Adding and Subtracting Rational Expressions and Solving Rational Equation

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Learning Objectives

- Solve linear equations using the inverse property
- Solve equations involving square roots and squares
- Add and subtract fractions and rational expressions using the butterfly technique and LCD
- Solve rational equations using cross multiplication and simplify rational expressions

Simplify each expression or solve each equation completely, showing all work and stating any restrictions on the variable.

1. Solve the linear equation using the inverse property.

$3x - 7 = 14$

Answer: _____

2. Solve the equation containing a square root.

$\sqrt{x+5} = 4$

Answer: _____

3. Solve the equation containing a square term.

$x^2 = 49$

Answer: _____

4. Add the fractions using the butterfly technique.

$\frac{2}{3} + \frac{4}{5}$

Answer: _____

5. Subtract the rational expressions and simplify.

$\frac{5}{x} - \frac{3}{x+2}$

Answer: _____

6. Add the rational expressions with the same denominator.

$\frac{3x}{x-1} + \frac{2}{x-1}$

Answer: _____

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7. Add the rational expressions by factoring the denominators first.

$$\frac{1}{x^2-4}+\frac{2}{x+2}$$

Answer: _____

8. Solve the rational equation using cross multiplication.

$$\frac{x+1}{4}=\frac{3}{2}$$

Answer: _____

9. Solve the rational equation using cross multiplication.

$$\frac{2}{x-3}=\frac{5}{x+1}$$

Answer: _____

10. Simplify the rational expression completely.

$$\frac{x^2-9}{x^2+5x+6}$$

Answer: _____

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ANSWER KEY & SOLUTIONS

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This worksheet covers the full scope of Week 2: review of solving linear equations with the inverse property, solving equations with square roots and squares, adding and subtracting numerical and rational fractions (including the butterfly technique), special cases for rational expressions requiring factoring, solving rational equations via cross multiplication, and additional practice simplifying rational expressions.

Solutions

1. Solve the linear equation using the inverse property.

$3x - 7 = 14$

→ *Add seven to both sides to undo the subtraction, giving three x equals twenty-one.*
→ *Divide both sides by three to isolate x, yielding x equals seven.*

Answer: $x = 7$

2. Solve the equation containing a square root.

$\sqrt{x+5} = 4$

→ *Square both sides to eliminate the radical, giving x plus five equals sixteen.*
→ *Subtract five from both sides to obtain x equals eleven.*
→ *Check by substituting back into the original equation, which gives the square root of sixteen equals four.*

Answer: $x = 11$

3. Solve the equation containing a square term.

$x^2 = 49$

→ *Take the square root of both sides to undo the square.*
→ *Remember to include both the positive and negative roots, giving x equals plus or minus seven.*

Answer: $x = \pm 7$

4. Add the fractions using the butterfly technique.

$\frac{2}{3} + \frac{4}{5}$

→ *Multiply the first numerator by the second denominator to get ten.*
→ *Multiply the second numerator by the first denominator to get twelve.*
→ *Add the two products to get twenty-two for the new numerator.*
→ *Multiply the denominators to get fifteen, giving twenty-two over fifteen.*

Answer: $\frac{22}{15}$

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5. Subtract the rational expressions and simplify.

$$\frac{5}{x}-\frac{3}{x+2}$$

→ *Use the butterfly technique by cross multiplying to find a common denominator of x times the quantity x plus two.*
→ *The first numerator becomes five times the quantity x plus two, which equals five x plus ten.*
→ *The second numerator becomes three x.*
→ *Subtract the numerators to get five x plus ten minus three x, which simplifies to two x plus ten over x times the quantity x plus two.*

Answer: $\frac{2x+10}{x(x+2)}$

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6. Add the rational expressions with the same denominator.

$$\frac{3x}{x-1}+\frac{2}{x-1}$$

→ *Since the denominators are identical, keep the common denominator x minus one.*
→ *Add the numerators directly to get three x plus two over x minus one.*

Answer: $\frac{3x+2}{x-1}$

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7. Add the rational expressions by factoring the denominators first.

$$\frac{1}{x^2-4}+\frac{2}{x+2}$$

→ *Factor x squared minus four as the product of x minus two and x plus two.*
→ *The least common denominator is the quantity x minus two times x plus two.*
→ *Rewrite the second fraction by multiplying its numerator and denominator by x minus two, giving two times x minus two over the common denominator.*
→ *Combine the numerators to get one plus two x minus four, which simplifies to two x minus three over the product of x minus two and x plus two.*

Answer: $\frac{2x-3}{(x-2)(x+2)}$

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8. Solve the rational equation using cross multiplication.

$$\frac{x+1}{4}=\frac{3}{2}$$

→ *Cross multiply to get two times the quantity x plus one equals four times three.*
→ *Simplify to two x plus two equals twelve.*
→ *Subtract two from both sides to get two x equals ten.*
→ *Divide both sides by two to obtain x equals five.*

Answer: $x=5$

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9. Solve the rational equation using cross multiplication.

$$\frac{2}{x-3}=\frac{5}{x+1}$$

→ *Cross multiply to get two times the quantity x plus one equals five times the quantity x minus three.*
→ *Expand both sides to get two x plus two equals five x minus fifteen.*
→ *Subtract two x from both sides to get two equals three x minus fifteen.*
→ *Add fifteen to both sides to get seventeen equals three x, so x equals seventeen over three.*

Answer: $x=\frac{17}{3}$

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10. Simplify the rational expression completely.

$$\frac{x^2 - 9}{x^2 + 5x + 6}$$

→ *Factor the numerator as a difference of squares into the quantity x minus three times x plus three.*
→ *Factor the denominator into the quantity x plus two times x plus three.*
→ *Cancel the common factor of x plus three from the numerator and denominator.*
→ *The simplified expression is x minus three over x plus two.*

Answer: $\frac{x - 3}{x + 2}$