Simplifying Rational Expressions: Factoring, Multiplying, and Dividing

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Learning Objectives

- Find the prime factorization of integers using a factor tree
- Simplify rational expressions by factoring numerator and denominator
- Multiply rational expressions and reduce to lowest terms
- Divide rational expressions using the reciprocal rule

Factor completely and simplify each rational expression, stating any restrictions where appropriate.

1. Warm-up: Factor the trinomial completely.

$x^2 + 7x + 12$

Answer: _____

2. Warm-up: Factor the difference of squares.

$9x^2 - 25$

Answer: _____

3. Use a factor tree to find the prime factorization.

72

Answer: _____

4. Use a factor tree to find the prime factorization.

180

Answer: _____

5. State the values of x for which the rational expression is undefined.

$$\frac{x+2}{x^2-9}$$

Answer: _____

6. Simplify the rational expression.

$$\frac{x^2-16}{x^2+8x+16}$$

Answer: _____

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7. Simplify the rational expression.

$$\frac{2x^2 - 8}{x^2 - x - 2}$$

Answer: _____

8. Multiply the rational expressions and simplify.

$$\frac{x^2 - 1}{x + 3} \cdot \frac{x + 3}{x - 1}$$

Answer: _____

9. Multiply the rational expressions and simplify.

$$\frac{x^2 + 5x + 6}{x^2 - 4} \cdot \frac{x - 2}{x + 3}$$

Answer: _____

10. Divide the rational expressions and simplify.

$$\frac{x^2 - 9}{x^2 + 2x} \div \frac{x - 3}{x + 2}$$

Answer: _____

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ANSWER KEY & SOLUTIONS

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This worksheet mirrors the video lesson: a factoring warm-up, prime factorization via factor trees, identifying rational expressions, simplifying rationals, multiplying rationals, and dividing rationals. Problems progress from numerical factoring to polynomial operations on rational expressions.

Solutions

1. Warm-up: Factor the trinomial completely.

$x^2 + 7x + 12$

→ *Find two numbers whose product is 12 and whose sum is 7.*
→ *The numbers 3 and 4 work since 3 times 4 equals 12 and 3 plus 4 equals 7.*
→ *Write the trinomial as the product of two binomials.*

Answer: $(x + 3)(x + 4)$

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2. Warm-up: Factor the difference of squares.

$9x^2 - 25$

→ *Recognize the expression as a difference of two perfect squares.*
→ *The square root of 9x squared is 3x and the square root of 25 is 5.*
→ *Apply the difference of squares pattern to write the factors.*

Answer: $(3x - 5)(3x + 5)$

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3. Use a factor tree to find the prime factorization.

72

→ *Split 72 into 8 times 9.*
→ *Break 8 into 2 times 2 times 2 and break 9 into 3 times 3.*
→ *Collect the prime factors and write them using exponents.*

Answer: $2^3 \cdot 3^2$

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4. Use a factor tree to find the prime factorization.

180

→ *Split 180 into 18 times 10.*
→ *Break 18 into 2 times 3 times 3 and break 10 into 2 times 5.*
→ *Combine the prime factors and write them using exponents.*

Answer: $2^2 \cdot 3^2 \cdot 5$

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5. State the values of x for which the rational expression is undefined.

$\frac{x+2}{x^2-9}$

→ *Set the denominator equal to zero.*
→ *Factor x squared minus 9 as the product of x minus 3 and x plus 3.*
→ *Solve each factor for x to find the restricted values.*

Answer: $x = 3,\ x = -3$

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6. Simplify the rational expression.

$$\frac{x^2-16}{x^2+8x+16}$$

→ *Factor the numerator as a difference of squares into x minus 4 times x plus 4.*
→ *Factor the denominator as a perfect square trinomial into x plus 4 squared.*
→ *Cancel the common factor of x plus 4 from numerator and denominator.*

Answer: $\frac{x-4}{x+4}$

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7. Simplify the rational expression.

$$\frac{2x^2-8}{x^2-x-2}$$

→ *Factor the numerator: pull out 2 to get 2 times the quantity x squared minus 4, then factor as 2 times x minus 2 times x plus 2.*
→ *Factor the denominator into x minus 2 times x plus 1.*
→ *Cancel the common factor x minus 2 to obtain the simplified expression.*

Answer: $\frac{2(x-2)}{x-2}$ reduces to $\frac{2(x+2)}{x+1}$

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8. Multiply the rational expressions and simplify.

$$\frac{x^2-1}{x+3}\cdot\frac{x+3}{x-1}$$

→ *Factor the numerator x squared minus 1 as x minus 1 times x plus 1.*
→ *Multiply across, then cancel the common factor of x plus 3 and the common factor of x minus 1.*
→ *The remaining factor x plus 1 is the simplified product.*

Answer: $x+1$

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9. Multiply the rational expressions and simplify.

$$\frac{x^2+5x+6}{x^2-4}\cdot\frac{x-2}{x+3}$$

→ *Factor each polynomial: the first numerator becomes x plus 2 times x plus 3 and the first denominator becomes x minus 2 times x plus 2.*
→ *Write the product as one fraction and cancel the common factors x plus 2, x plus 3, and x minus 2.*
→ *All factors cancel, leaving a value of 1.*

Answer: 1

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10. Divide the rational expressions and simplify.

$$\frac{x^2-9}{x^2+2x}\div\frac{x-3}{x+2}$$

→ *Rewrite the division as multiplication by the reciprocal of the second fraction.*
→ *Factor each polynomial: x squared minus 9 becomes x minus 3 times x plus 3, and x squared plus 2x becomes x times x plus 2.*
→ *Cancel the common factors x minus 3 and x plus 2 to obtain the simplified quotient.*

Answer: $\frac{(x+3)(x+2)}{x(x+2)}$ reduces to $\frac{x+3}{x}$