

MATH 110: Probability — Worksheet

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Learning Objectives

- Identify the sample space of a random experiment
- Compute classical (theoretical) probability
- Express probabilities as fractions and decimals

Simplify each expression completely. Show all steps and circle your final answer.

Binomial Probability

1. In 15 independent trials each with success probability 0.2, find $P(X = 1)$.

Answer: _____

2. In 10 independent trials each with success probability 0.5, find $P(X = 1)$.

Answer: _____

3. In 9 independent trials each with success probability 0.5, find $P(X = 1)$.

Answer: _____

4. In 5 independent trials each with success probability 0.2, find $P(X = 3)$.

Answer: _____

5. In 14 independent trials each with success probability 0.2, find $P(X = 2)$.

Answer: _____

6. In 6 independent trials each with success probability 0.5, find $P(X = 1)$.

Answer: _____

Classical (theoretical) probability

7. A bag contains 2 red, 6 blue, and 4 green marbles. One marble is drawn at random. Find $P(\text{red})$.

Answer: _____

8. A bag contains 7 red, 4 blue, and 6 green marbles. One marble is drawn at random. Find $P(\text{red})$.

Answer: _____

9. A bag contains 5 red, 9 blue, and 9 green marbles. One marble is drawn at random. Find $P(\text{red})$.

Answer: _____

10. A bag contains 5 red, 5 blue, and 5 green marbles. One marble is drawn at random. Find $P(\text{red})$.

Answer: _____

11. A bag contains 9 red, 9 blue, and 6 green marbles. One marble is drawn at random. Find $P(\text{red})$.

Answer: _____

12. A bag contains 9 red, 3 blue, and 5 green marbles. One marble is drawn at random. Find $P(\text{red})$.

Answer: _____

Geometric Distribution

13. Each trial has probability 0.5 of success. Find the probability that the first success occurs on trial 4.

Answer: _____

14. Each trial has probability 0.3 of success. Find the probability that the first success occurs on trial 1.

Answer: _____

15. Each trial has probability 0.4 of success. Find the probability that the first success occurs on trial 2.

Answer: _____

16. Each trial has probability 0.2 of success. Find the probability that the first success occurs on trial 2.

Answer: _____

17. Each trial has probability 0.2 of success. Find the probability that the first success occurs on trial 4.

Answer: _____

18. Each trial has probability 0.2 of success. Find the probability that the first success occurs on trial 6.

Answer: _____

Expected Value of a Discrete Random Variable

19. A random variable X has the following probability distribution. Values: [2, 4, 6, 8], Probabilities: [0.3, 0.3, 0.2, 0.2]. Find $E(X)$.

Answer: _____

20. A random variable X has the following probability distribution. Values: [0, 2, 4, 6], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find $E(X)$.

Answer: _____

21. A random variable X has the following probability distribution. Values: [0, 1, 2, 3], Probabilities: [0.2, 0.3, 0.3, 0.2]. Find $E(X)$.

Answer: _____

22. A random variable X has the following probability distribution. Values: [2, 4, 6, 8], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find $E(X)$.

Answer: _____

23. A random variable X has the following probability distribution. Values: [0, 2, 4, 6], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find $E(X)$.

Answer: _____

24. A random variable X has the following probability distribution. Values: [0, 1, 2, 3], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find $E(X)$.

Answer: _____

Conditional Probability from a Two-Way Table

25. A two-way table shows: Group A has 7 successes and 17 failures; Group B has 6 successes and 23 failures. Find $P(\text{success} \mid \text{Group A})$.

Answer: _____

26. A two-way table shows: Group A has 12 successes and 21 failures; Group B has 22 successes and 10 failures. Find $P(\text{success} \mid \text{Group A})$.

Answer: _____

27. A two-way table shows: Group A has 22 successes and 5 failures; Group B has 20 successes and 25 failures. Find $P(\text{success} \mid \text{Group A})$.

Answer: _____

28. A two-way table shows: Group A has 20 successes and 10 failures; Group B has 22 successes and 11 failures. Find $P(\text{success} \mid \text{Group A})$.

