Conic Sections: Introduction & Parabolas

Analytic Geometry Worksheet · Grade 10–12

Name: ________________________________ Date: __________________

Learning Objectives

- Identify and describe the four conic sections (circle, ellipse, parabola, hyperbola) as intersections of a plane and a double-napped cone
- Recognize and apply the standard equations of a parabola opening vertically or horizontally
- Determine the vertex, focus, directrix, and axis of symmetry of a parabola from its equation or graph

Problems

1. Name all four conic sections formed by the intersection of a plane and a double-napped cone.

2. A parabola is defined as the set of all points that are equidistant from two special features. Name both features.

3. For the parabola with the equation shown, identify whether it opens upward, downward, left, or right, and state the value of p.

$$(x-0)^2 = 8(y-0)$$

4. Identify the vertex, focus, and directrix of the parabola given by the equation shown.

$$x^2 = 12y$$

5. Identify the vertex, focus, and directrix of the parabola given by the equation shown.

$$y^2 = -16x$$

6. Write the standard equation of a parabola that opens to the right with vertex at (2, -3) and p = 5.

7. Find the vertex, focus, directrix, and axis of symmetry for the parabola given by the equation shown.

Scan to watch

$$(x-3)^2 = -8(y+1)$$

8. Convert the equation shown to standard form, then find the vertex and the value of p.

$$y^2 - 6y - 4x + 1 = 0$$

9. A parabola has its focus at (1, 4) and its directrix at y = -2. Write the standard equation of the parabola.

10. Convert the equation shown to standard form, identify all key features (vertex, focus, directrix, axis of symmetry, and direction of opening), and determine the length of the latus rectum.

$$x^2 + 8x + 12y - 20 = 0$$

Conic Sections: Introduction & Parabolas — Answer Key

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Answer Key

1. Answer: Circle, Ellipse, Parabola, Hyperbola

- A double-napped cone cut by a plane produces four distinct curves.
- Slicing horizontally → Circle; slicing at an angle through one nappe → Ellipse; slicing parallel to the slant edge → Parabola; slicing through both nappes → Hyperbola.

2. Answer: The focus (fixed point) and the directrix (fixed line)

- By definition, every point P on the parabola satisfies: distance from P to the focus = distance from P to the directrix.
- The focus is a fixed interior point; the directrix is a fixed line outside the parabola.

3. Answer: Opens upward; p = 2

- The equation is in the form $(x - h)^2 = 4p(y - k)$, which describes a vertical parabola.
- Here $4p = 8$, so $p = 2$. Since $p > 0$, the parabola opens upward.

4. Answer: Vertex (0,0); Focus (0,3); Directrix y = -3

- The equation is $x^2 = 4py$, so $4p = 12$ and $p = 3$.
- Vertex is at $(h, k) = (0, 0)$.
- Focus is at $(0, 0 + 3) = (0, 3)$.
- Directrix is $y = 0 - 3 = -3$.

5. Answer: Vertex (0,0); Focus (-4,0); Directrix x = 4

- The equation is $y^2 = 4px$, so $4p = -16$ and $p = -4$.
- Vertex is at $(0, 0)$.
- Since $p < 0$, the parabola opens to the left.
- Focus is at $(0 + (-4), 0) = (-4, 0)$; Directrix is $x = 0 - (-4) = 4$.

6. Answer: (y + 3)^2 = 20(x - 2)

- A parabola opening sideways uses the form $(y - k)^2 = 4p(x - h)$.
- Substitute $h = 2$, $k = -3$, and $p = 5$: $(y - (-3))^2 = 4(5)(x - 2)$.
- Simplify: $(y + 3)^2 = 20(x - 2)$.

7. Answer: Vertex (3,-1); Focus (3,-3); Directrix y = 1; Axis of symmetry x = 3

- Form is $(x - h)^2 = 4p(y - k)$, so $h = 3$, $k = -1$, and $4p = -8$, giving $p = -2$.
- Since $p < 0$, the parabola opens downward.
- Vertex: $(3, -1)$.
- Focus: $(3, -1 + (-2)) = (3, -3)$.
- Directrix: $y = -1 - (-2) = 1$.

Scan to watch

- Axis of symmetry: $x = 3$ (vertical line through the vertex).

8. Answer: $(y - 3)^2 = 4(x + 2)$; Vertex $(-2, 3)$; $p = 1$

- Group y-terms: $y^2 - 6y = 4x - 1$.
- Complete the square: $y^2 - 6y + 9 = 4x - 1 + 9$, so $(y - 3)^2 = 4x + 8$.
- Factor right side: $(y - 3)^2 = 4(x + 2)$.
- Standard form $(y - k)^2 = 4p(x - h)$: $h = -2$, $k = 3$, $4p = 4$, so $p = 1$.
- Vertex is $(-2, 3)$; parabola opens to the right.

9. Answer: $(x - 1)^2 = 12(y - 1)$

- The vertex lies midway between the focus and directrix: $k = (4 + (-2))/2 = 1$, $h = 1$. Vertex: $(1, 1)$.
- The value of p equals the distance from vertex to focus: $p = 4 - 1 = 3$.
- Since the focus is above the directrix, the parabola opens upward, using $(x - h)^2 = 4p(y - k)$.
- Substitute: $(x - 1)^2 = 4(3)(y - 1) = 12(y - 1)$.

10. Answer: $(x+4)^2 = -12(y-3)$; Vertex $(-4,3)$; Focus $(-4,0)$; Directrix $y=6$; Axis $x=-4$; Opens downward; Latus rectum = 12

- Rearrange: $x^2 + 8x = -12y + 20$.
- Complete the square: $x^2 + 8x + 16 = -12y + 20 + 16$, so $(x + 4)^2 = -12y + 36$.
- Factor: $(x + 4)^2 = -12(y - 3)$.
- Standard form $(x - h)^2 = 4p(y - k)$: $h = -4$, $k = 3$, $4p = -12$, $p = -3$.
- Since $p < 0$, the parabola opens downward.
- Vertex: $(-4, 3)$; Focus: $(-4, 3 + (-3)) = (-4, 0)$; Directrix: $y = 3 - (-3) = 6$.
- Axis of symmetry: $x = -4$.
- Length of latus rectum $= |4p| = |-12| = 12$.