 - 4.212, 99 + 4.212) = (94.788, 103.212).

Answer: _____

25. A sample of n = 5 gives x-bar = 114 and s = 20. Build a 95% confidence interval for the population mean using the t-distribution.

- Degrees of freedom: df = n - 1 = 5 - 1 = 4.
- Critical value: t* = 2.776 (95% CI, df=4).
- Standard error: SE = s / sqrt(n) = 20 / sqrt(5) = 8.9443.
- Margin of error: E = t* * SE = 2.776 * 8.9443 = 24.829.

- 95% CI: (114 - 24.829, 114 + 24.829) = (89.171, 138.829).

Answer: _____

26. A sample of n = 30 gives x-bar = 74 and s = 5. Build a 95% confidence interval for the population mean using the t-distribution.

- Degrees of freedom: df = n - 1 = 30 - 1 = 29.
- Critical value: t* = 2.045 (95% CI, df=29).
- Standard error: SE = s / sqrt(n) = 5 / sqrt(30) = 0.9129.
- Margin of error: E = t* * SE = 2.045 * 0.9129 = 1.867.
- 95% CI: (74 - 1.867, 74 + 1.867) = (72.133, 75.867).

Answer: _____

27. A sample of n = 5 gives x-bar = 119 and s = 8. Build a 95% confidence interval for the population mean using the t-distribution.

- Degrees of freedom: df = n - 1 = 5 - 1 = 4.
- Critical value: t* = 2.776 (95% CI, df=4).
- Standard error: SE = s / sqrt(n) = 8 / sqrt(5) = 3.5777.
- Margin of error: E = t* * SE = 2.776 * 3.5777 = 9.932.
- 95% CI: (119 - 9.932, 119 + 9.932) = (109.068, 128.932).

Answer: _____

28. A sample of n = 15 gives x-bar = 86 and s = 12. Build a 95% confidence interval for the population mean using the t-distribution.

- Degrees of freedom: df = n - 1 = 15 - 1 = 14.
- Critical value: t* = 2.145 (95% CI, df=14).
- Standard error: SE = s / sqrt(n) = 12 / sqrt(15) = 3.0984.
- Margin of error: E = t* * SE = 2.145 * 3.0984 = 6.646.
- 95% CI: (86 - 6.646, 86 + 6.646) = (79.354, 92.646).

Answer: _____

29. A sample of n = 30 gives x-bar = 107 and s = 17. Build a 95% confidence interval for the population mean using the t-distribution.

- Degrees of freedom: df = n - 1 = 30 - 1 = 29.
- Critical value: t* = 2.045 (95% CI, df=29).
- Standard error: SE = s / sqrt(n) = 17 / sqrt(30) = 3.1038.
- Margin of error: E = t* * SE = 2.045 * 3.1038 = 6.347.
- 95% CI: (107 - 6.347, 107 + 6.347) = (100.653, 113.347).

Answer: _____

30. A sample of n = 25 gives x-bar = 80 and s = 19. Build a 95% confidence interval for the population mean using the t-distribution.

- Degrees of freedom: df = n - 1 = 25 - 1 = 24.
- Critical value: t* = 2.064 (95% CI, df=24).
- Standard error: SE = s / sqrt(n) = 19 / sqrt(25) = 3.8.
- Margin of error: E = t* * SE = 2.064 * 3.8 = 7.843.
- 95% CI: (80 - 7.843, 80 + 7.843) = (72.157, 87.843).

Answer: _____