Answer: _____

29. A two-way table shows: Group A has 7 successes and 14 failures; Group B has 18 successes and 12 failures. Find $P(\text{success} \mid \text{Group A})$.

Answer: _____

30. A two-way table shows: Group A has 15 successes and 21 failures; Group B has 24 successes and 30 failures. Find $P(\text{success} \mid \text{Group A})$.

Answer: _____

Answer Key & Solutions

Topics: Expected Value of a Discrete Random Variable, Binomial Probability, Classical (theoretical) probability, Geometric Distribution, Conditional Probability from a Two-Way Table. All answers verified by independent computation.

Solutions

Binomial Probability

1. In 15 independent trials each with success probability 0.2, find $P(X = 1)$.

- Identify: $n=15$, $k=1$, $p=0.2$, $q=1-p=0.8$.
- Compute $C(n,k) = C(15,1) = 15$.
- Compute $p^k = 0.2^1$.
- Compute $q^{(n-k)} = 0.8^{(15-1)}$.
- $P(X=1) = 15 * 0.2^1 * 0.8^{(15-1)} = 0.1319$.

Answer: _____

2. In 10 independent trials each with success probability 0.5, find $P(X = 1)$.

- Identify: $n=10$, $k=1$, $p=0.5$, $q=1-p=0.5$.
- Compute $C(n,k) = C(10,1) = 10$.
- Compute $p^k = 0.5^1$.
- Compute $q^{(n-k)} = 0.5^{(10-1)}$.
- $P(X=1) = 10 * 0.5^1 * 0.5^{(10-1)} = 0.0098$.

Answer: _____

3. In 9 independent trials each with success probability 0.5, find $P(X = 1)$.

- Identify: $n=9$, $k=1$, $p=0.5$, $q=1-p=0.5$.
- Compute $C(n,k) = C(9,1) = 9$.

- Compute $p^k = 0.5^1$.
- Compute $q^{(n-k)} = 0.5^{(9-1)}$.
- $P(X=1) = 9 * 0.5^1 * 0.5^{(9-1)} = 0.0176$.

Answer: _____

4. In 5 independent trials each with success probability 0.2, find $P(X = 3)$.

- Identify: $n=5, k=3, p=0.2, q=1-p=0.8$.
- Compute $C(n,k) = C(5,3) = 10$.
- Compute $p^k = 0.2^3$.
- Compute $q^{(n-k)} = 0.8^{(5-3)}$.
- $P(X=3) = 10 * 0.2^3 * 0.8^{(5-3)} = 0.0512$.

Answer: _____

5. In 14 independent trials each with success probability 0.2, find $P(X = 2)$.

- Identify: $n=14, k=2, p=0.2, q=1-p=0.8$.
- Compute $C(n,k) = C(14,2) = 91$.
- Compute $p^k = 0.2^2$.
- Compute $q^{(n-k)} = 0.8^{(14-2)}$.
- $P(X=2) = 91 * 0.2^2 * 0.8^{(14-2)} = 0.2501$.

Answer: _____

6. In 6 independent trials each with success probability 0.5, find $P(X = 1)$.

- Identify: $n=6, k=1, p=0.5, q=1-p=0.5$.
- Compute $C(n,k) = C(6,1) = 6$.
- Compute $p^k = 0.5^1$.
- Compute $q^{(n-k)} = 0.5^{(6-1)}$.
- $P(X=1) = 6 * 0.5^1 * 0.5^{(6-1)} = 0.0938$.

Answer: _____

Classical (theoretical) probability

7. A bag contains 2 red, 6 blue, and 4 green marbles. One marble is drawn at random. Find $P(\text{red})$.

- Count the total outcomes: $2 + 6 + 4 = 12$.
- $P(\text{red}) = \text{favorable} / \text{total} = 2/12 = 1/6$.
- As a decimal: $P(\text{red}) = 0.1667$ (approx).

