Conic Sections: Parabolas

Analytic Geometry Worksheet · Grade 10–12

Name: _____ Date: _____

Learning Objectives

- Identify the vertex, focus, and directrix of a parabola from its equation or graph
- Write the standard equation of a parabola opening vertically or horizontally
- Determine the direction a parabola opens based on the sign of p

Problems

1. A parabola has its vertex at the origin and opens upward with p = 2. Write its standard equation.

$$(x-0)^2 = 4(2)(y-0)$$

2. Identify the direction a parabola opens if its equation is given below and p is negative.

$$(x-h)^2 = 4p(y-k), \quad p<0$$

3. A parabola opens to the right with vertex at (0, 0) and p = 3. Write its standard equation.

$$(y-k)^2 = 4p(x-h)$$

4. Find the focus and directrix of the parabola given below.

$$x^2 = 16y$$

5. Find the vertex, focus, and directrix of the parabola given below.

$$(x-2)^2 = -8(y-3)$$

6. Find the vertex, focus, and directrix of the parabola given below.

$$(y+1)^2 = -12(x-4)$$

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7. Write the standard equation of a parabola with vertex at (3, -2) and focus at (3, 2).

$$(x-h)^2 = 4p(y-k)$$

8. Convert the equation below to standard form and identify the vertex, focus, and directrix.

$$x^2 - 6x - 8y + 1 = 0$$

9. A parabola has its vertex at (-1, 4) and its directrix at x = 2. Write its standard equation.

$$(y-k)^2 = 4p(x-h)$$

10. Convert the equation below to standard form, then find the vertex, focus, directrix, and axis of symmetry.

$$y^2 + 10y - 4x + 29 = 0$$

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Conic Sections: Parabolas — Answer Key

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Answer Key

1. Answer: x^2 = 8y

- The parabola opens vertically upward, so use (x - h)^2 = 4p(y - k).
- Vertex is (0, 0), so h = 0 and k = 0.
- p = 2, so 4p = 8.
- Standard equation: x^2 = 8y.

2. Answer: Opens downward

- The equation (x - h)^2 = 4p(y - k) represents a vertical parabola.
- When p > 0, it opens upward; when p < 0, it opens downward.
- Since p is negative, the parabola opens downward.

3. Answer: y^2 = 12x

- The parabola opens horizontally to the right, so use (y - k)^2 = 4p(x - h).
- Vertex is (0, 0), so h = 0 and k = 0.
- p = 3, so 4p = 12.
- Standard equation: y^2 = 12x.

4. Answer: Focus: (0, 4); Directrix: y = -4

- Compare x^2 = 16y with x^2 = 4py: 4p = 16, so p = 4.
- Since p > 0 the parabola opens upward with vertex at (0, 0).
- Focus is at (h, k + p) = (0, 0 + 4) = (0, 4).
- Directrix is y = k - p = 0 - 4 = -4.

5. Answer: Vertex: (2, 3); Focus: (2, 1); Directrix: y = 5

- Vertex is (h, k) = (2, 3).
- 4p = -8, so p = -2. Negative p means it opens downward.
- Focus is at (h, k + p) = (2, 3 + (-2)) = (2, 1).
- Directrix is y = k - p = 3 - (-2) = 5.

6. Answer: Vertex: (4, -1); Focus: (1, -1); Directrix: x = 7

- Vertex is (h, k) = (4, -1).
- 4p = -12, so p = -3. Negative p means it opens to the left.
- Focus is at (h + p, k) = (4 + (-3), -1) = (1, -1).
- Directrix is x = h - p = 4 - (-3) = 7.

7. Answer: (x - 3)^2 = 16(y + 2)

- Vertex is (3, -2) and focus is (3, 2); same x-coordinate means vertical parabola.
- p = distance from vertex to focus = 2 - (-2) = 4.
- 4p = 16.

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- Equation: (x - 3)^2 = 16(y + 2).

8. Answer: (x - 3)^2 = 8(y + 1); Vertex: (3, -1); Focus: (3, 1); Directrix: y = -3

- Group and complete the square: (x^2 - 6x) = 8y - 1.
- Complete the square: (x - 3)^2 - 9 = 8y - 1.
- (x - 3)^2 = 8y - 1 + 9 = 8y + 8 = 8(y + 1).
- Standard form: (x - 3)^2 = 8(y + 1); vertex (3, -1), p = 2, focus (3, 1), directrix y = -3.

9. Answer: (y - 4)^2 = -12(x + 1)

- The directrix is a vertical line x = 2, so the parabola opens horizontally.
- Vertex is (-1, 4). The directrix x = 2 is to the right of vertex, so it opens left.
- p = distance from vertex to directrix = -1 - 2 = -3.
- 4p = -12. Equation: (y - 4)^2 = -12(x + 1).

10. Answer: (y + 5)^2 = 4(x - 1); Vertex: (1, -5); Focus: (2, -5); Directrix: x = 0; Axis: y = -5

- Group y terms: (y^2 + 10y) = 4x - 29.
- Complete the square: (y + 5)^2 - 25 = 4x - 29.
- (y + 5)^2 = 4x - 29 + 25 = 4x - 4 = 4(x - 1).
- Standard form: (y + 5)^2 = 4(x - 1); vertex (1, -5), 4p = 4 so p = 1.
- Focus: (1 + 1, -5) = (2, -5). Directrix: x = 1 - 1 = 0. Axis of symmetry: y = -5.

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