Parabolas in Conic Sections

Pre-Calculus / Conic Sections Worksheet · Grade 10–12

Name: ________________________________ Date: ____________________

Learning Objectives

- Identify the vertex, focus, and directrix of a parabola from its equation or given points
- Write the standard and general equation of a parabola given key information
- Determine the direction of opening of a parabola and solve for unknown parameters

Problems

1. A parabola has its vertex at the origin and opens upward with p = 3. Write the standard equation of the parabola.

$$(x-0)^2 = 4(3)(y-0)$$

2. A parabola opens to the right with vertex at the origin and passes through the point (4, 4). Write the standard equation.

$$y^2 = 4p \cdot x$$

3. Identify the vertex and the value of p for the parabola given in standard form below.

$$(x-2)^2 = 8(y+1)$$

4. A parabola has a minimum point (vertex) at (1, 3) and passes through the point (5, 7). Find the value of p.

$$(x-1)^2 = 4p(y-3)$$

5. Write the standard equation of the parabola with minimum point at (-2, 6) passing through (2, 8). Then find the focus.

$$(x+2)^2 = 8(y-6)$$

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6. A parabola opens downward with vertex at (0, 5) and p = 3. Write the standard equation and find the directrix.

$$x^2 = -12(y-5)$$

7. Convert the standard equation below to general form by expanding and setting equal to zero.

$$(x-3)^2 = 4(y+2)$$

8. A parabola opens to the left with vertex at (4, -1) and passes through (0, 3). Write the standard equation and find the directrix.

$$(y+1)^2 = -4p(x-4)$$

9. Convert the general equation below to standard form, then identify the vertex, direction of opening, focus, and directrix.

$$x^2 - 6x - 8y + 1 = 0$$

10. A parabola has its focus at (2, 5) and directrix y = 1. Find the vertex, the value of p, write the standard equation, and then convert to general form.

$$(x-2)^2 = 8(y-3)$$

Parabolas in Conic Sections — Answer Key

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Answer Key

1. Answer: $x^2 = 12y$

- Use the formula $(x - h)^2 = 4p(y - k)$ for a parabola opening upward.
- Since the vertex is at the origin, $h = 0$ and $k = 0$.
- Substitute $p = 3$: $x^2 = 4(3)y = 12y$.
- Standard equation: $x^2 = 12y$.

2. Answer: $y^2 = 4x$

- Use the formula $y^2 = 4px$ for a parabola opening to the right with vertex at origin.
- Substitute the point (4, 4): $(4)^2 = 4p(4)$.
- $16 = 16p$, so $p = 1$.
- Standard equation: $y^2 = 4(1)x = 4x$.

3. Answer: Vertex: (2, -1); p = 2

- Compare $(x - h)^2 = 4p(y - k)$ with the given equation.
- $h = 2$, $k = -1$, so the vertex is (2, -1).
- $4p = 8$, so $p = 2$.
- The parabola opens upward since $p > 0$.

4. Answer: p = 4

- The minimum point is the vertex, so $h = 1$ and $k = 3$.
- Substitute the point (5, 7): $(5 - 1)^2 = 4p(7 - 3)$.
- $16 = 4p(4) = 16p$.
- Divide both sides by 16: $p = 1$.
- Wait — recalculate: $16 = 16p$ gives $p = 1$. (Answer corrected: $p = 1$.)

5. Answer: Standard: $(x+2)^2 = 8(y-6)$; Focus: (–2, 8)

- Vertex is (-2, 6), so $h = -2$ and $k = 6$.
- Substitute (2, 8): $(2-(-2))^2 = 4p(8-6) \rightarrow 16 = 4p(2) = 8p \rightarrow p = 2$.
- Standard equation: $(x + 2)^2 = 4(2)(y - 6) = 8(y - 6)$.
- Focus is p units above vertex: $(-2, 6 + 2) = (-2, 8)$.

6. Answer: Standard: $x^2 = -12(y-5)$; Directrix: $y = 8$

- For a downward-opening parabola: $(x - h)^2 = -4p(y - k)$.
- $h = 0$, $k = 5$, $p = 3$: $x^2 = -4(3)(y - 5) = -12(y - 5)$.
- Directrix is p units above the vertex (opposite direction of opening): $y = 5 + 3 = 8$.

7. Answer: $x^2 - 6x - 4y + 1 = 0$

- Expand $(x - 3)^2$: $x^2 - 6x + 9$.
- Expand the right side: $4y + 8$.

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- Equation: $x^2 - 6x + 9 = 4y + 8$.
- Rearrange to zero: $x^2 - 6x - 4y + 9 - 8 = 0 \rightarrow x^2 - 6x - 4y + 1 = 0$.

8. Answer: Standard: $(y+1)^2 = -4(x-4)$; Directrix: $x = 5$

- For a left-opening parabola: $(y - k)^2 = -4p(x - h)$.
- Vertex (4, -1): $h = 4$, $k = -1$.
- Substitute (0, 3): $(3 - (-1))^2 = -4p(0 - 4) \rightarrow 16 = 16p \rightarrow p = 1$.
- Standard equation: $(y + 1)^2 = -4(1)(x - 4) = -4(x - 4)$.
- Directrix is p units to the right (opposite of left opening): $x = 4 + 1 = 5$.

9. Answer: Standard: $(x-3)^2 = 8(y+1)$; Vertex: (3,–1); Opens upward; Focus: (3, 1); Directrix: $y = -3$

- Group and complete the square: $x^2 - 6x = 8y - 1$.
- Complete the square on x: $(x - 3)^2 - 9 = 8y - 1$.
- $(x - 3)^2 = 8y - 1 + 9 = 8y + 8 = 8(y + 1)$.
- Standard form: $(x - 3)^2 = 8(y + 1)$. Vertex: (3, -1).
- $4p = 8 \rightarrow p = 2$. Opens upward.
- Focus: (3, -1 + 2) = (3, 1). Directrix: $y = -1 - 2 = -3$.

10. Answer: Vertex: (2, 3); $p = 2$; Standard: $(x-2)^2 = 8(y-3)$; General: $x^2 - 4x - 8y + 28 = 0$

- The vertex lies midway between the focus and directrix: $k = (5 + 1)/2 = 3$, $h = 2$. Vertex: (2, 3).
- p = distance from vertex to focus = $5 - 3 = 2$.
- Parabola opens upward (focus above directrix): $(x - 2)^2 = 4(2)(y - 3) = 8(y - 3)$.
- Expand to general form: $x^2 - 4x + 4 = 8y - 24$.
- Rearrange: $x^2 - 4x - 8y + 4 + 24 = 0 \rightarrow x^2 - 4x - 8y + 28 = 0$.

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