Answer: _____

8. A bag contains 7 red, 4 blue, and 6 green marbles. One marble is drawn at random. Find $P(\text{red})$.

- Count the total outcomes: $7 + 4 + 6 = 17$.
- $P(\text{red}) = \text{favorable} / \text{total} = 7/17 = 7/17$.
- As a decimal: $P(\text{red}) = 0.4118$ (approx).

Answer: _____

9. A bag contains 5 red, 9 blue, and 9 green marbles. One marble is drawn at random. Find $P(\text{red})$.

- Count the total outcomes: $5 + 9 + 9 = 23$.
- $P(\text{red}) = \text{favorable} / \text{total} = 5/23 = 5/23$.
- As a decimal: $P(\text{red}) = 0.2174$ (approx).

Answer: _____

10. A bag contains 5 red, 5 blue, and 5 green marbles. One marble is drawn at random. Find $P(\text{red})$.

- Count the total outcomes: $5 + 5 + 5 = 15$.
- $P(\text{red}) = \text{favorable} / \text{total} = 5/15 = 1/3$.
- As a decimal: $P(\text{red}) = 0.3333$ (approx).

Answer: _____

11. A bag contains 9 red, 9 blue, and 6 green marbles. One marble is drawn at random. Find $P(\text{red})$.

- Count the total outcomes: $9 + 9 + 6 = 24$.
- $P(\text{red}) = \text{favorable} / \text{total} = 9/24 = 3/8$.
- As a decimal: $P(\text{red}) = 0.375$ (approx).

Answer: _____

12. A bag contains 9 red, 3 blue, and 5 green marbles. One marble is drawn at random. Find $P(\text{red})$.

- Count the total outcomes: $9 + 3 + 5 = 17$.
- $P(\text{red}) = \text{favorable} / \text{total} = 9/17 = 9/17$.
- As a decimal: $P(\text{red}) = 0.5294$ (approx).

Answer: _____

Geometric Distribution

13. Each trial has probability 0.5 of success. Find the probability that the first success occurs on trial 4.

- Identify: $p=0.5$, $q=1-p=0.5$, $k=4$.
- $P(X=4) = q^{(k-1)} * p = 0.5^{(4-1)} * 0.5$.
- $P(X=4) = 0.0625$.

Answer: _____

14. Each trial has probability 0.3 of success. Find the probability that the first success occurs on trial 1.

- Identify: $p=0.3$, $q=1-p=0.7$, $k=1$.
- $P(X=1) = q^{(k-1)} * p = 0.7^{(1-1)} * 0.3$.
- $P(X=1) = 0.3$.

Answer: _____

15. Each trial has probability 0.4 of success. Find the probability that the first success occurs on trial 2.

- Identify: $p=0.4$, $q=1-p=0.6$, $k=2$.
- $P(X=2) = q^{(k-1)} * p = 0.6^{(2-1)} * 0.4$.
- $P(X=2) = 0.24$.

Answer: _____

16. Each trial has probability 0.2 of success. Find the probability that the first success occurs on trial 2.

- Identify: $p=0.2$, $q=1-p=0.8$, $k=2$.
- $P(X=2) = q^{(k-1)} * p = 0.8^{(2-1)} * 0.2$.
- $P(X=2) = 0.16$.

Answer: _____

17. Each trial has probability 0.2 of success. Find the probability that the first success occurs on trial 4.

- Identify: $p=0.2$, $q=1-p=0.8$, $k=4$.
- $P(X=4) = q^{(k-1)} * p = 0.8^{(4-1)} * 0.2$.
- $P(X=4) = 0.1024$.

Answer: _____

18. Each trial has probability 0.2 of success. Find the probability that the first success occurs on trial 6.

- Identify: $p=0.2$, $q=1-p=0.8$, $k=6$.
- $P(X=6) = q^{(k-1)} * p = 0.8^{(6-1)} * 0.2$.
- $P(X=6) = 0.0655$.

Answer: _____

Expected Value of a Discrete Random Variable

19. A random variable X has the following probability distribution. Values: [2, 4, 6, 8], Probabilities: [0.3, 0.3, 0.2, 0.2]. Find $E(X)$.

