Analyzing Ellipses: Standard Form & Graphing

Precalculus Worksheet · Grade 10–12

Name: _____ Date: _____

Learning Objectives

- Find the center, vertices, and foci of an ellipse from given information
- Derive the standard equation of an ellipse using the values of a, b, and c
- Graph an ellipse by identifying key features including center, vertices, and foci

Problems

1. An ellipse has a major axis of length 10. Find the value of a.

$$2a = 10$$

2. Find the center of an ellipse whose foci are at (1, 3) and (7, 3).

$$\text{Center} = \left(\frac{x_1 + x_2}{2},\ \frac{y_1 + y_2}{2}\right)$$

3. An ellipse has foci at (0, 0) and (6, 0). Find the value of c, which is the distance from the center to each focus.

$$c = \frac{\text{distance between foci}}{2}$$

4. Given a = 5 and c = 3 for an ellipse, find the value of b squared using the relationship between a, b, and c.

$$c^2 = a^2 - b^2$$

5. Write the standard equation of an ellipse with center (2, 1), a squared equal to 9, and b squared equal to 5, where the major axis is horizontal.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Scan to watch

6. An ellipse has foci at (0, 1) and (4, 1) and a major axis of length 6. Find a, c, and b squared.

$$c^2 = a^2 - b^2$$

7. Find the vertices of the ellipse with center (2, 1) and a squared equal to 9, b squared equal to 5, where the major axis is horizontal.

$$\frac{(x-2)^2}{9} + \frac{(y-1)^2}{5} = 1$$

8. Given the foci at (2, 0) and (2, 4) and a major axis of length 8, write the standard equation of the ellipse. The major axis is vertical.

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

9. Identify the center, foci, and vertices of the ellipse given by the equation below.

$$\frac{(x+3)^2}{25} + \frac{(y-4)^2}{9} = 1$$

10. An ellipse has vertices at (–1, 5) and (–1, –3) and foci at (–1, 4) and (–1, –2). Write the standard equation of the ellipse and identify b squared.

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Scan to watch

Analyzing Ellipses: Standard Form & Graphing — Answer Key

Precalculus Worksheet · Grade 10–12

Answer Key

1. Answer: a = 5
- The relationship between the major axis length and a is: 2a = major axis
- Substitute: 2a = 10
- Divide both sides by 2: a = 5

2. Answer: Center = (4, 3)
- The center is the midpoint of the two foci
- x-coordinate of center: (1 + 7) / 2 = 4
- y-coordinate of center: (3 + 3) / 2 = 3
- Center = (4, 3)

3. Answer: c = 3
- Distance between foci = 6 – 0 = 6
- c is half that distance: c = 6 / 2 = 3

4. Answer: $b^2 = 16$
- Use the formula: $c^2 = a^2 – b^2$
- Substitute: $9 = 25 – b^2$
- Solve: $b^2 = 25 – 9 = 16$

5. Answer: $(x–2)^2/9 + (y–1)^2/5 = 1$
- Center (h, k) = (2, 1), $a^2 = 9$, $b^2 = 5$
- Since the major axis is horizontal, a^2 goes under $(x – h)^2$
- Equation: $(x – 2)^2 / 9 + (y – 1)^2 / 5 = 1$

6. Answer: a = 3, c = 2, $b^2 = 5$
- Major axis = 6, so 2a = 6 → a = 3
- Distance between foci = 4, so c = 2
- $b^2 = a^2 – c^2 = 9 – 4 = 5$

7. Answer: Major vertices: (–1, 1) and (5, 1); Minor vertices: (2, 1+√5) and (2, 1–√5)
- $a = \sqrt{9} = 3$, $b = \sqrt{5} \approx 2.24$
- Major vertices (horizontal): (2 – 3, 1) = (–1, 1) and (2 + 3, 1) = (5, 1)
- Minor vertices (vertical): $(2, 1 + \sqrt{5})$ and $(2, 1 – \sqrt{5})$

8. Answer: $(x–2)^2/12 + (y–2)^2/16 = 1$
- Center = midpoint of foci = ((2+2)/2, (0+4)/2) = (2, 2)

Scan to watch

- $2a = 8 \rightarrow a = 4$, so $a^2 = 16$
- c = distance from center to focus = 2, so $c^2 = 4$
- $b^2 = a^2 – c^2 = 16 – 4 = 12$
- Since major axis is vertical, a^2 goes under $(y – k)^2$
- Equation: $(x – 2)^2 / 12 + (y – 2)^2 / 16 = 1$

9. Answer: Center: (–3, 4); Foci: (–3±4, 4) = (1, 4) and (–7, 4); Vertices: (2, 4) and (–8, 4)

- Center (h, k) = (–3, 4)
- $a^2 = 25 \rightarrow a = 5$; $b^2 = 9 \rightarrow b = 3$
- $c^2 = a^2 – b^2 = 25 – 9 = 16 \rightarrow c = 4$
- Foci (horizontal major axis): (–3 + 4, 4) = (1, 4) and (–3 – 4, 4) = (–7, 4)
- Major vertices: (–3 + 5, 4) = (2, 4) and (–3 – 5, 4) = (–8, 4)

10. Answer: $(x+1)^2/7 + (y–1)^2/16 = 1$

- Center = midpoint of vertices = ((–1+(–1))/2, (5+(–3))/2) = (–1, 1)
- a = distance from center to vertex = |5 – 1| = 4, so $a^2 = 16$
- c = distance from center to focus = |4 – 1| = 3, so $c^2 = 9$
- $b^2 = a^2 – c^2 = 16 – 9 = 7$
- Major axis is vertical, so a^2 goes under $(y – k)^2$
- Equation: $(x + 1)^2 / 7 + (y – 1)^2 / 16 = 1$

Scan to watch