- Multiply each value by its probability: $2(0.3) + 4(0.3) + 6(0.2) + 8(0.2)$.
- Sum the products: $E(X) = 4.6$.

Answer: _____

20. A random variable X has the following probability distribution. Values: [0, 2, 4, 6], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find $E(X)$.

- Multiply each value by its probability: $0(0.25) + 2(0.25) + 4(0.25) + 6(0.25)$.
- Sum the products: $E(X) = 3.0$.

Answer: _____

21. A random variable X has the following probability distribution. Values: [0, 1, 2, 3], Probabilities: [0.2, 0.3, 0.3, 0.2]. Find $E(X)$.

- Multiply each value by its probability: $0(0.2) + 1(0.3) + 2(0.3) + 3(0.2)$.
- Sum the products: $E(X) = 1.5$.

Answer: _____

22. A random variable X has the following probability distribution. Values: [2, 4, 6, 8], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find $E(X)$.

- Multiply each value by its probability: $2(0.25) + 4(0.25) + 6(0.25) + 8(0.25)$.
- Sum the products: $E(X) = 5.0$.

Answer: _____

23. A random variable X has the following probability distribution. Values: [0, 2, 4, 6], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find $E(X)$.

- Multiply each value by its probability: $0(0.25) + 2(0.25) + 4(0.25) + 6(0.25)$.
- Sum the products: $E(X) = 3.0$.

Answer: _____

24. A random variable X has the following probability distribution. Values: [0, 1, 2, 3], Probabilities: [0.25, 0.25, 0.25, 0.25]. Find $E(X)$.

- Multiply each value by its probability: $0(0.25) + 1(0.25) + 2(0.25) + 3(0.25)$.
- Sum the products: $E(X) = 1.5$.

Answer: _____

Conditional Probability from a Two-Way Table

25. A two-way table shows: Group A has 7 successes and 17 failures; Group B has 6 successes and 23 failures. Find $P(\text{success} \mid \text{Group A})$.

- Column total for Group A: $7 + 6 = 13$.
- Successes in Group A: 7.
- $P(\text{success} \mid \text{Group A}) = 7 / 13 = 0.5385$.

Answer: _____

26. A two-way table shows: Group A has 12 successes and 21 failures; Group B has 22 successes and 10 failures. Find $P(\text{success} \mid \text{Group A})$.

- Column total for Group A: $12 + 22 = 34$.
- Successes in Group A: 12.
- $P(\text{success} \mid \text{Group A}) = 12 / 34 = 0.3529$.

Answer: _____

27. A two-way table shows: Group A has 22 successes and 5 failures; Group B has 20 successes and 25 failures. Find $P(\text{success} \mid \text{Group A})$.

- Column total for Group A: $22 + 20 = 42$.
- Successes in Group A: 22.
- $P(\text{success} \mid \text{Group A}) = 22 / 42 = 0.5238$.

Answer: _____

28. A two-way table shows: Group A has 20 successes and 10 failures; Group B has 22 successes and 11 failures. Find $P(\text{success} \mid \text{Group A})$.

- Column total for Group A: $20 + 22 = 42$.
- Successes in Group A: 20.
- $P(\text{success} \mid \text{Group A}) = 20 / 42 = 0.4762$.

Answer: _____

29. A two-way table shows: Group A has 7 successes and 14 failures; Group B has 18 successes and 12 failures. Find $P(\text{success} \mid \text{Group A})$.

- Column total for Group A: $7 + 18 = 25$.
- Successes in Group A: 7.
- $P(\text{success} \mid \text{Group A}) = 7 / 25 = 0.28$.

Answer: _____

30. A two-way table shows: Group A has 15 successes and 21 failures; Group B has 24 successes and 30 failures. Find $P(\text{success} \mid \text{Group A})$.

- Column total for Group A: $15 + 24 = 39$.
- Successes in Group A: 15.
- $P(\text{success} \mid \text{Group A}) = 15 / 39 = 0.3846$.

Answer: _____